What is the basic structural unit of cumulus convection, and why does it matter?

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Workshop on Convection Parameterization: Progress and Challenges

Exeter, U. K.
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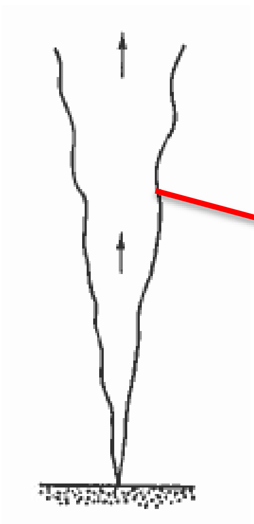
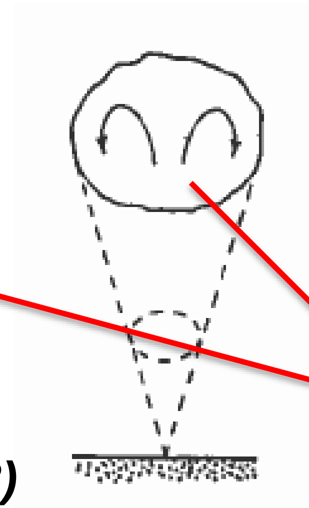
The dynamics of cumulus drafts are *far* from a solved problem!

<u>Entrainment</u> (and detrainment) is a particularly challenging problem that seems depends strongly on the structure of convective updrafts.

Which provides a better framework for understanding cumulus updraft structure?

Steady-state plume? ***Rising thermal?*** ***Something else?***





Turner (1973)

?

Lab studies of dry plumes and thermals suggest factor of ~2 greater fractional entrainment for thermals (Morton et al. 1956; Scorer 1957)

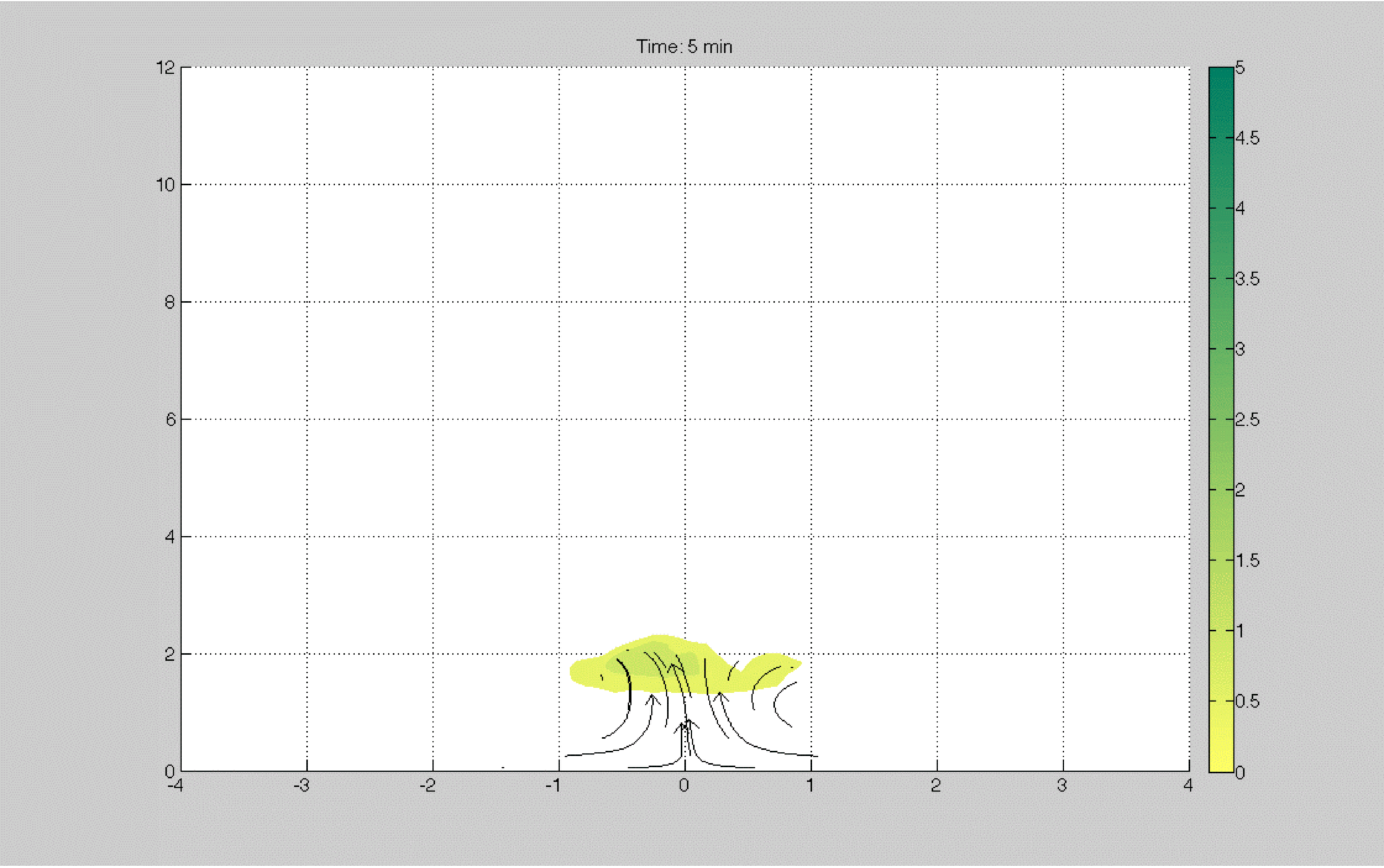
• The earliest convection parameterizations and most current ones are based on the *steady state plume* framework

- <u>straightforward</u> to implement

- unlike the simple R^{-1} scaling of entrainment in the traditional plume model, schemes use a wide variety of methods to formulate entrainment (and detrainment) rates (functions of w, z, RH, etc.)

→ ***there is no simple, agreed upon formulation for representing entrainment in schemes!***

LES of growing cumulus convection – unsheared flow

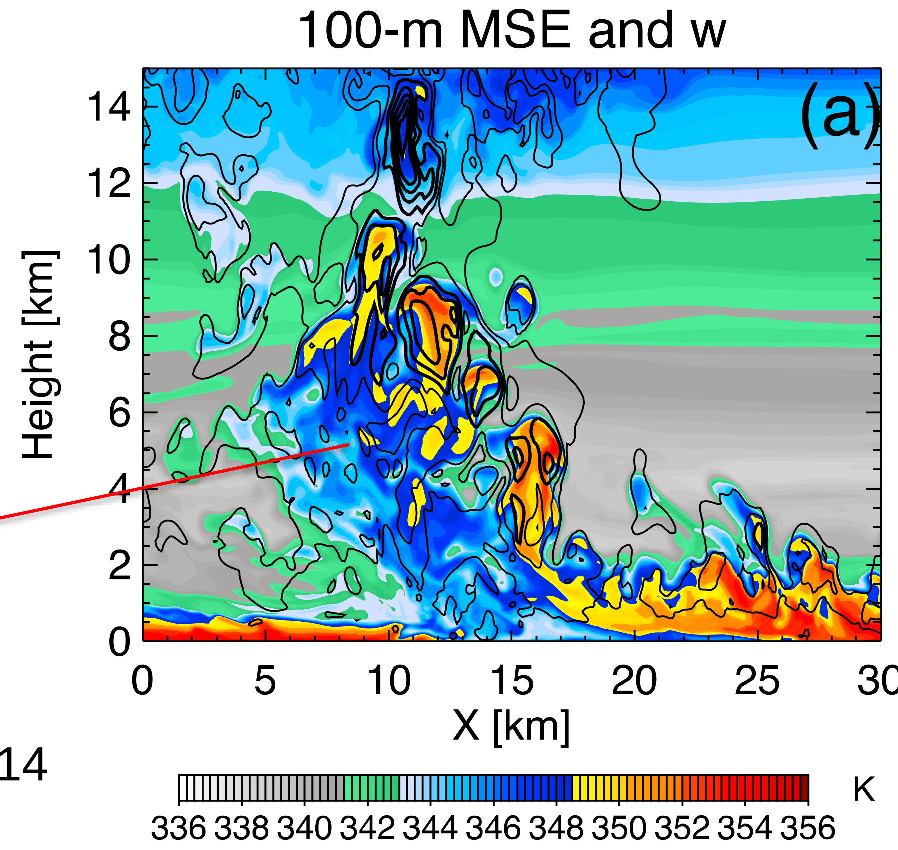


• Observations of moist updrafts comprised of "bubbles" (thermal-like structures) go back at least to Scorer and Ludlam in the 1950's.

• More recent examples: Blyth et al. (1988), Damiani et al. (2006)

• Recent LES ($\Delta x \sim 100$ m) support a thermal-like view, even for deep convection…

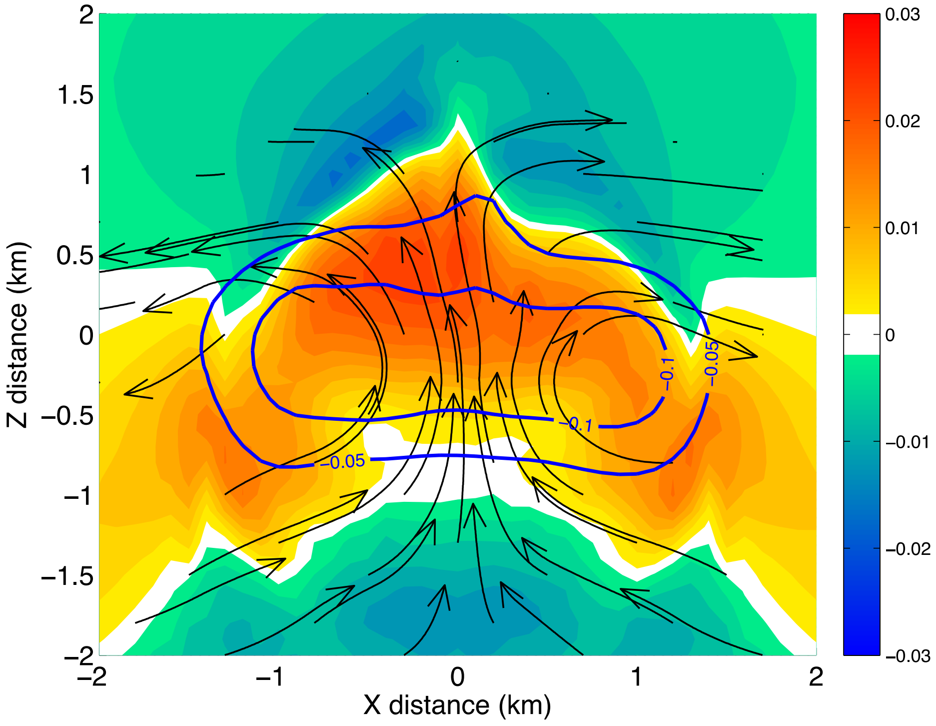
"Thermal chain"



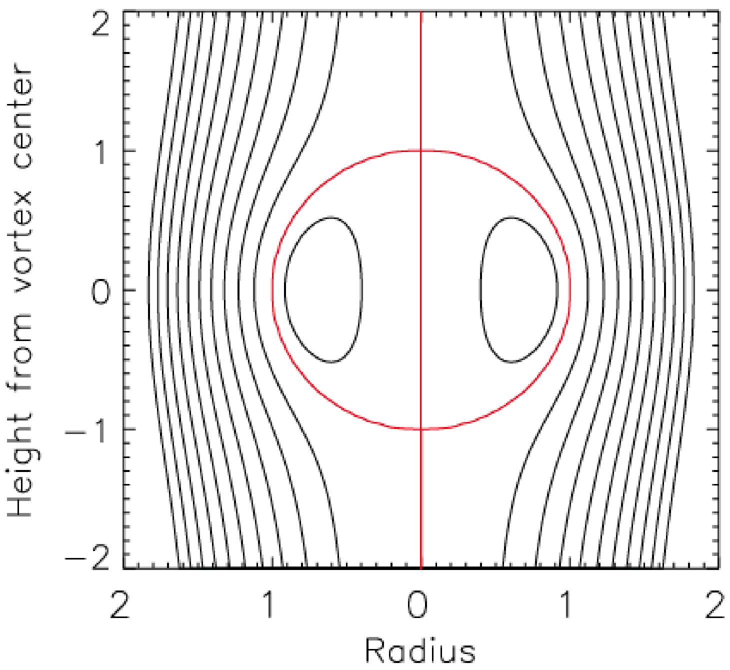
Varble et al. 2014

- The flow within individual thermals in high-resolution LES resembles *Hill's analytic spherical vortex* (Sherwood et al. 2013; Romps and Charn 2015):

CM1 simulations (Δx = 100 m) with no environmental shear



Streamlines; Hill's vortex



Observations and LES suggest moist convection often occurs as a succession of rising thermals, especially congestus/deep → "*thermal chain*"

Neither *thermal* nor *plume* models adequately describe this structure…

1. *Why* does this structure occur, and *what* are the driving mechanisms?
2. What are the entrainment and detrainment behaviors?
3. Are there linkages to the traditional plume or thermal models?

A simple theoretical model (analytic)

- **Approximate solutions to axisymmetric momentum, mass continuity, and cloud thermodynamic equations; environmental shear is neglected for tractability.**

- **Reynolds averaging applied, turbulent fluxes represented by first-order Smagorinsky-type approach.**

- **Use previous approach (*Morrison 2016a,b*) to estimate p_B *forcing.***

- **Condensate loading neglected in buoyancy, and within cloudy updrafts it is assumed that $q_v = q_s$ and sufficient cloud water is always available to retain saturated conditions when evaporation occurs.**

- **Solutions are first obtained for a scalar *C*, buoyancy *B*, and vertical velocity *w* at $r = 0$ and the top/center of the primary ascending thermal, next at the thermal bottom, then at additional heights lower in the updraft.**

Morrison (2017), JAS
Morrison et al. (2019), JAS (submitted)

Expressions for the *top/center* of first thermal (at $r = 0$)

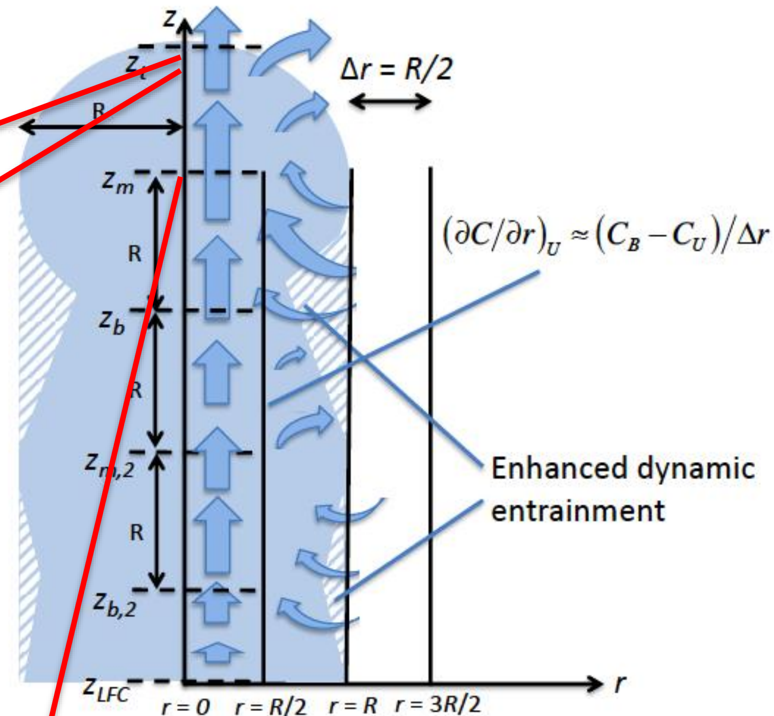
$$\varepsilon = \frac{2k^2 L}{P_r R^2}$$

$$C = \frac{1 - \frac{\varepsilon z_t}{2}}{1 + \frac{\varepsilon z_t}{2}} C_{LFC}$$

$$B = \frac{B_{AD} - \varepsilon \Omega}{1 + \frac{\varepsilon z_t}{2}}$$

with the exception of the facto

ant pressure term. These differe



$$w_m = \sqrt{2\left(1 + \frac{2\alpha^2 R^2}{z_t^2}\right)^{-1}\left(1 + \frac{2}{3\sigma} P_r \varepsilon z_m\right)^{-1} \int_{z_{LFC}}^{z_t} B dz_t^*}$$

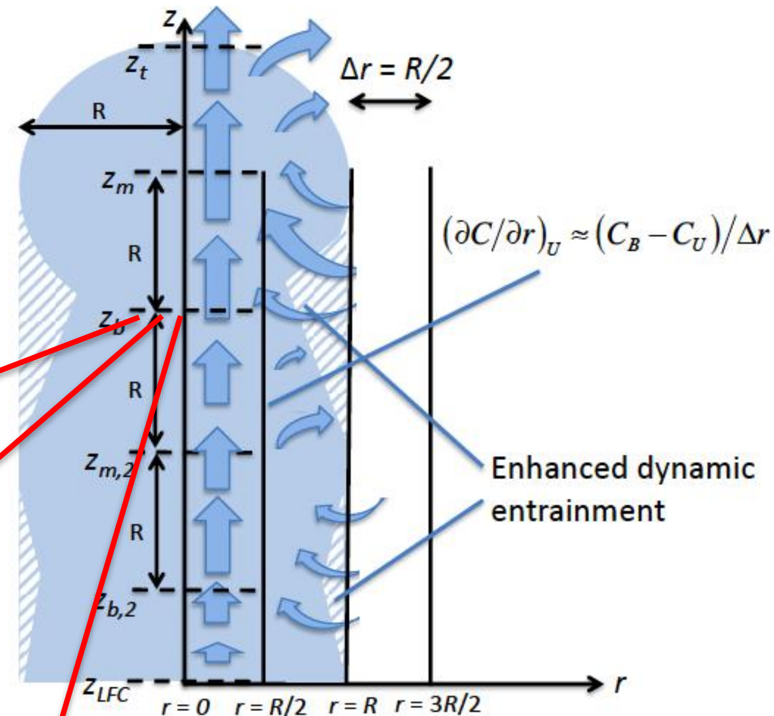
Expressions for the *bottom* of the first thermal (at $r = 0$)
<u>Enhanced mixing</u> from dynamic entrainment!

