A simple model of a balanced boundary layer coupled to a large-scale convective circulation

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Idealised Walker circulation

Figure: Schematics of the flows and balances in the simple model
Components of simple model

1. Maintenance of Weak Temperature Gradient
2. Mass balance
3. Boundary-layer momentum balance
4. Moisture balance
Maintenance of Weak Temperature Gradient

- Convection tries to relax to moist adiabat from the SST, red ($T_s$).
- Equal and opposite relaxation back to WTG, black ($T_w$).
Convective relaxation to WTG

A Bett’s-Miller relaxation formula for mass flux and precipitation.

\[ M_c = \gamma_c \frac{T_s - T_w}{\tau_c} \quad |x| \leq L_c/2, \]
\[ \frac{P}{L\rho_0 H} = \gamma_q \frac{q_s - q_w}{\tau_c} \quad |x| \leq L_c/2, \]

where \( M_c \) is the mass flux divided by density, \( P \) the precipitation flux, \( \tau_c \) the relaxation timescale.

Thermodynamic timescales from convection (\( \tau_{\text{conv}} \)) and the boundary layer (\( \tau_{\text{boun}} \)) are combined as

\[ \frac{1}{\tau_c} = \frac{A_c}{\tau_{\text{conv}}} + \frac{A_b}{\tau_{\text{boun}}}. \]

The relaxation time scale is in the range of 0 to 10 hours.
Mass balance and SST

A constant radiatively-driven subsidence velocity \((w_s)\) gives mass balance

\[ L_x w_s + L_c \langle M_c \rangle = 0 \]

where \(L_x\) domain length and \(L_c\) width of convection. Angle brackets are horizontal average over the convecting region. The SST is defined as

\[ T_s = T_{s0} \exp \left( -\frac{x^2}{L_s^2} \right) + \theta_0, \]

The WTG profile temperature at the edge of the convecting region is

\[ T_w = T_s(x = \pm L_c/2) \]
Sensitivities to SST, subsidence and relaxation timescale

The maximum mass flux and horizontal length scale vary as (small values of $L_c/L_s$):

$$M_{c0} \propto \left( \frac{-w_s L_x}{L_s} \right)^{2/3} \left( \frac{\gamma_c T_{s0}}{\tau_c} \right)^{1/3},$$

$$L_c \propto \left( \frac{-w_s L_x L_s^2 \tau_c}{\gamma_c T_{s0}} \right)^{1/3}$$

- Increasing SST, increases mass flux, decreases horizontal length.
- Increasing subsidence, increases both mass flux and length scale.
- Increasing relaxation timescale, decreases mass flux, increases length scale.
Figure: The sum of subsidence and mass flux for the WTG layer. Shown are profiles for the control ($\tau_c = 2$ h, black) and $\tau_c = 0.4$ h (red). The convective width for $\tau_c = 2$ h is marked by the horizontal arrow.
Contraction of convection width

Figure: The convective width (normalised by width of SST) plotted against convective relaxation timescale. Also shown is a $\tau_c^{1/3}$ power law (dotted).
Coupling to boundary-layer momentum balance

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Coupling to boundary-layer momentum balance

Horizontal thermal gradients are significant within the boundary layer, so we need a momentum balance

\[
\frac{d\phi_b}{dx} = - \frac{u_b}{\tau_b},
\]

where \(u_b\) is boundary-layer wind, \(\phi_b\) geopotential, \(\tau_b\) the Rayleigh boundary-layer timescale. Boundary-layer top vertical velocity (\(w_b\)) is calculated using continuity and hydrostatic balance is given by

\[
w_b = - \frac{du_b}{dx} h, \quad \phi_b = - \frac{h}{2 \theta_0} g (\theta_b - \theta_0)
\]

where \(h\) is the boundary-layer depth. The boundary layer potential temperature matches the ascent in the convection region.

\[
- \frac{\tau_b g h^2}{2 \theta_0} \frac{d^2 \theta_b}{dx^2} = w_b = M_c + w_s.
\]
Vertical profiles of potential temperature for the boundary layer (black) with respect to the WTG moist adiabat based on $T_w$ (red) for the: (left) convective boundary layer or (right) stable boundary layer.
Boundary-layer potential temperature

Figure: The distribution of SST (black) and boundary-layer potential temperature for $\tau_c = 2$ h (black dot), $\tau_c = 0.4$ h (red) and $\tau_c = 10$ h (red dot).
The key moisture balance is

\[
\frac{E_{nc}}{\rho_0 Lh} = \frac{2u_{bc} q_{bc}}{L_x} + \frac{H - h 2u_{uc} q_{uc}}{h L_x},
\]

and evaporation balances precipitation when averaged over the domain

\[E_c + E_{nc} = P_d.\]
Balanced diagnosis in Met Office UM, latitude 1 degrees

Summary

- A simple model coupling the boundary layer to large-scale convection.

- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).

- Model predicts a $\tau_1/c$ variation of convective width, where $\tau_c$ is the relaxation timescale.

- Boundary layer momentum balance limits the minimum convective width.

- The evaporation in the non-convecting region sets the horizontal advection of moisture.

- Principles to test in GCMs:
  - How well is WTG maintained?
  - How close to balance is the vertical velocity in the tropics?
  - Does the convective width decrease with increased efficiency of convection scheme?

- Beare and Cullen (2019), JAS.
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