A simple model of a balanced boundary layer coupled to a large-scale convective circulation

Bob Beare [1] and Mike Cullen [2]

[1]University of Exeter.

[2]Met Office, Exeter.

CPPC, Met Office, July 2019

Idealised Walker circulation

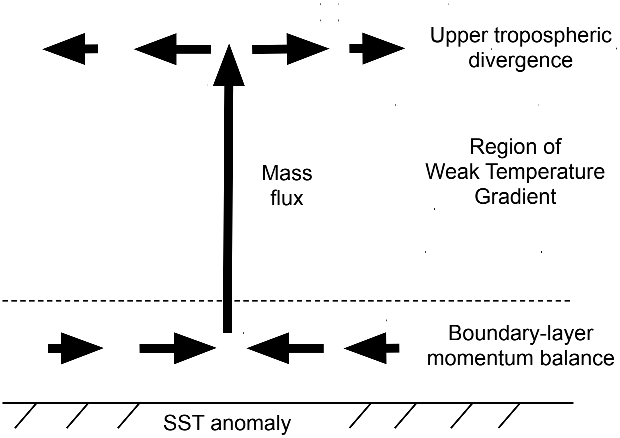
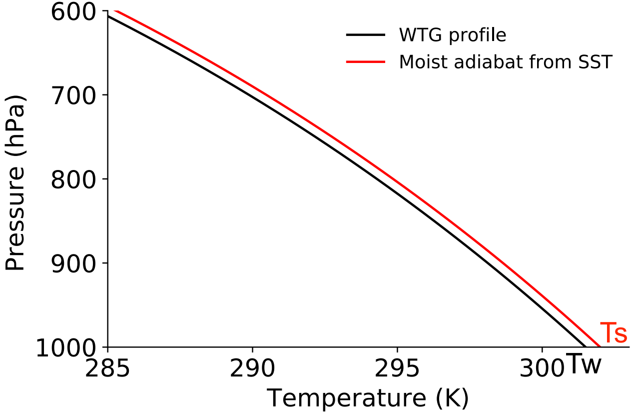


Figure: Schematics of the flows and balances in the simple model

Components of simple model

1. Maintenance of Weak Temperature Gradient
2. Mass balance
3. Boundary-layer momentum balance
4. Moisture balance

Maintenance of Weak Temperature Gradient



- Convection tries to relax to moist adiabat from the SST, red (T_s).
- Equal and opposite relaxation back to WTG, black (T_w).

Convective relaxation to WTG

A Bett's-Miller relaxation formula for mass flux and precipitation.

$$M_c = \gamma_c \frac{T_s - T_w}{\tau_c} \quad |x| \leq L_c/2,$$
$$\frac{P}{L\rho_0 H} = \gamma_q \frac{q_s - q_w}{\tau_c} \quad |x| \leq L_c/2,$$

where M_c is the mass flux divided by density, P the precipitation flux, τ_c the relaxation timescale.
Thermodynamic timescales from convection (τ_{conv}) and the boundary layer (τ_{boun}) are combined as

$$\frac{1}{\tau_c} = \frac{A_c}{\tau_{\mathrm{conv}}} + \frac{A_b}{\tau_{\mathrm{boun}}}.$$

The relaxation time scale is in the range of 0 to 10 hours.

Mass balance and SST

A constant radiatively-driven subsidence velocity (w_s) gives mass balance

$$L_x w_s + L_c \langle M_c \rangle = 0$$

where L_x domain length and L_c width of convection. Angle brackets are horizontal average over the convecting region. The SST is defined as

$$T_s = T_{s0} \exp\left(-\frac{x^2}{L_s^2}\right) + \theta_0,$$

The WTG profile temperature at the edge of the convecting region is

$$T_w = T_s(x = \pm L_c/2)$$

Sensitivities to SST, subsidence and relaxation timescale

The maximum mass flux and horizontal length scale vary as (small values of L_c/L_s):

$$M_{c0} \propto \left(\frac{-w_s L_x}{L_s}\right)^{2/3} \left(\frac{\gamma_c T_{s0}}{\tau_c}\right)^{1/3},$$

$$L_c \propto \left(\frac{-w_s L_x L_s^2 \tau_c}{\gamma_c T_{s0}}\right)^{1/3}$$

- Increasing SST, increases mass flux, decreases horizontal length.
- Increasing subsidence, increases both mass flux and length scale.
- Increasing relaxation timescale, decreases mass flux, increases length scale.