Mixing enhanced by 9/4

$$\varepsilon_b = 2\frac{\xi_b k^2 L}{P_r R^2} = \frac{9k^2 L}{2P_r R^2}$$

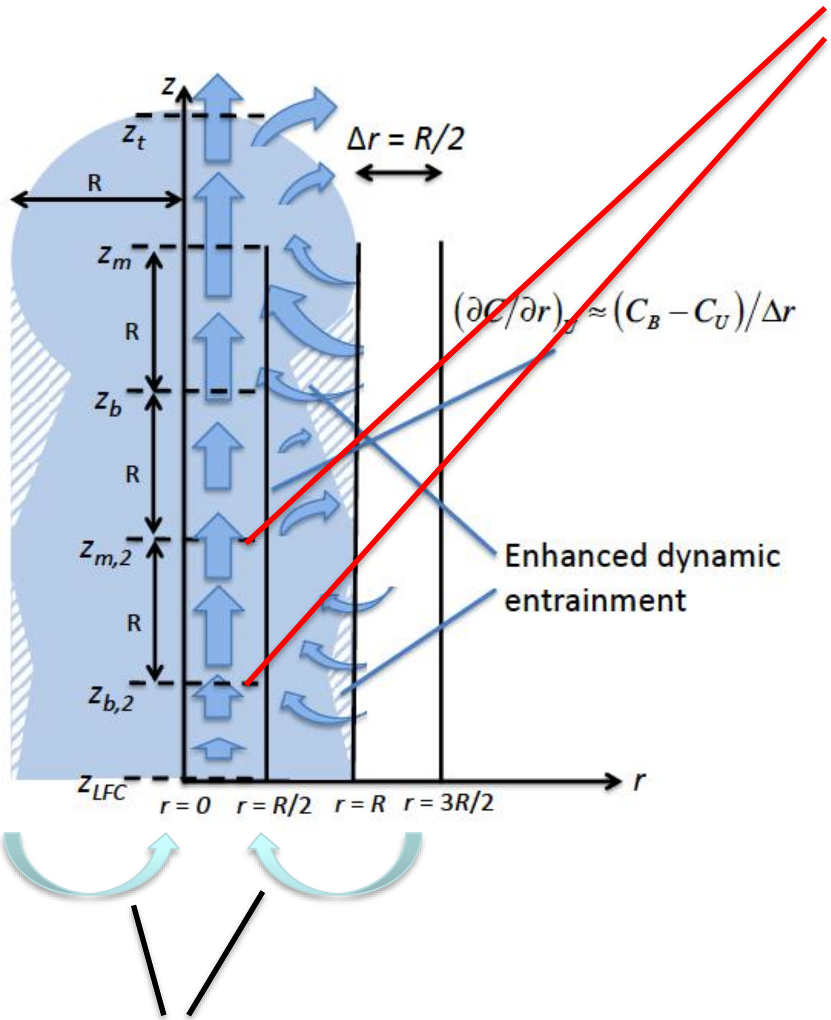
$$C = \frac{1 - \frac{\varepsilon_b z_b}{2}}{1 + \frac{\varepsilon_b z_b}{2}} C_{LFC}$$

$$B = \frac{B_{AD} - \varepsilon\Omega}{1 + \frac{\varepsilon_b z_b}{2}}$$



$$w_b = \sqrt{2\left(1 + \frac{2\alpha^2 R^2}{z_t^2}\right)^{-1}\left(1 + \frac{2}{3}P_r \varepsilon_b z_b\right)^{-1}\int_{z_{LFC}}^{z_b} B\,dz_b^*}$$

And so on at other heights…



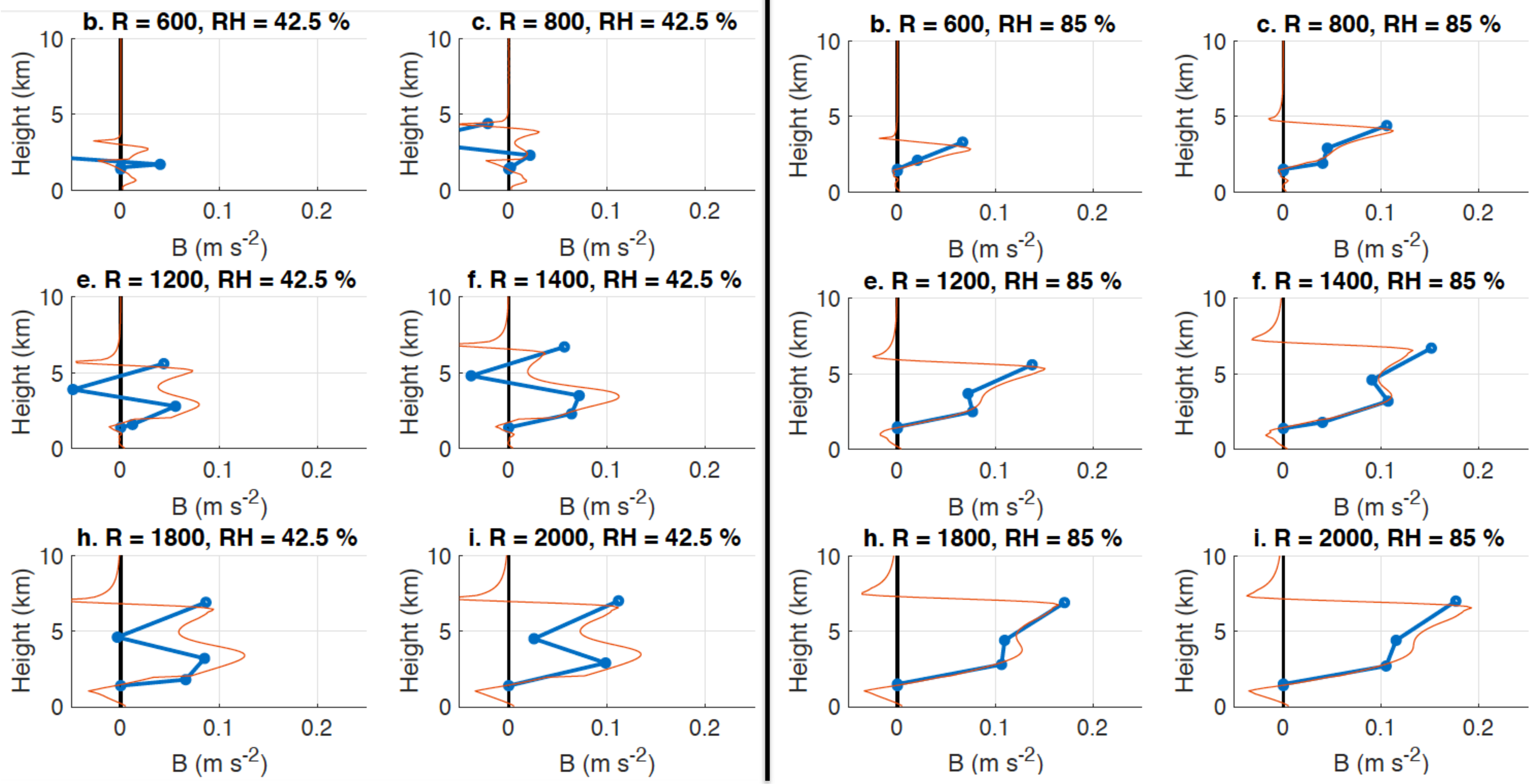
Sustained ascent to LFC by buoyant pressure forcing induced by updraft's B perturbation

Comparison with axisymmetric cloud updraft simulations
NOTE: Not LES

Analytic
Numerical

Buoyancy profiles

RH=0.425 *RH=0.85*



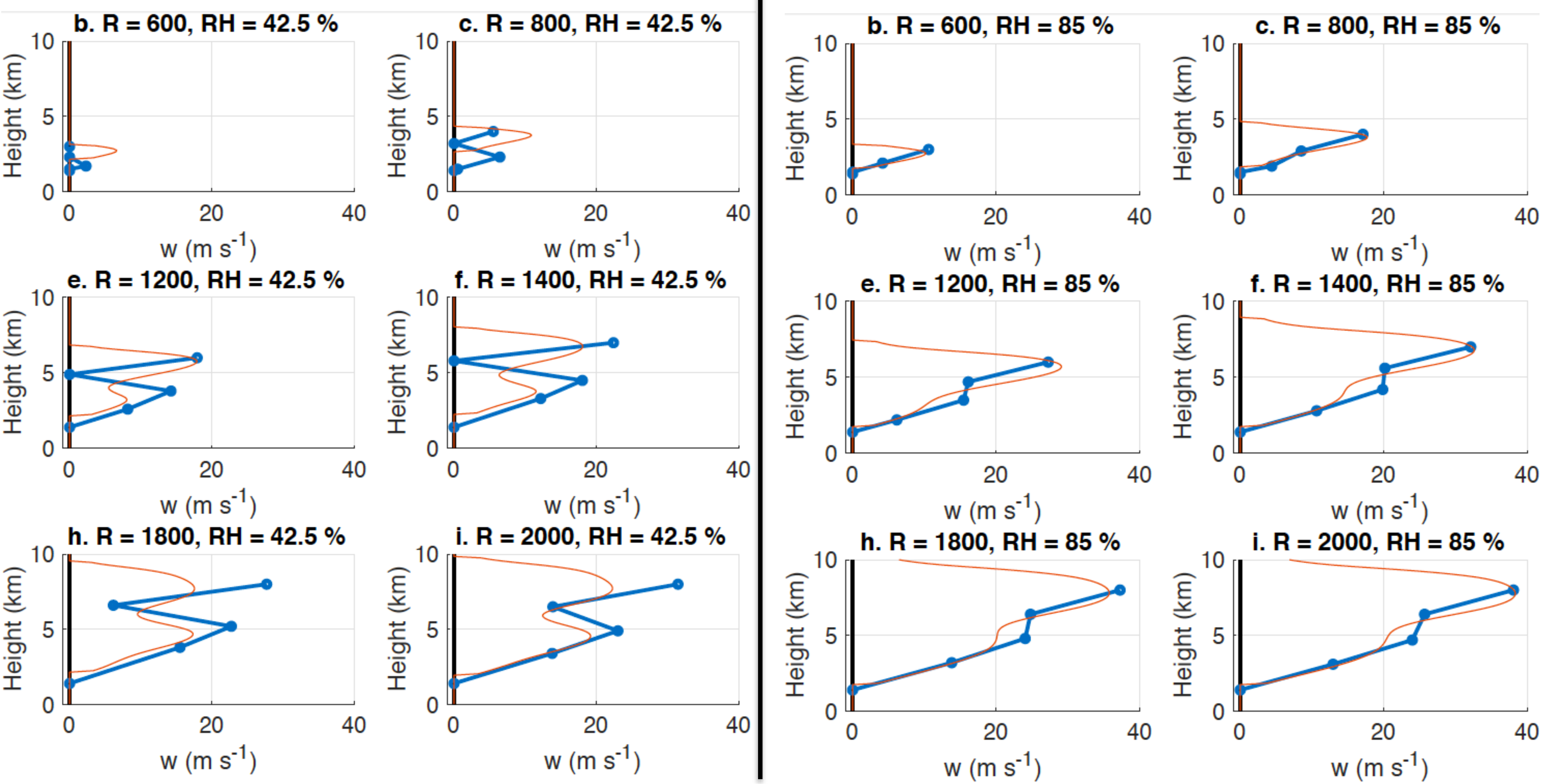
Peters et al. (2019), JAS (submitted)

Comparison with axisymmetric cloud updraft simulations
NOTE: Not LES

Analytic
Numerical

Vertical velocity profiles

RH=0.425 ***RH=0.85***



Peters et al. (2019), JAS (submitted)

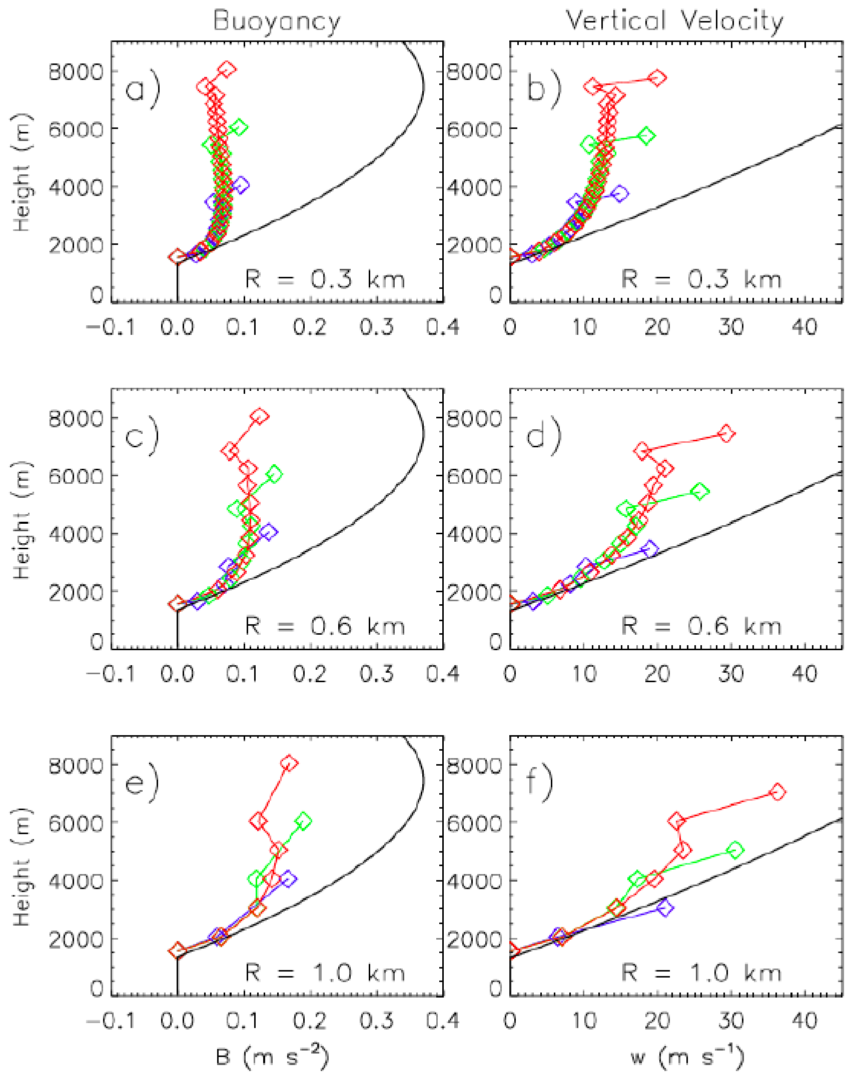
Why does the thermal chain structure occur?

- **Locally enhanced entrainment at the bottom of thermals leads to engulfment of *dry* environmental air.**

- **Reduced condensation and evaporation from this entrainment reduces buoyancy and *w* locally compared to above and below.**

Thermal chains are a unique feature of moist convection owing to interactions between dry air entrainment, cond/evap, buoyancy, and flow structure.

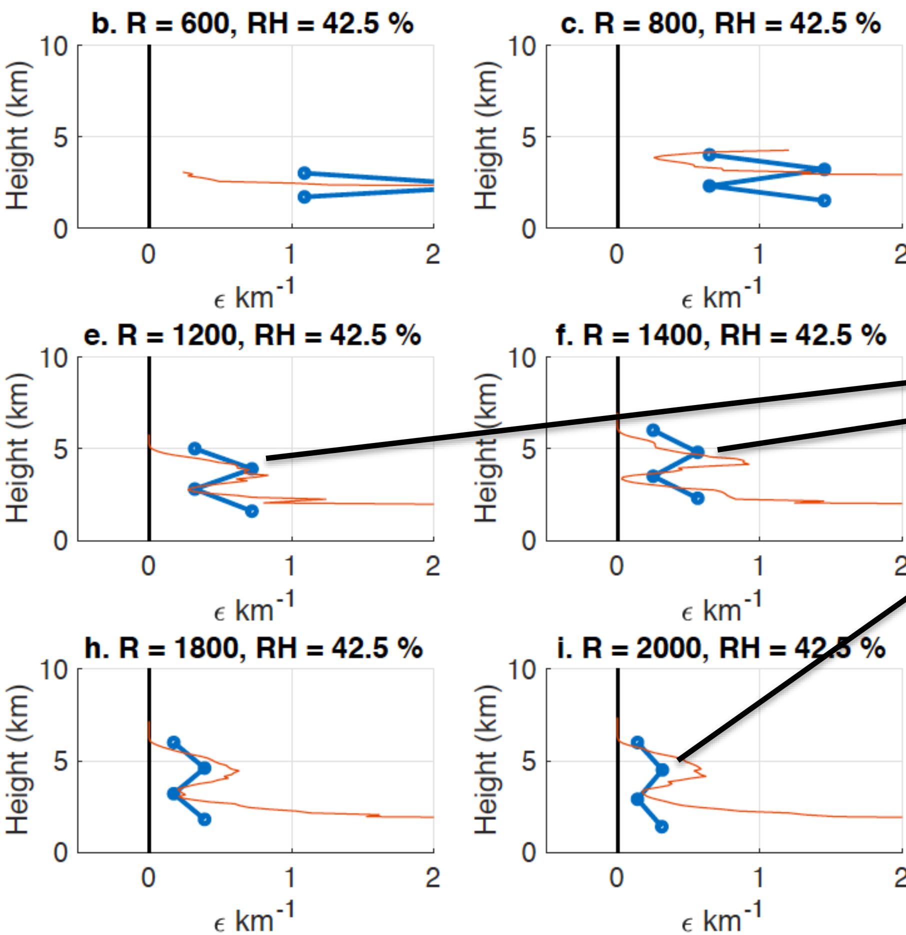
Theoretical expressions with RH set to 1



Comparison with axisymmetric cloud updraft simulations
NOTE: Not LES

Implied fractional entrainment rate vs. "direct" calculation from simulations (Romps 2010 method)

RH=0.425



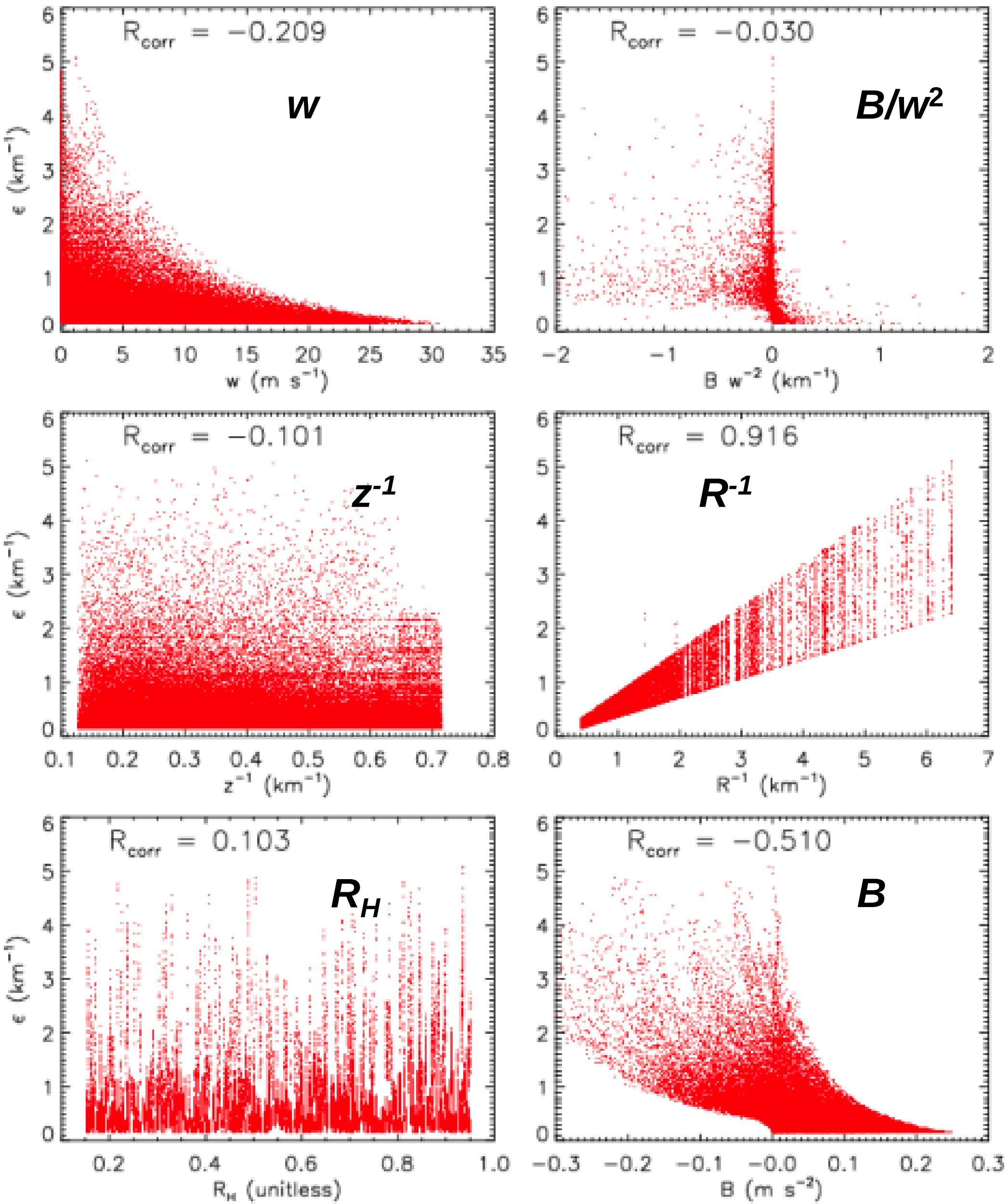
Analytic
Numerical

Pulses of high fractional entrainment rate at the bottom of individual thermals

Complicated behavior of entrainment!

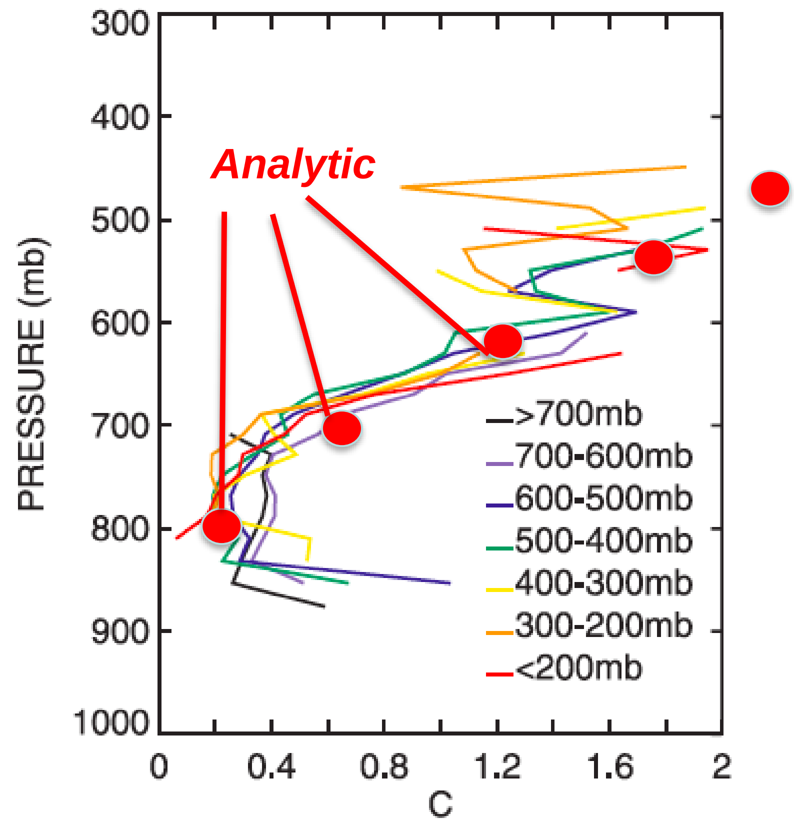
Peters et al. (2019), JAS (submitted)

Correlation of ε and various parameters



ε and Bw^{-2} and moderately correlated when partitioned by height

$C = \varepsilon / (Bw^{-2})$



Del Genio and Wu (2010)

Fine… But these are non-turbulent axisymmetric runs...

What about the behavior of LES???

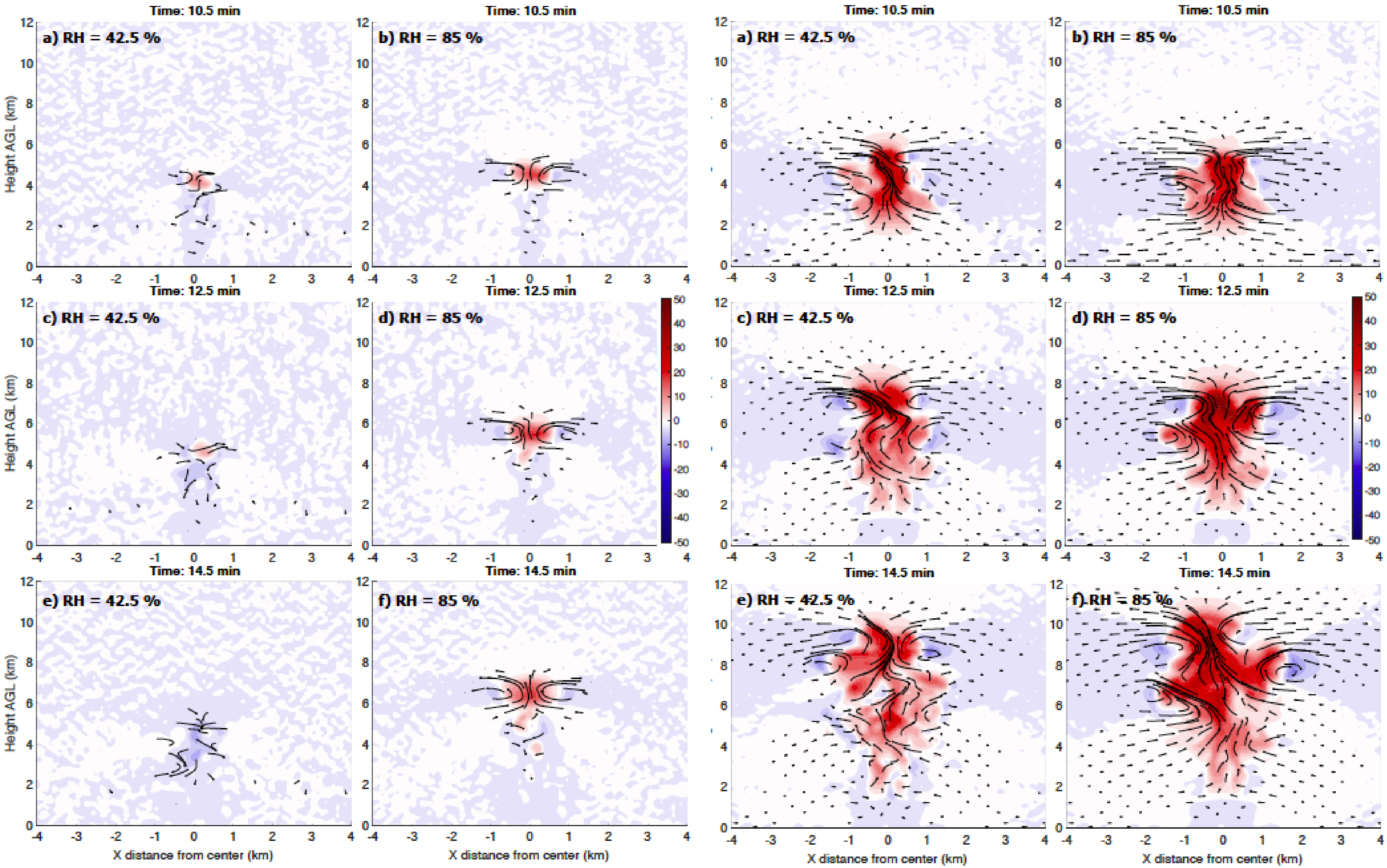
- **“Single cloud” simulations using CM1**
- **Convection initiated by applying warm bubbles of varying sizes; initial noise to theta field rapidly generates turbulent-like flow and -5/3 slope kinetic energy spectra (within ~5 min)**
- **Weisman-Klemp thermodynamic sounding but with RH modified above the level of free convection: 1) moist (constant 85% RH above LFC), 2) dry (constant 42.5% RH above LFC)**

Peters et al. (2019), JAS (submitted)

Vertical cross sections of vertical velocity

Initial bubble = 500 m — Initial bubble = 2000 m

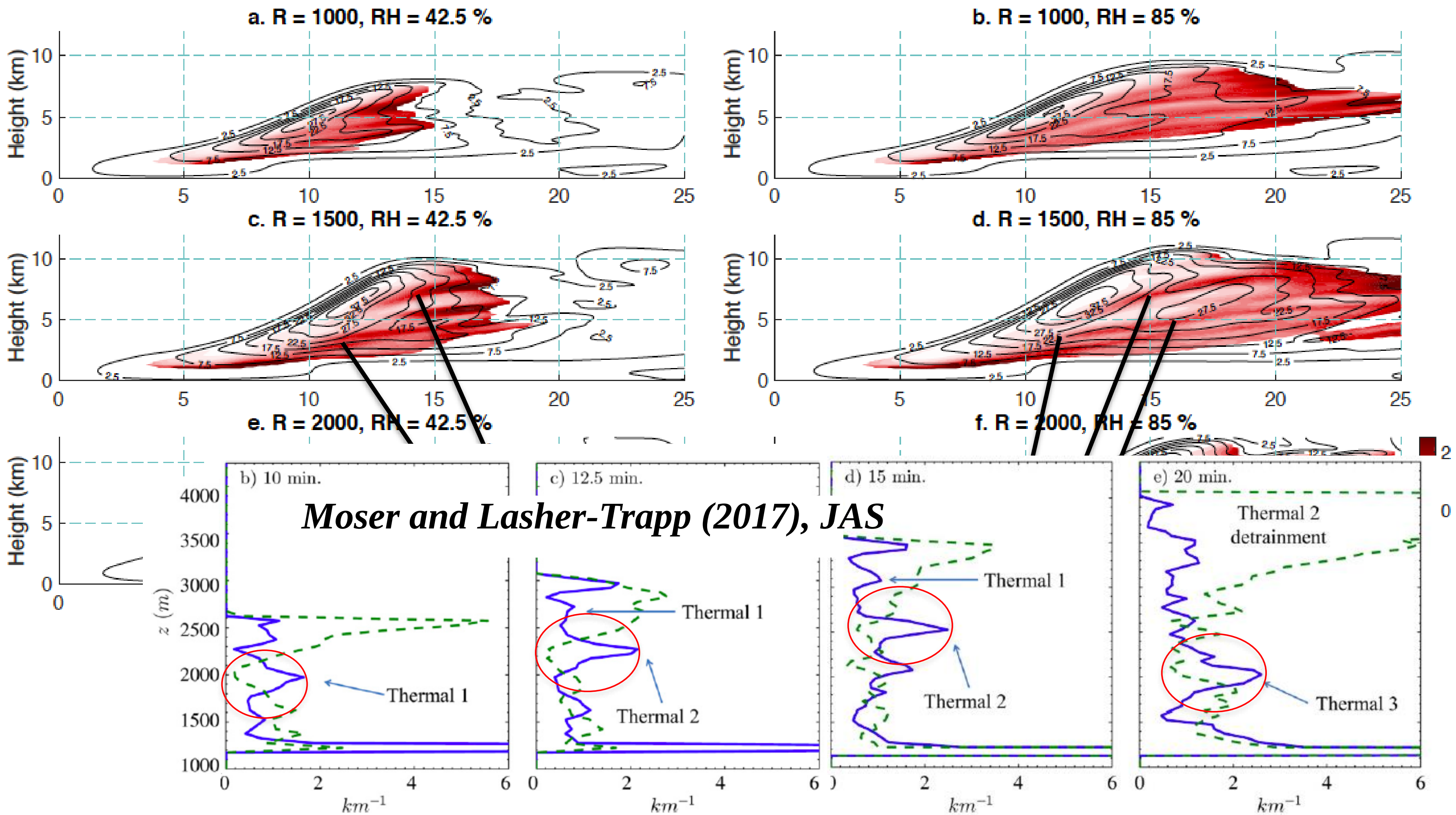
RH=0.425 — RH=0.85 — RH=0.425 — RH=0.85



Time/height plots of fractional entrainment rate

Peters et al. (2019), JAS (submitted)