Mass flux

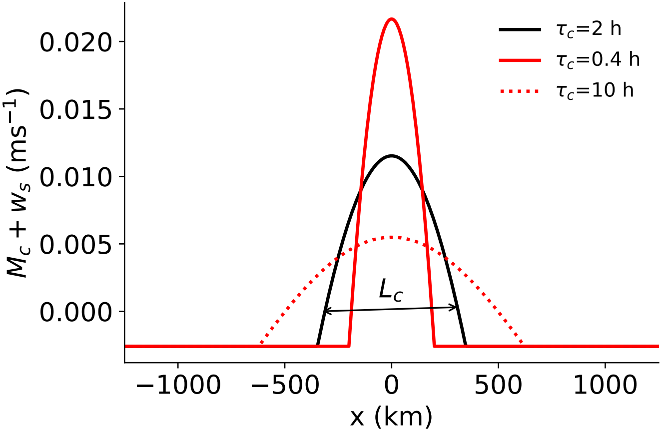


Figure: The sum of subsidence and mass flux for the WTG layer. Shown are profiles for the control ($\tau_c = 2$ h, black) and $\tau_c = 0.4$ h (red). The convective width for $\tau_c = 2$ h is marked by the horizontal arrow.

Contraction of convection width

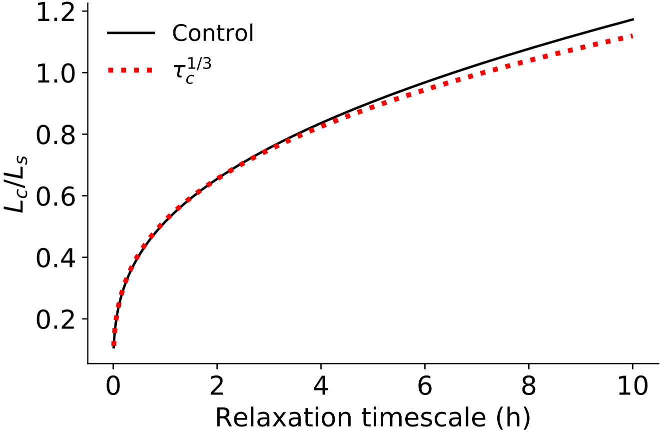


Figure: The convective width (normalised by width of SST) plotted against convective relaxation timescale. Also shown is a $\tau_c^{1/3}$ power law (dotted).

Coupling to boundary-layer momentum balance

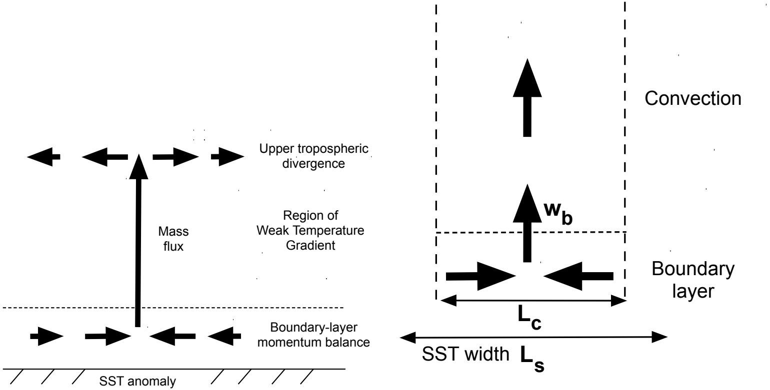


Figure: Schematics of the flows and balances in the simple model

Coupling to boundary-layer momentum balance

Horizontal thermal gradients are significant within the boundary layer, so we need a momentum balance

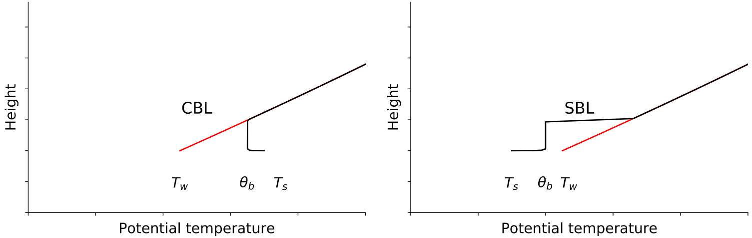
$$\overbrace{\frac{d\phi_b}{dx}}^{\text{Pressure gradient}} = -\overbrace{\frac{u_b}{\tau_b}}^{\text{Drag}},$$

where u_b is boundary-layer wind, ϕ_b geopotential, τ_b the Rayleigh boundary-layer timescale. Boundary-layer top vertical velocity (w_b) is calculated using continuity and hydrostatic balance is given by

$$w_b = -\frac{du_b}{dx}h, \quad \phi_b = -\frac{h}{2}\frac{g(\theta_b - \theta_0)}{\theta_0}$$

where h is the boundary-layer depth. The boundary layer potential temperature matches the ascent in the convection region.

$$-\frac{\tau_b g h^2}{2\theta_0}\frac{d^2\theta_b}{dx^2} = w_b = M_c + w_s.$$



Vertical profiles of potential temperature for the boundary layer (black) with respect to the WTG moist adiabat based on T_w (red) for the: (left) convective boundary layer or (right) stable boundary layer.