RH=0.425 | ***"Direct" calculation following Romps (2010) method*** | **RH=0.85**



Moser and Lasher-Trapp (2017), JAS

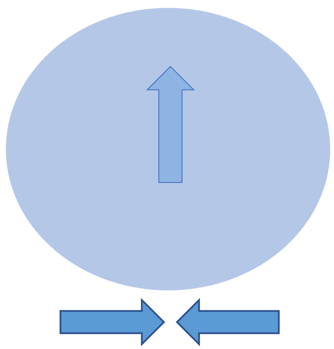
What is the basic structural unit of cumulus updrafts?

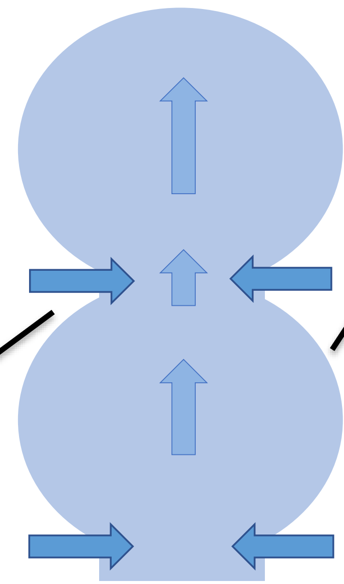
Rising thermal

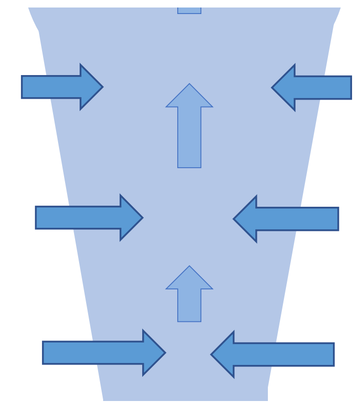
Thermal chain

Plume/Starting plume

Rise of newer thermals into wake of previous ones affects properties of entrained air, micro/dynamics interactions







Pulses of larger entrainment rate: Fractional entrainment rate factor of ~9/4 larger

Increasing $R^2/(zL)$
Increasing RH
Increasing CAPE

Summary

• **Are cumulus updrafts plumes or thermals? Often a bit of both, encapsulated by the *thermal chain* structure (succession of rising thermals), which occurs over a wide range of conditions.**

• **Thermal chains occur in solutions to a simple equation set describing moist updrafts → *intermediary regime in a continuum between the traditional thermal and plume/starting plume* models.**

• **Evidence for *transition* from isolated thermal to thermal chain to starting plume structure with increasing $R^2/(zL)$, increasing environmental *RH*, and increasing *CAPE*.**

Summary, cntd.

• ***Complicated entrainment behavior* → pulses of high fractional entrainment rate occur at the bottom of individual rising thermals in a chain. Perhaps this helps explain the challenge of obtaining simple entrainment rate scalings?**

• **<u>Locally enhanced entrainment</u> contributes significantly to overall *cloud dilution*, while local cloud regions can remain relatively *undilute*.**

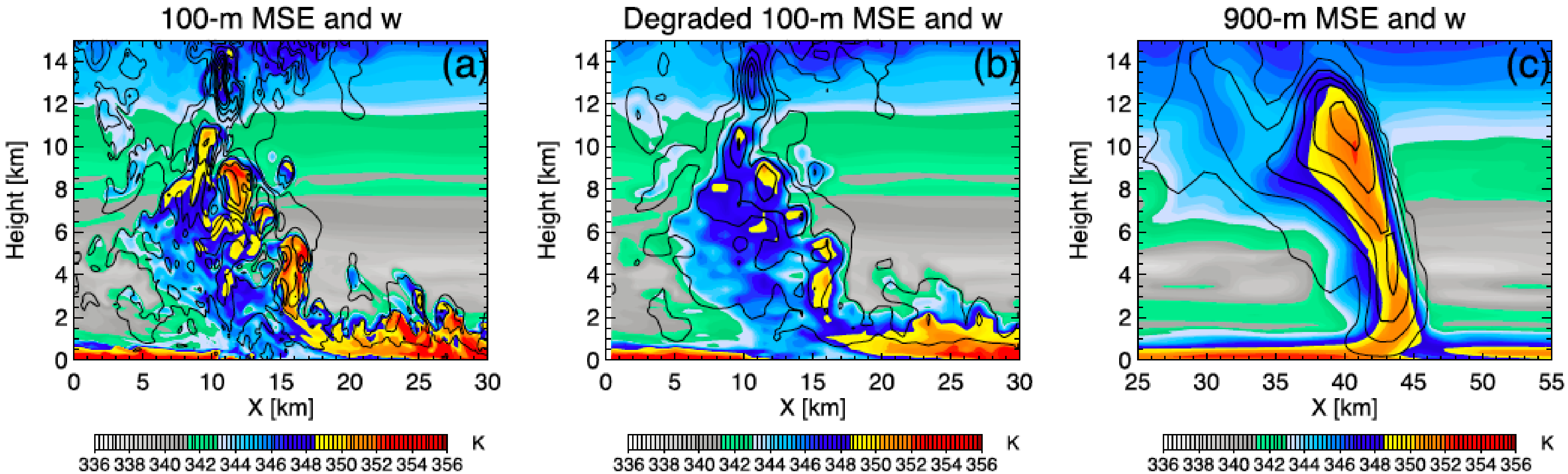
• **Theory and LES indicate local enhancement in entrainment rate of factor ~2 consistent with difference of fractional entrainment between dry thermals and plumes from lab studies (Morton et al. 1956; Scorer 1957).**

- **Probably not a coincidence, though mechanisms of entrainment appear to be quite different for *moist* convection than *dry*.**

Implications for modeling

- **Is this work relevant to convection schemes?**
  - I think so…?
- **How can these ideas be implemented into schemes?**
  - I have no idea…

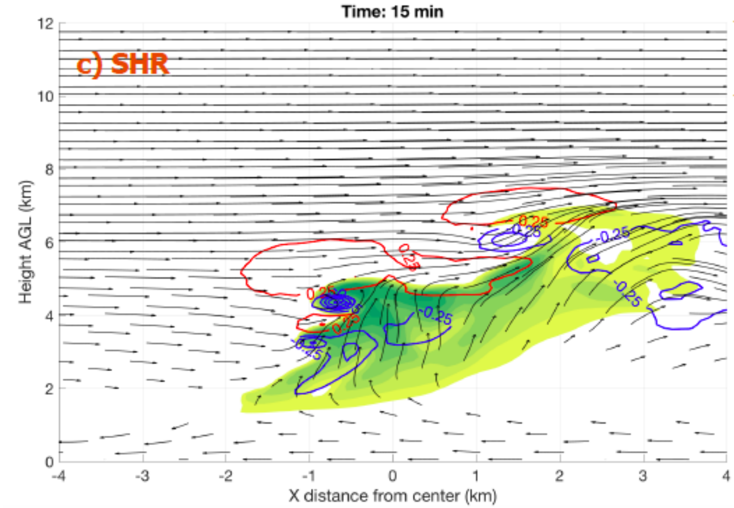
- **Because models in the "gray zone" under-resolve updrafts and hence over-predict $R^2/(ZL)$, we might expect them to produce *plume-like updrafts* contributing to *under-dilution*.**



Varble et al. (2014)

Current and future work

- **Role of environmental shear (Peters et al. 2019 JAS)**



- **Detrainment and downdrafts**

- **Observations of dynamics (Doppler or radar wind profiler)**

- **Large-domain, multi-cloud LES with "natural" convective initiation**

- **Detailed analysis (theory + LES) of interactions between thermal flow structure, ascent and volume growth rates, and entrainment/detrainment → *why is R nearly constant for moist thermals but increases sharply for dry ones?***

- **What controls *R*? (lots of relevant length scales…)**

Thank you!

Questions?

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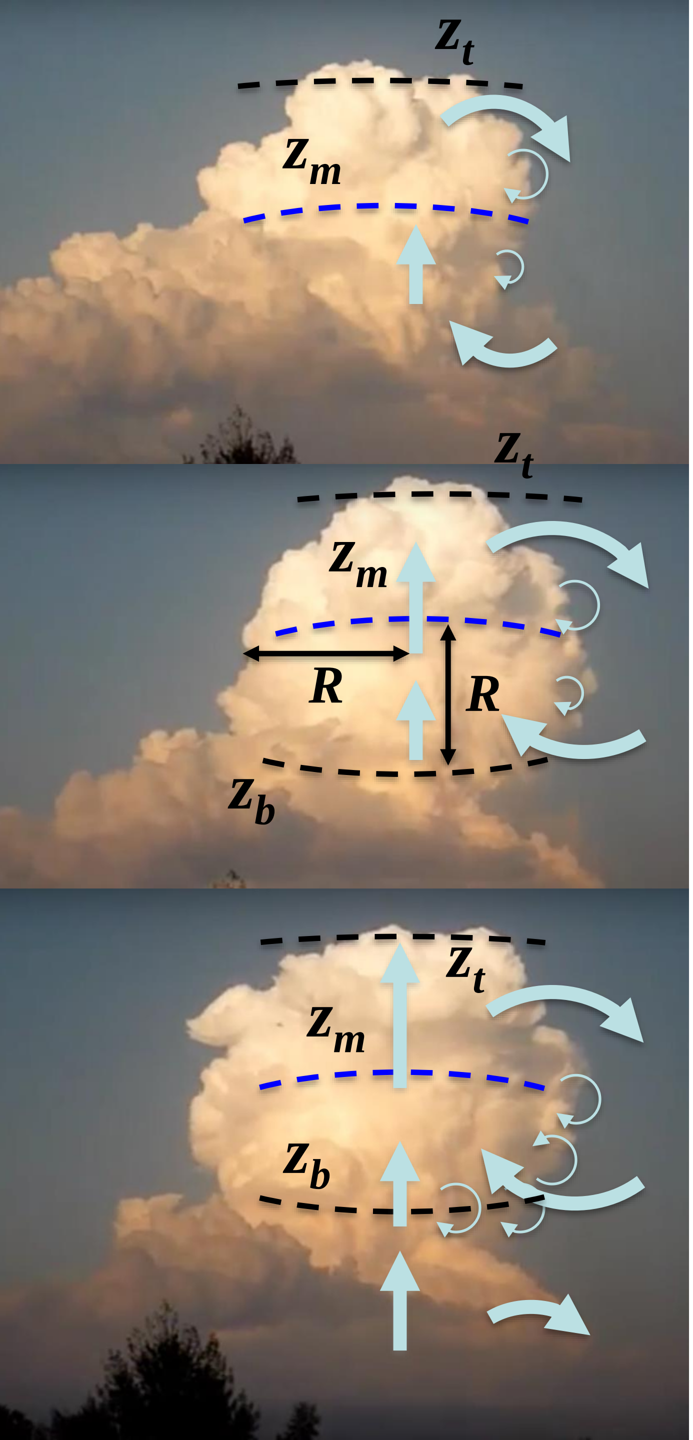
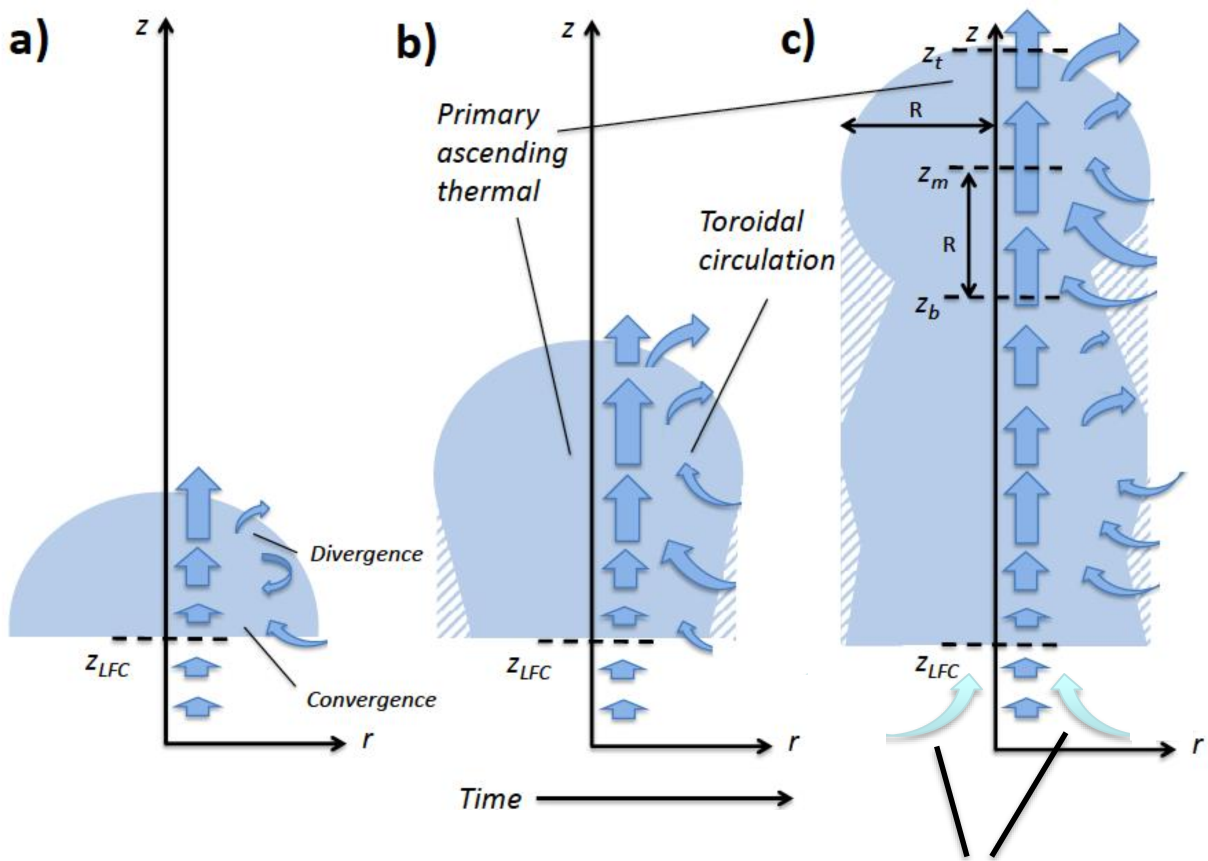


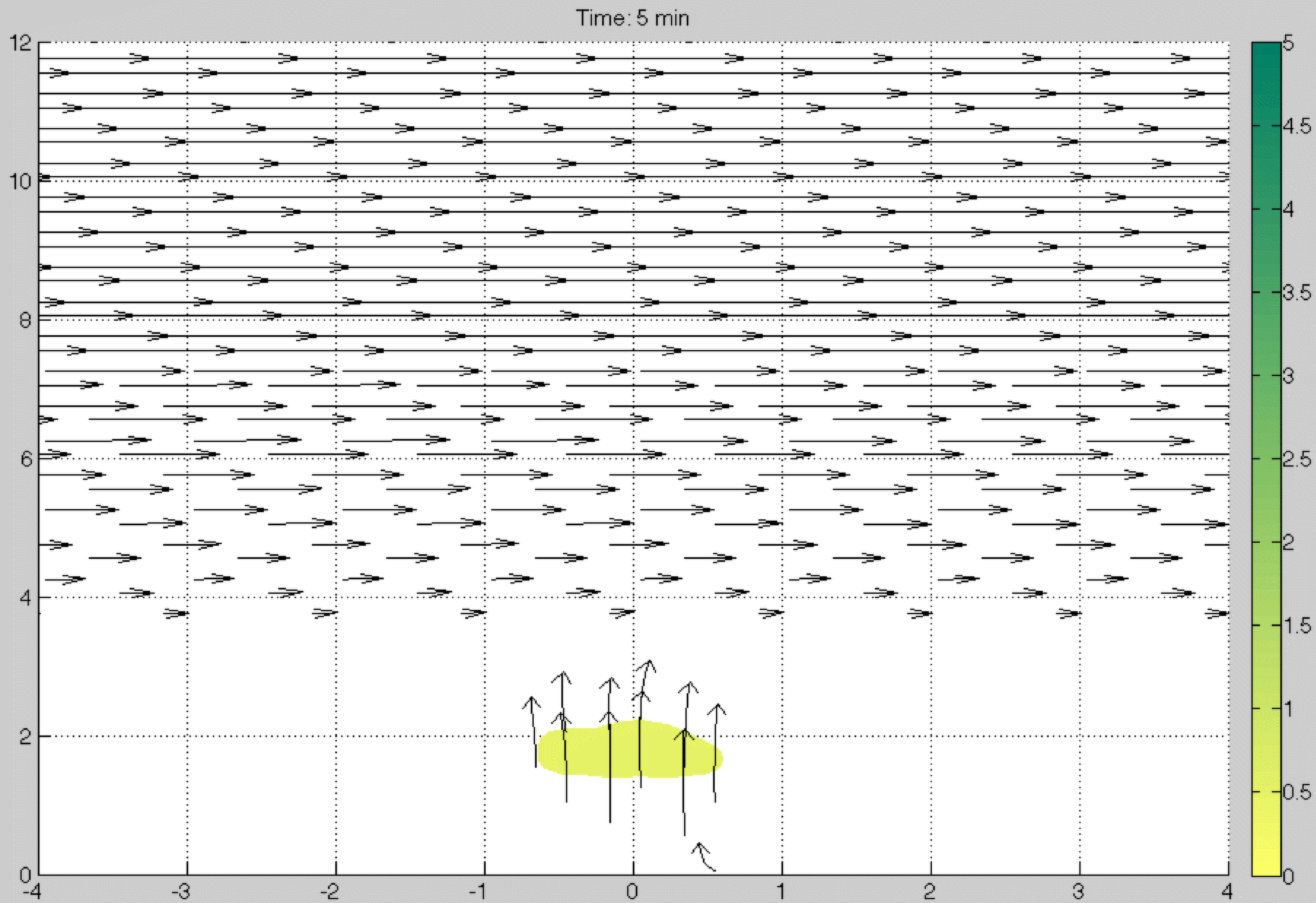
Conceptual model for “thermal chains”

Morrison et al. (2019), JAS (submitted)

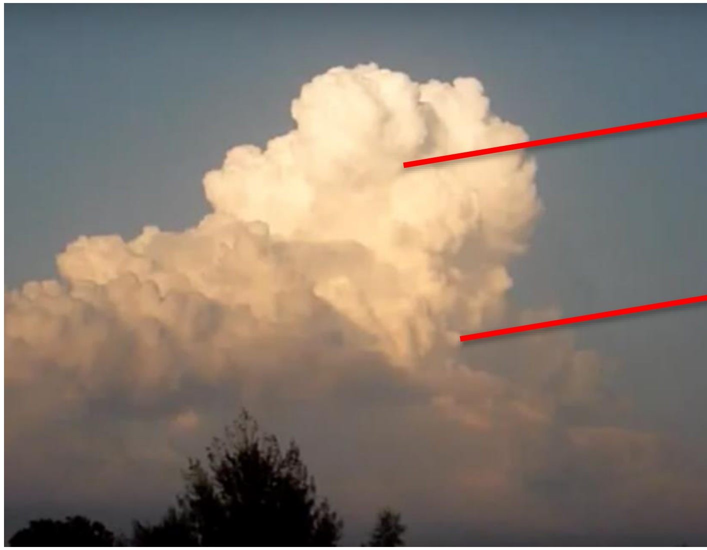


Sustained ascent to LFC by buoyant pressure forcing induced by updraft’s B perturbation

LES of growing cumulus convection – sheared flow



• Two critical features for *moist* convection not accounted for by these conceptual models:



Increase of buoyancy from condensation and latent heating aloft

Decrease of buoyancy from entrainment and evaporation

• Modification to account for these challenges, e.g.,

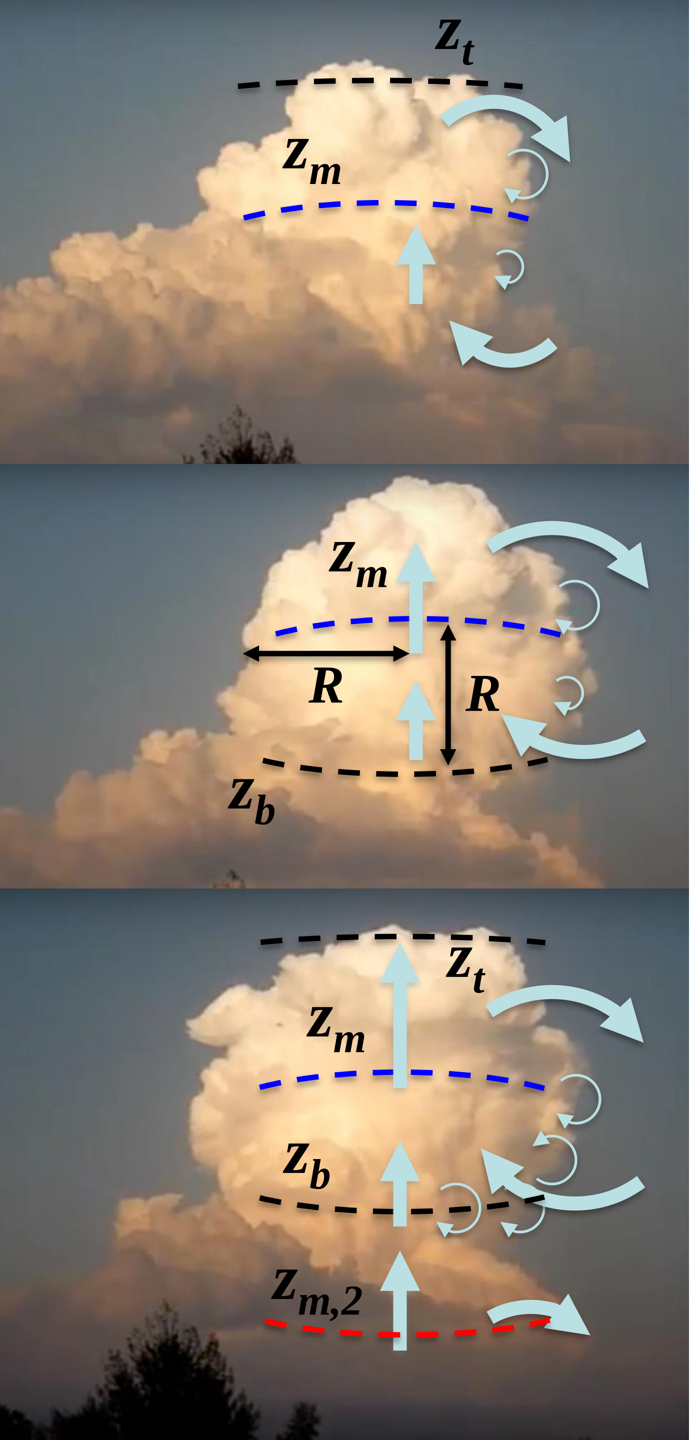
- separation of <u>*dynamic*</u> and smaller-scale <u>*turbulent*</u> entrainment
- buoyancy sorting

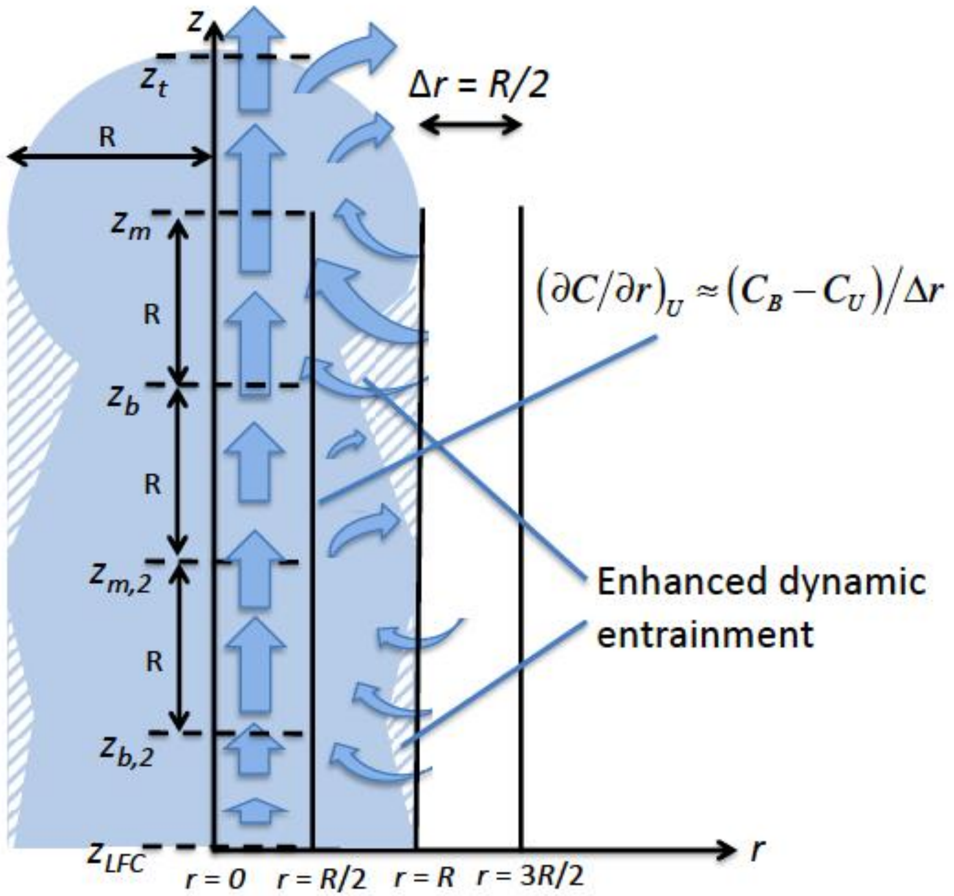
Quote from Koenig
(Koenig, 1963, JAS)

"Commonly, these clouds were found to have life-spans of the order of one hour and a pulsating growth habit similar to that described in Scorer and Ludlam's (1953) bubble theory of convection. Each bubble or turret comprising the uppermost portion of the cloud was visible generally for 5 to 10 min: initially, as an active, hard appearing, ascending cloud mass; later, as a dissipating fibrous cloud mass whose place at the cloud summit was soon to be lost to a younger, active bubble."

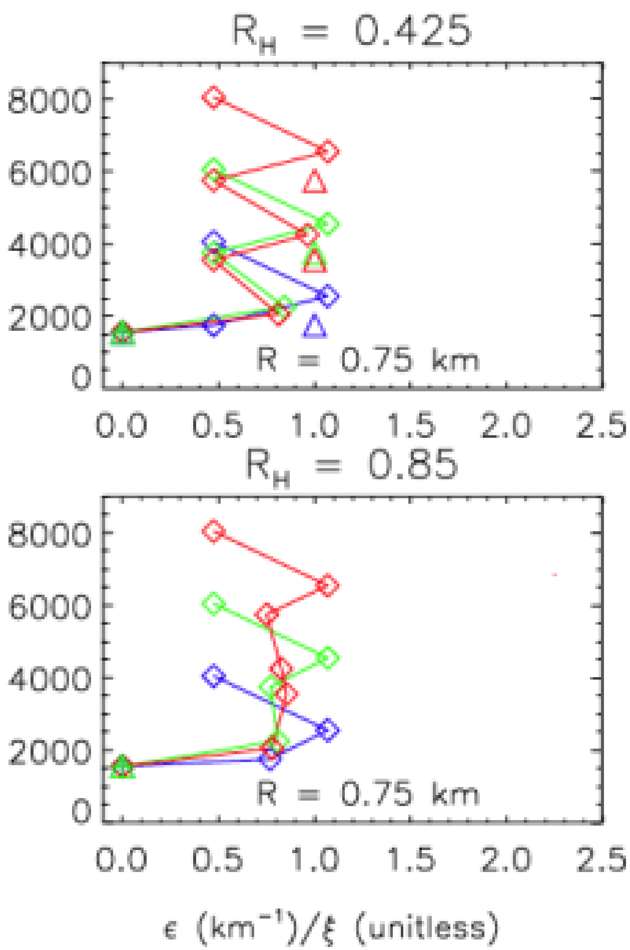
Schematic of the theoretical analytic model

Morrison (2017), JAS,
Morrison et al. (2019), JAS (submitted)





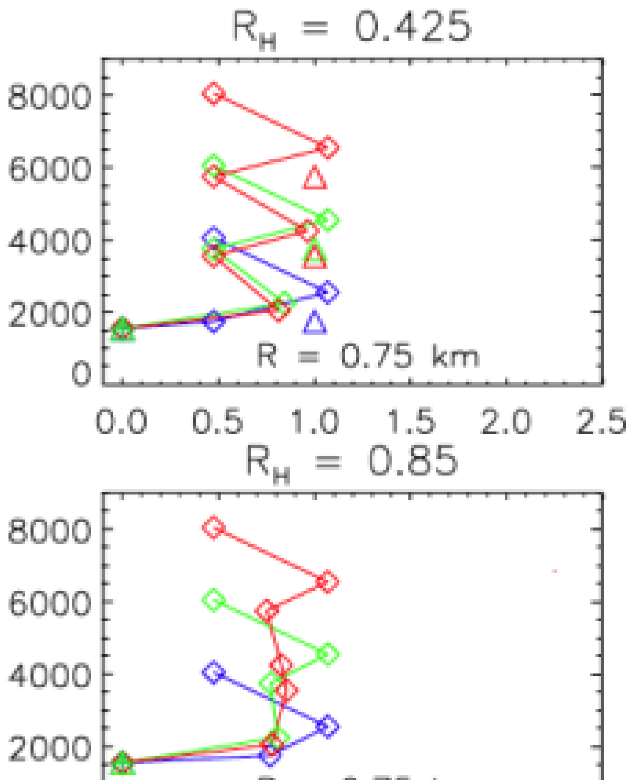
Implied fractional entrainment rate ε



Profiles of analytic ε for various R and environmental R_H.

A complicated structure of ε, even in this simple analytic model!