Boundary-layer potential temperature

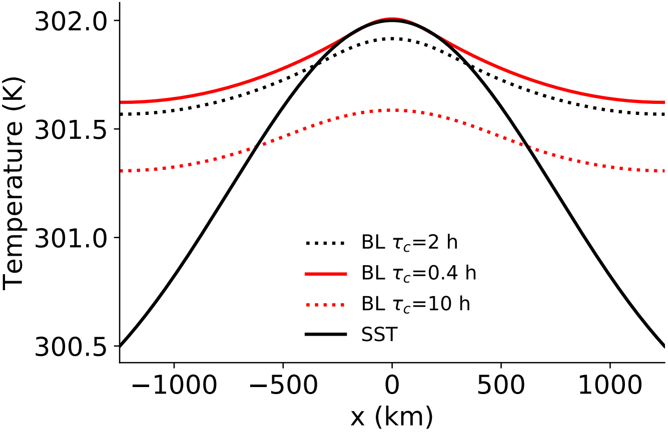


Figure: The distribution of SST (black) and boundary-layer potential temperature for $\tau_c = 2$ h (black dot), $\tau_c = 0.4$ h (red) and $\tau_c = 10$ h (red dot).

Moisture budget

The key moisture balance is

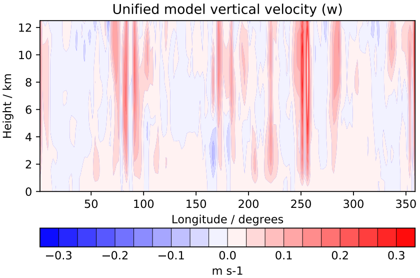
$$\overbrace{\frac{E_{nc}}{\rho_0 L h}}^{\text{Non-convective evaporation}} = \overbrace{\frac{2u_{bc}q_{bc}}{L_x}}^{\text{BL advection}} + \overbrace{\frac{H-h}{h}\frac{2u_{uc}q_{uc}}{L_x}}^{\text{Upper trop. advection}} ,$$

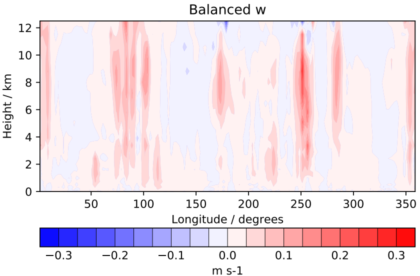
and evaporation balances precipitation when averaged over the domain

$$E_c + E_{nc} = P_d .$$

Balanced diagnosis in Met Office UM, latitude 1 degrees

See Cullen (2018) *Fluids*.





Summary

- A simple model coupling the boundary layer to large-scale convection.

Summary

- A simple model coupling the boundary layer to large-scale convection.
- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).

Summary

- A simple model coupling the boundary layer to large-scale convection.
- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).
- Model predicts a ${\tau_c}^{1/3}$ variation of convective width, where τ_c is the relaxation timescale.

Summary

- A simple model coupling the boundary layer to large-scale convection.
- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).
- Model predicts a $\tau_c^{1/3}$ variation of convective width, where τ_c is the relaxation timescale.
- Boundary layer momentum balance limits the minimum convective width.

Summary

- A simple model coupling the boundary layer to large-scale convection.
- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).
- Model predicts a $\tau_c^{1/3}$ variation of convective width, where τ_c is the relaxation timescale.
- Boundary layer momentum balance limits the minimum convective width.
- The evaporation in the non-convecting region sets the horizontal advection of moisture.

Summary

- A simple model coupling the boundary layer to large-scale convection.
- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).
- Model predicts a $\tau_c^{1/3}$ variation of convective width, where τ_c is the relaxation timescale.
- Boundary layer momentum balance limits the minimum convective width.
- The evaporation in the non-convecting region sets the horizontal advection of moisture.
- Principles to test in GCMs:
  - How well is WTG maintained?
  - How close to balance is the vertical velocity in the tropics?
  - Does the convective width decrease with increased efficiency of convection scheme?

Summary

- A simple model coupling the boundary layer to large-scale convection.
- Model based on relaxation to WTG, in the place of the moist static energy budget, e.g. Emanuel (2019).
- Model predicts a $\tau_c^{1/3}$ variation of convective width, where τ_c is the relaxation timescale.
- Boundary layer momentum balance limits the minimum convective width.
- The evaporation in the non-convecting region sets the horizontal advection of moisture.
- Principles to test in GCMs:
  - How well is WTG maintained?
  - How close to balance is the vertical velocity in the tropics?
  - Does the convective width decrease with increased efficiency of convection scheme?
- Beare and Cullen (2019), *JAS*.