Implied fractional entrainment rate ε



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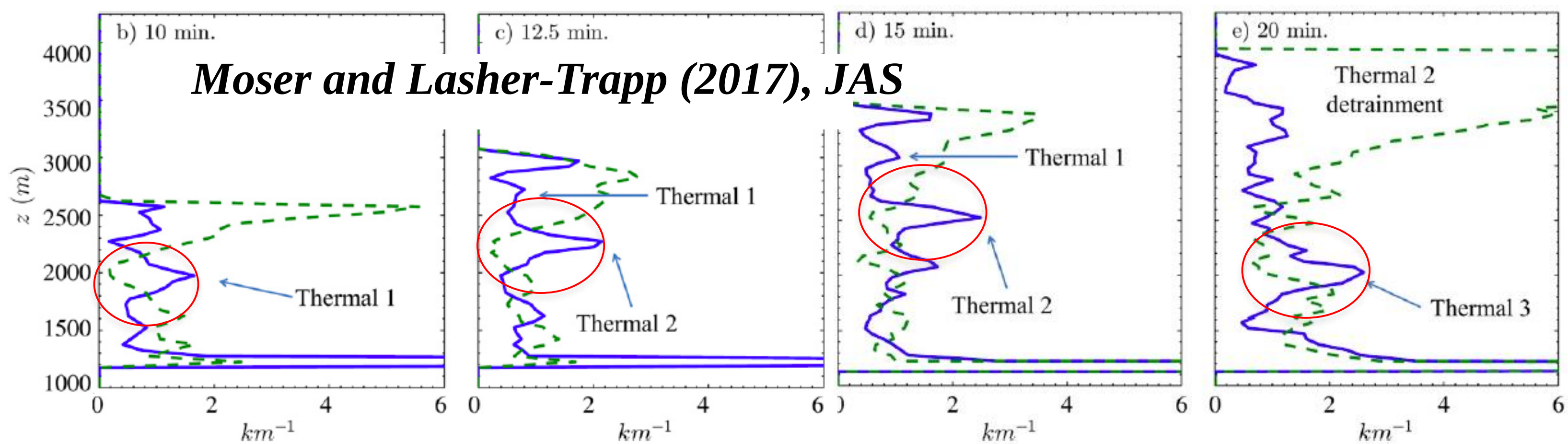

Moser and Lasher-Trapp (2017), JAS

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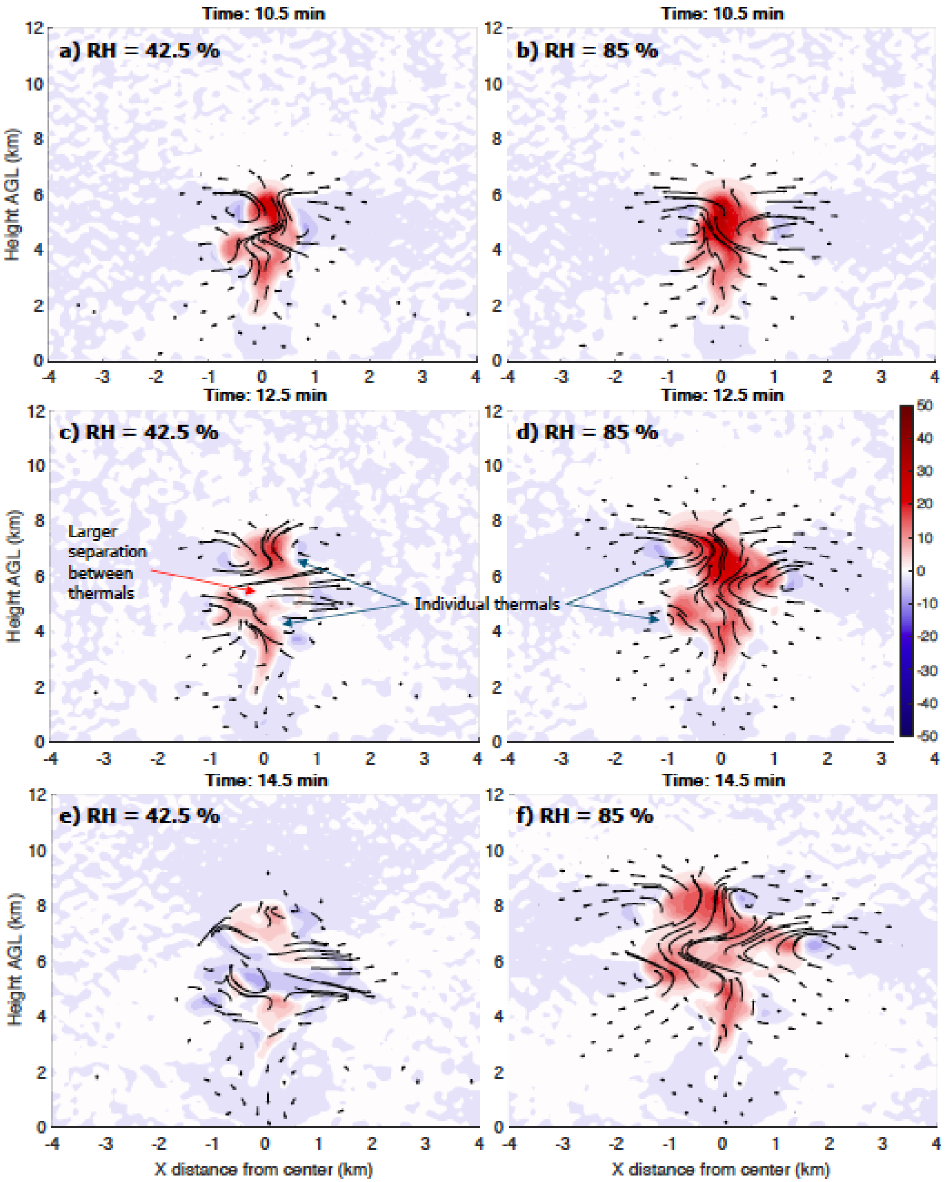
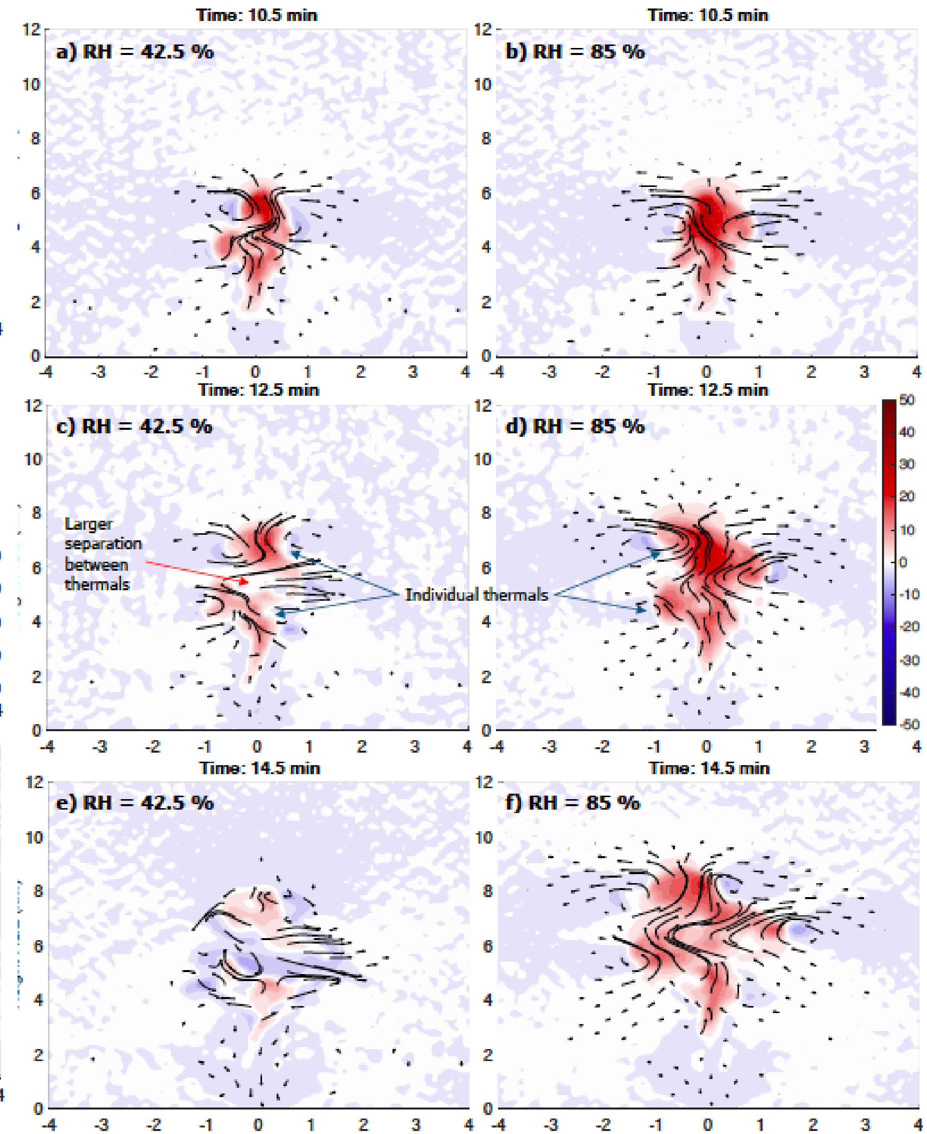
Peters et al. (2019), JAS (submitted)

Initial bubble = 1000 m

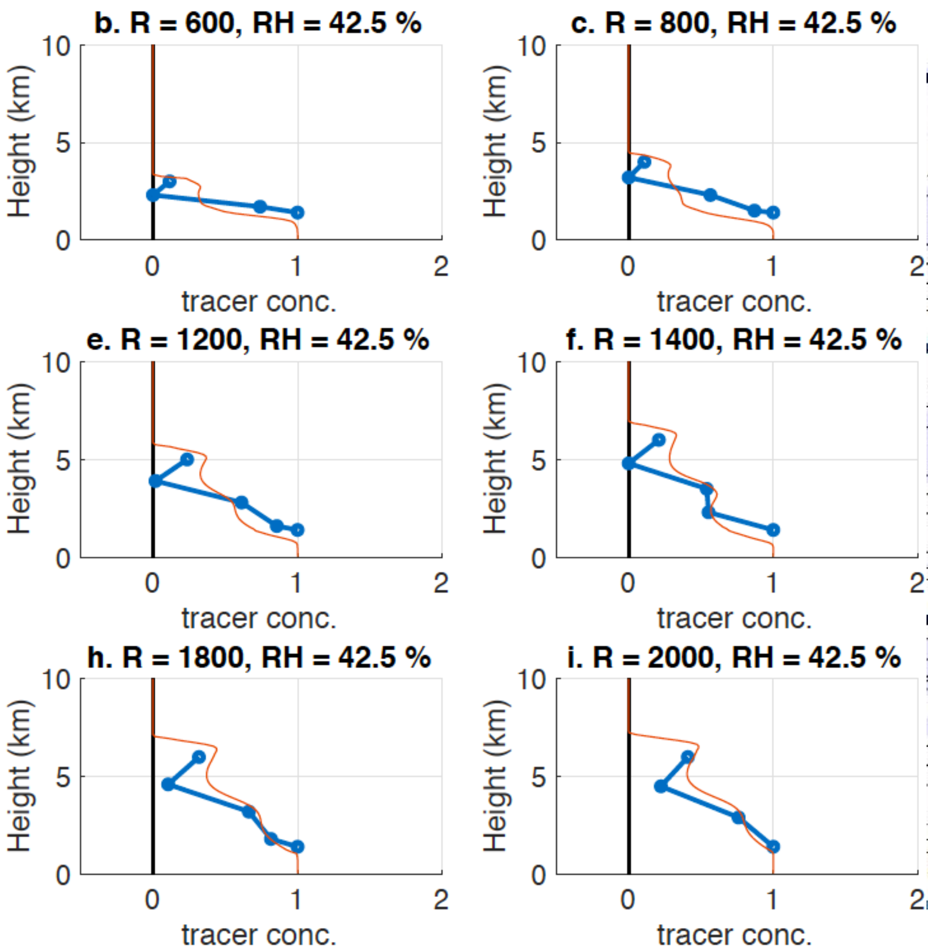


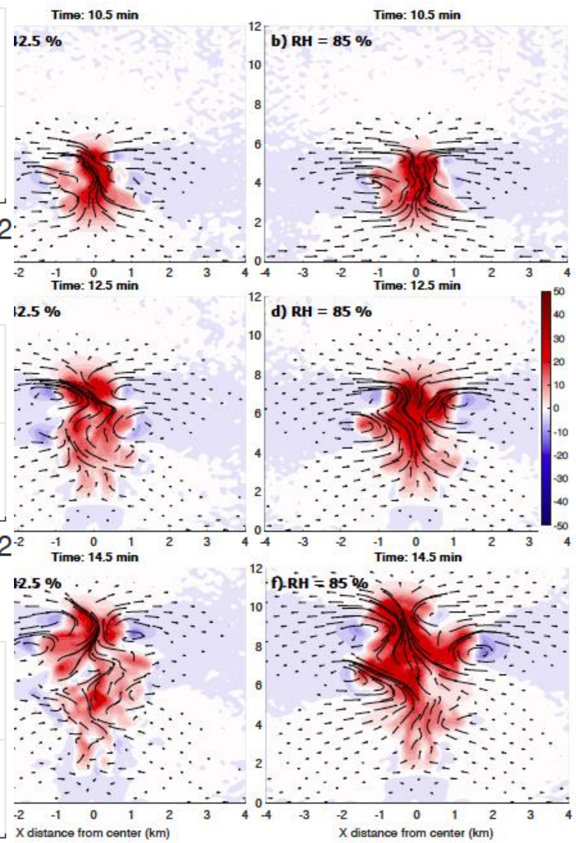
Initial bubble = 1500 m



Peters et al. (2019), JAS (submitted)

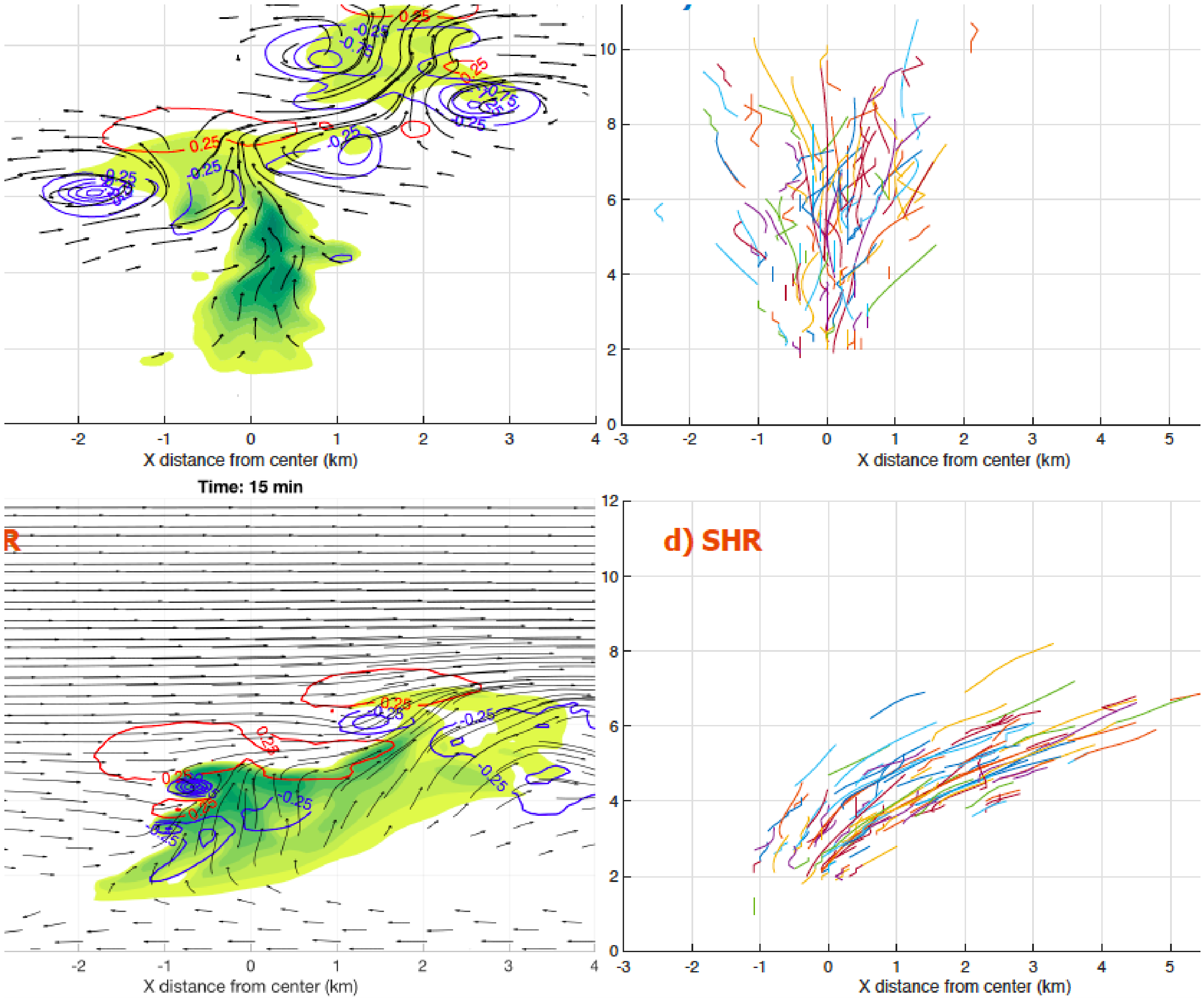
***Tracer** profiles*

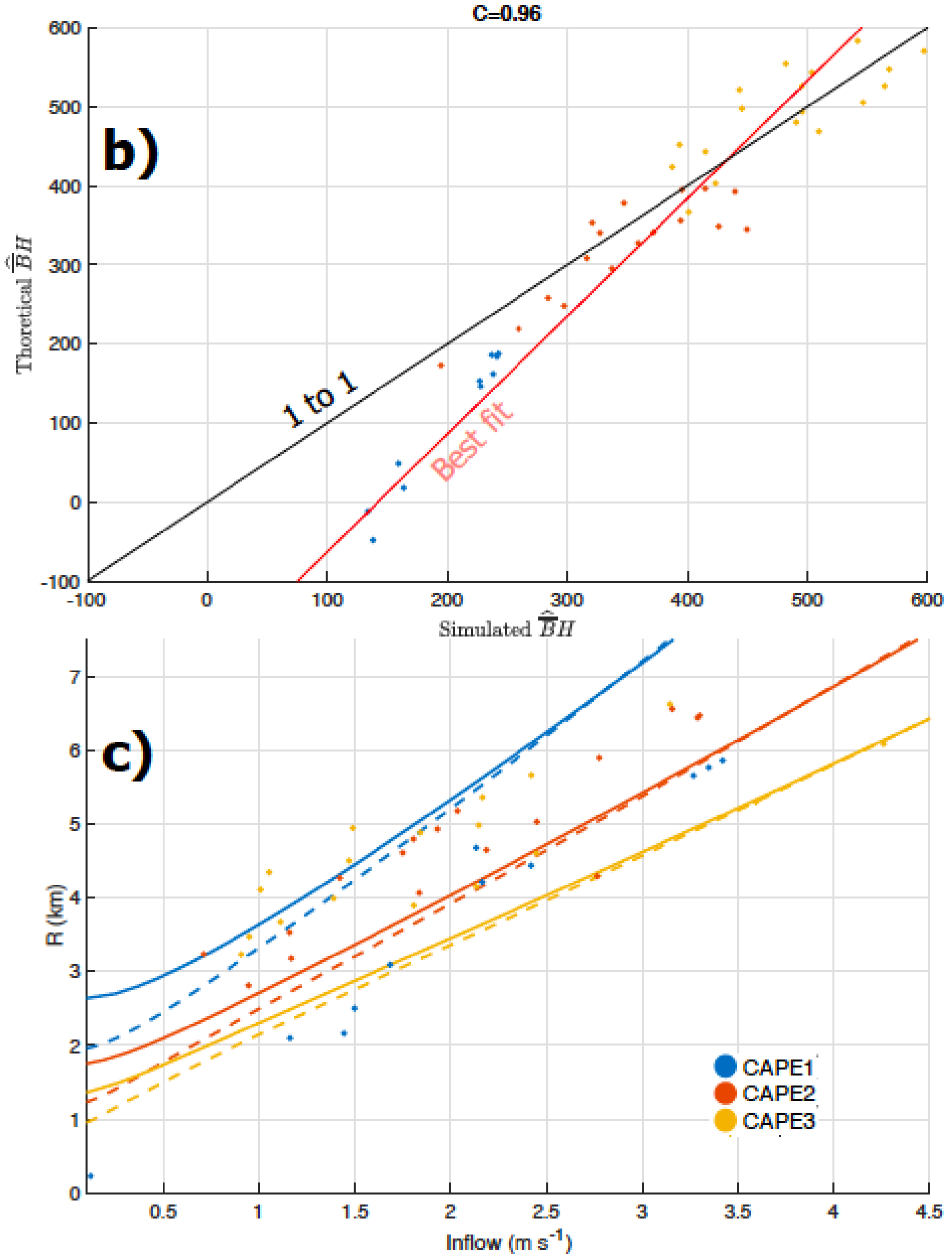




IG. 21. As in Fig. 18, but for the runs with an initial bubble radius of 2000 m.

Peters et al. (2019), JAS

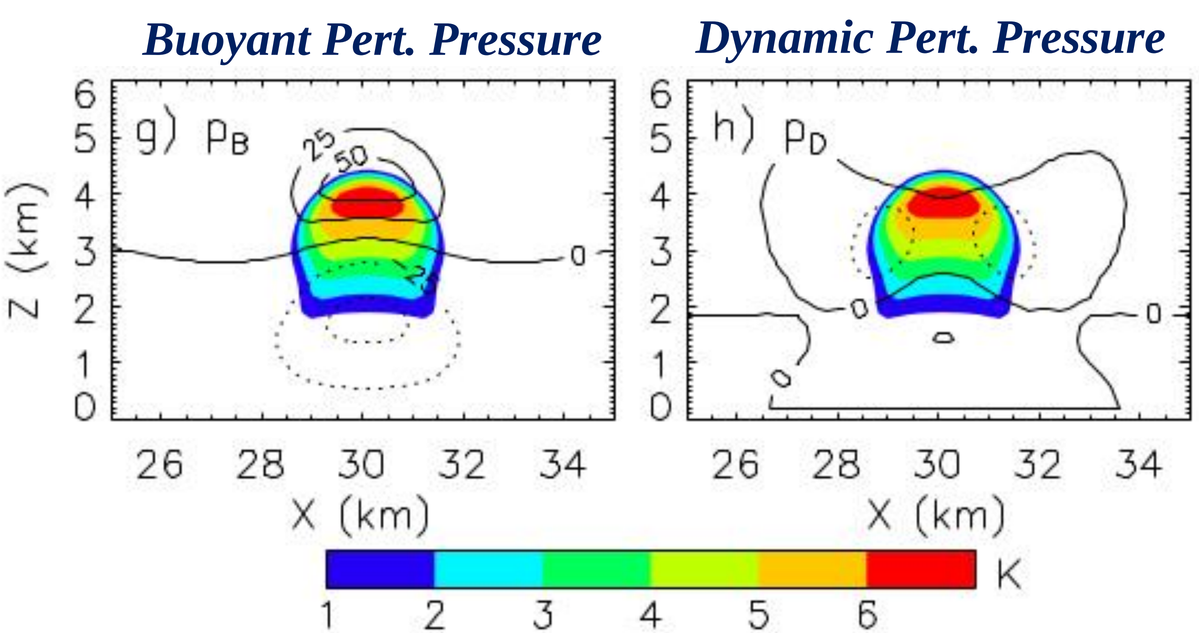




Peters et al. (2019), JAS (accepted)

In weakly sheared environments the *dynamic* perturbation pressure is fairly <u>symmetric</u> between the top and bottom of updrafts.

However, the *buoyant* perturbation pressure is <u>not</u> (it's closer to being <u>anti-symmetric</u>)!

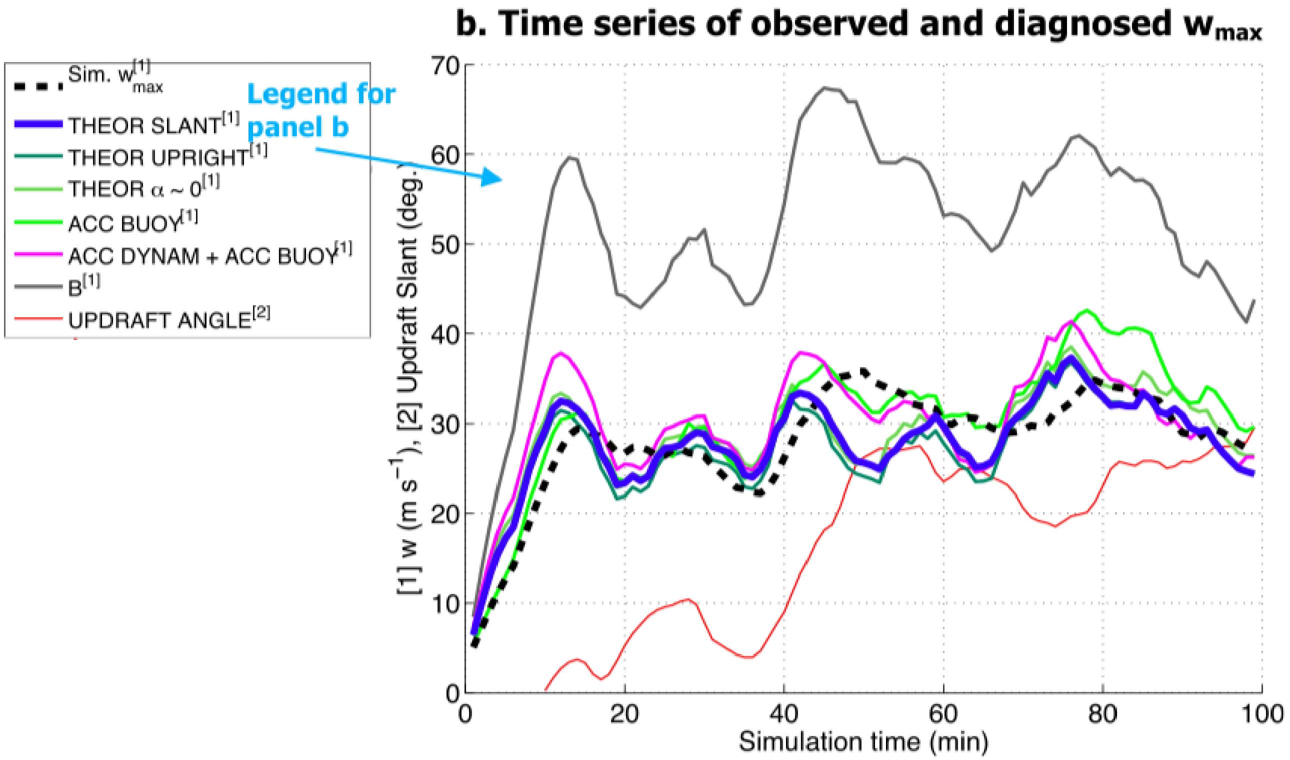


Idealized 3D simulations using CM1 with an unsheared environment

Morrison (2016b) JAS, similar to Markowski and Richardson (2010)

This implies that we can estimate w_{max} *including perturbation pressure effects* by integrating the w momentum equation using $p \sim p_B$! Actually works well for sheared flow too…

Thus, we can integrate $\frac{Dw}{Dt} \approx -\rho^{-1}\frac{\partial p_B}{\partial z} + B \equiv B_{eff}$ to estimate w_{max}.



Peters (2016) JAS

Idealized squall line simulation, 0-1.5 km Δu of ~19 m/s.

Even in a unsheared environments, p_D is essential to explain *where* w_{max} occurs relative to the buoyancy profile.

A simple analytic model for p_B…

1) Derive a theoretical scaling of p_B and *w* based on approximate analytic solutions to the governing momentum and continuity equations assuming Boussinesq flow:

•2D slab and axisymmetric cylindrical coordinates are used to compare 2D versus "3D" updrafts

•Buoyancy distributions are calculated from real and idealized soundings with various *R* → *entrainment is not explicitly included*

2) Analytic solutions are compared to direct numerical solutions of the Poisson p_B equation and (steady state) vertical velocity at the updraft center for the same buoyancy distributions.

Morrison (2016a,b), JAS

3D

$$w_{max} = \sqrt{\frac{2CAPE}{1 + \frac{2\alpha^2 R^2}{H^2}}}$$

$$\alpha \equiv \frac{\overline{w}}{w_0} = \frac{1}{w_0} \int_0^{2\pi} \int_0^R \frac{w}{\pi R^2} r dr d\theta$$

For *R* → 0:

$$w_{max} \rightarrow \sqrt{2CAPE}$$

For *R* → infinity:

$$w_{max} \rightarrow 0$$

Hydrostatic regime $(\alpha R/H)^2 >> 1$:

$$w_{max} \approx \frac{H}{\alpha R} \sqrt{2CAPE}$$

2D

$$w_{max} = \sqrt{\frac{2CAPE}{1 + \frac{8\alpha^2 R^2}{H^2}}}$$

$$\alpha \equiv \frac{\overline{w}}{w_0} = \frac{1}{w_0} \int_0^R \frac{w}{R} dr$$

For *R* → 0:

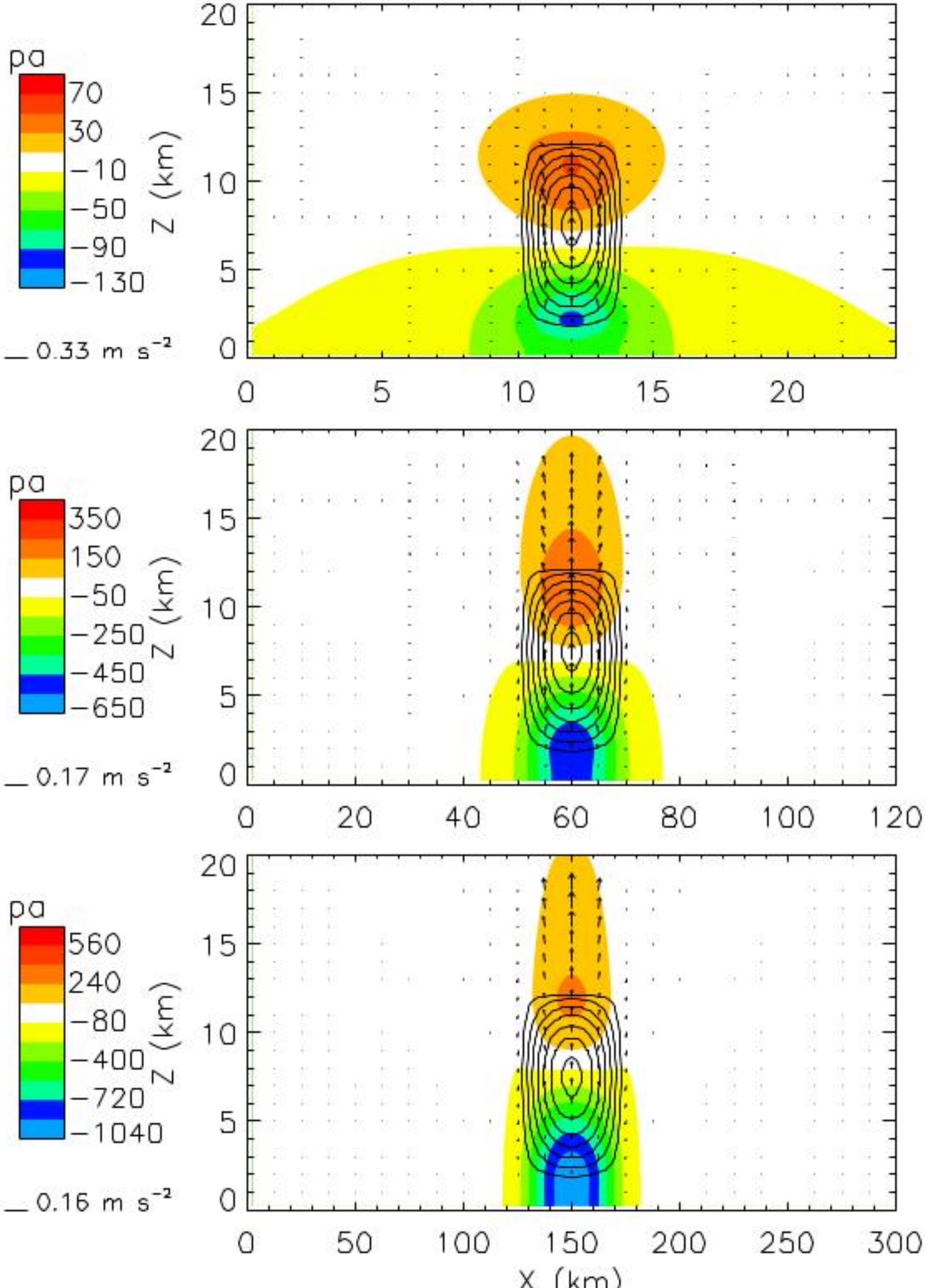
$$w_{max} \rightarrow \sqrt{2CAPE}$$

For *R* → infinity:

$$w_{max} \rightarrow 0$$

Hydrostatic regime $(\alpha R/H)^2 >> 1$:

$$w_{max} \approx \frac{H}{2\alpha R} \sqrt{2CAPE}$$



Direct numerical solution

(similar to Parker 2010)

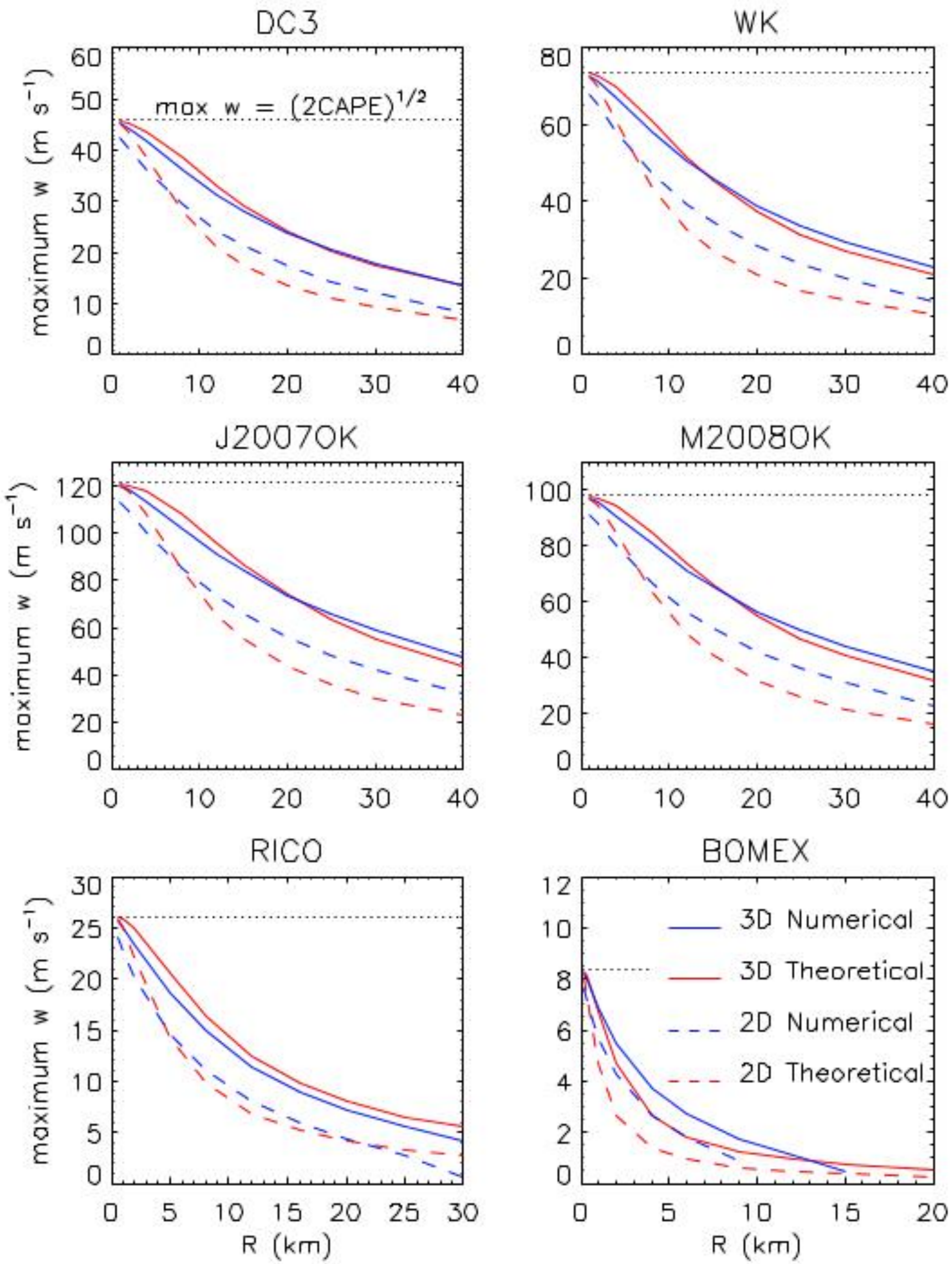
$$\nabla^2 p_B = \frac{\partial(r\,B)}{\partial z}$$

W-K idealized sounding

(*Weisman and Klemp 1982*)

Horizontal buoyancy distribution specified as cosine function from updraft center to edge.

Integrate rising parcel from bottom to top using the calculated p_B field to obtain "numerical" *w* at the updraft center (r = 0).

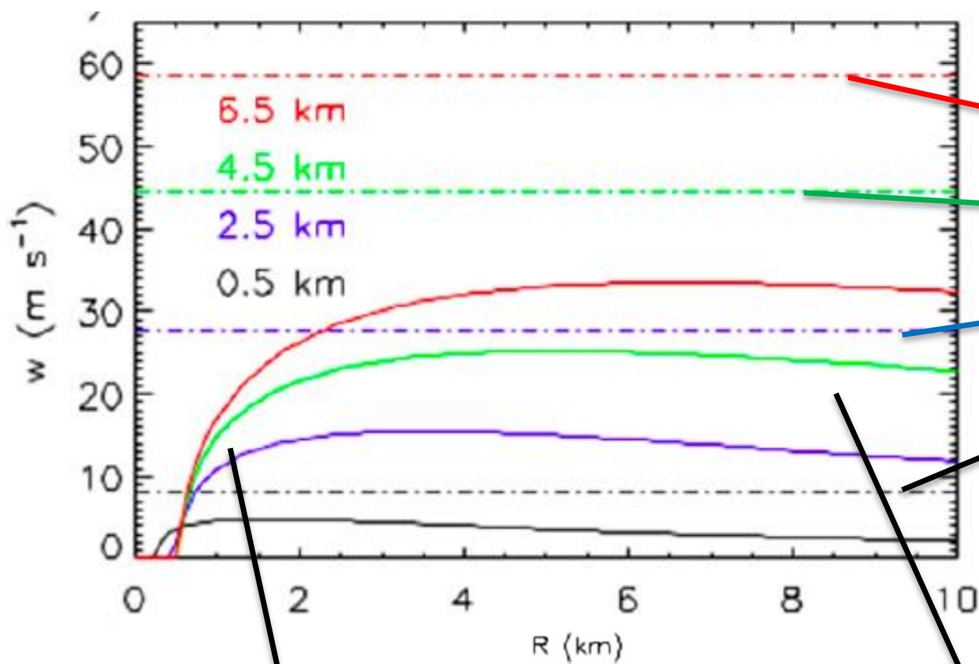


Comparison of w_{max}

Results shown for 6 different vertical buoyancy distributions based on various soundings.

Can this help explain the factor of 2 over-prediction of w_{max} from $\sqrt{2CAPE}$?

Analytic expression for w at height z_m

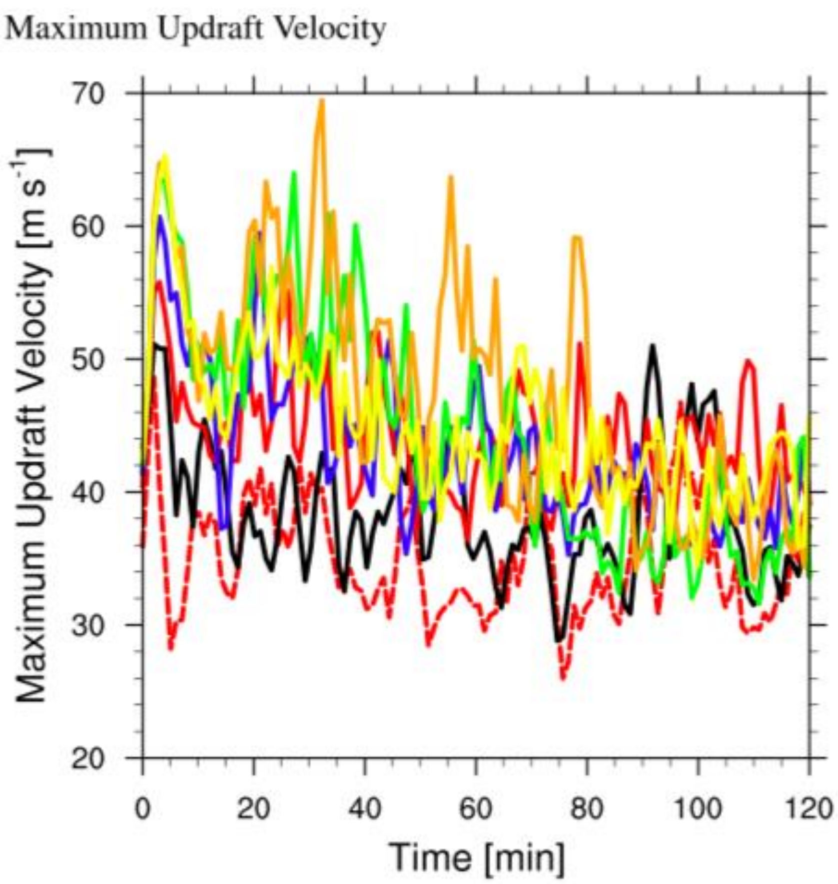


$\sqrt{2CAPE}$ (at different heights)

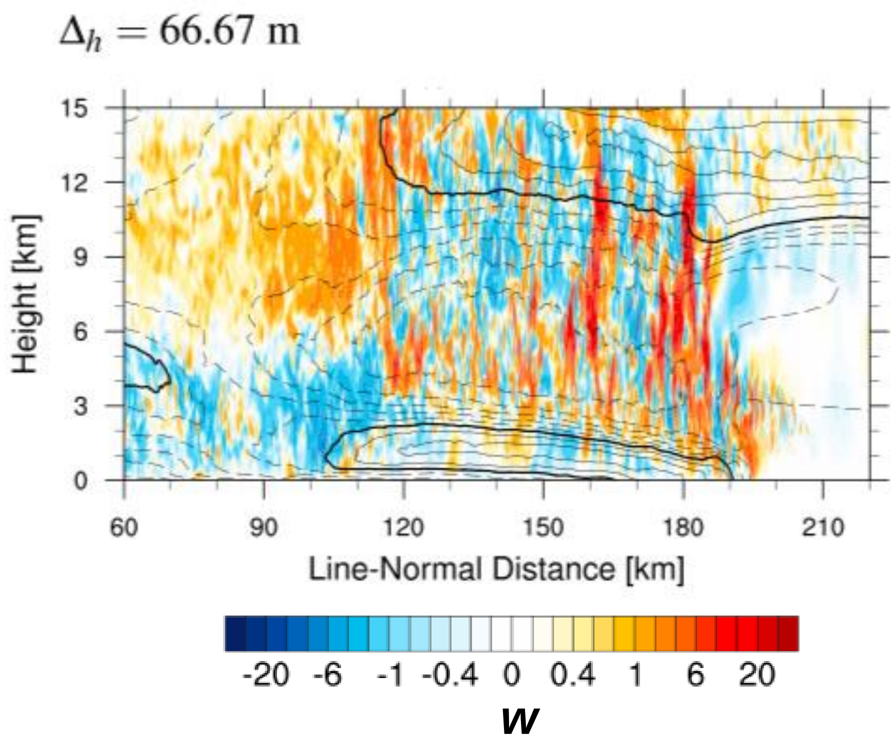
Effects of <u>mixing/ dilution</u> at small *R*

Effects of <u>perturbation pressure</u> at large *R*

A rule of thumb is about a factor of 2 over-estimation of w_{max} from neglecting perturbation pressure effects and entrainment/mixing…



Lebo and Morrison (2015), MWR



CAPE **~ 4200 J kg^{-1} → *w_{max}* ~ 130 m s^{-1}**

3D

$$\Delta p = \frac{\rho_0 \alpha^2 R^2}{H^2} w_{LNB}^2 = \frac{2\rho_0 \alpha^2 R^2}{H^2}\left(1 + \frac{2\alpha^2 R^2}{H^2}\right)^{-1} CAPE$$

For *R* → 0:

$$\Delta p = 0$$

Hydrostatic regime ($\alpha R/H$)^2 >> 1:

$$\Delta p \approx \Delta p_h = \rho_0 CAPE$$

2D

$$\Delta p = \frac{4\rho_0 \alpha^2 R^2}{H^2} w_{LNB}^2 = \frac{8\rho_0 \alpha^2 R^2}{H^2}\left(1 + \frac{8\alpha^2 R^2}{H^2}\right)^{-1} CAPE$$

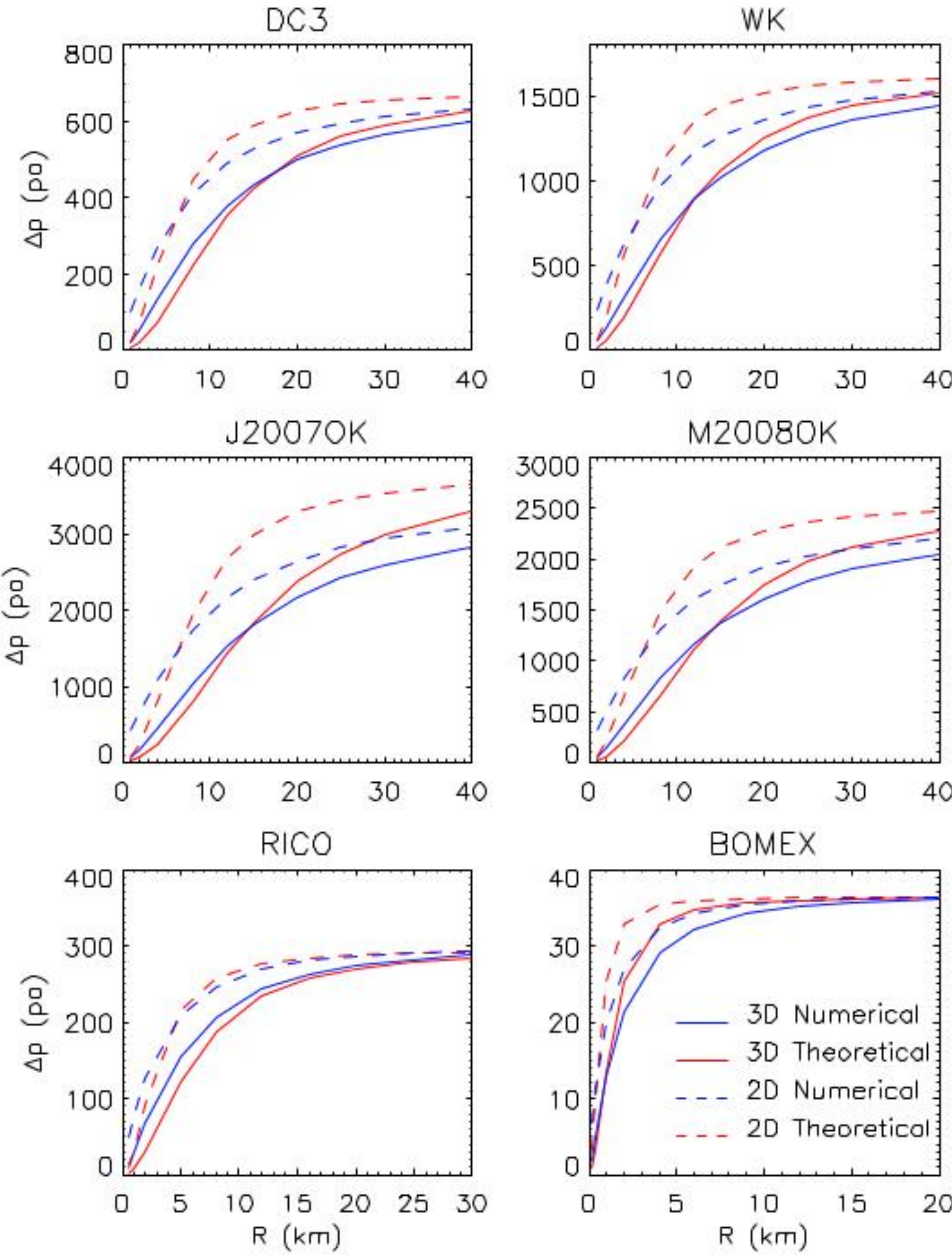
For *R* → 0:

$$\Delta p = 0$$

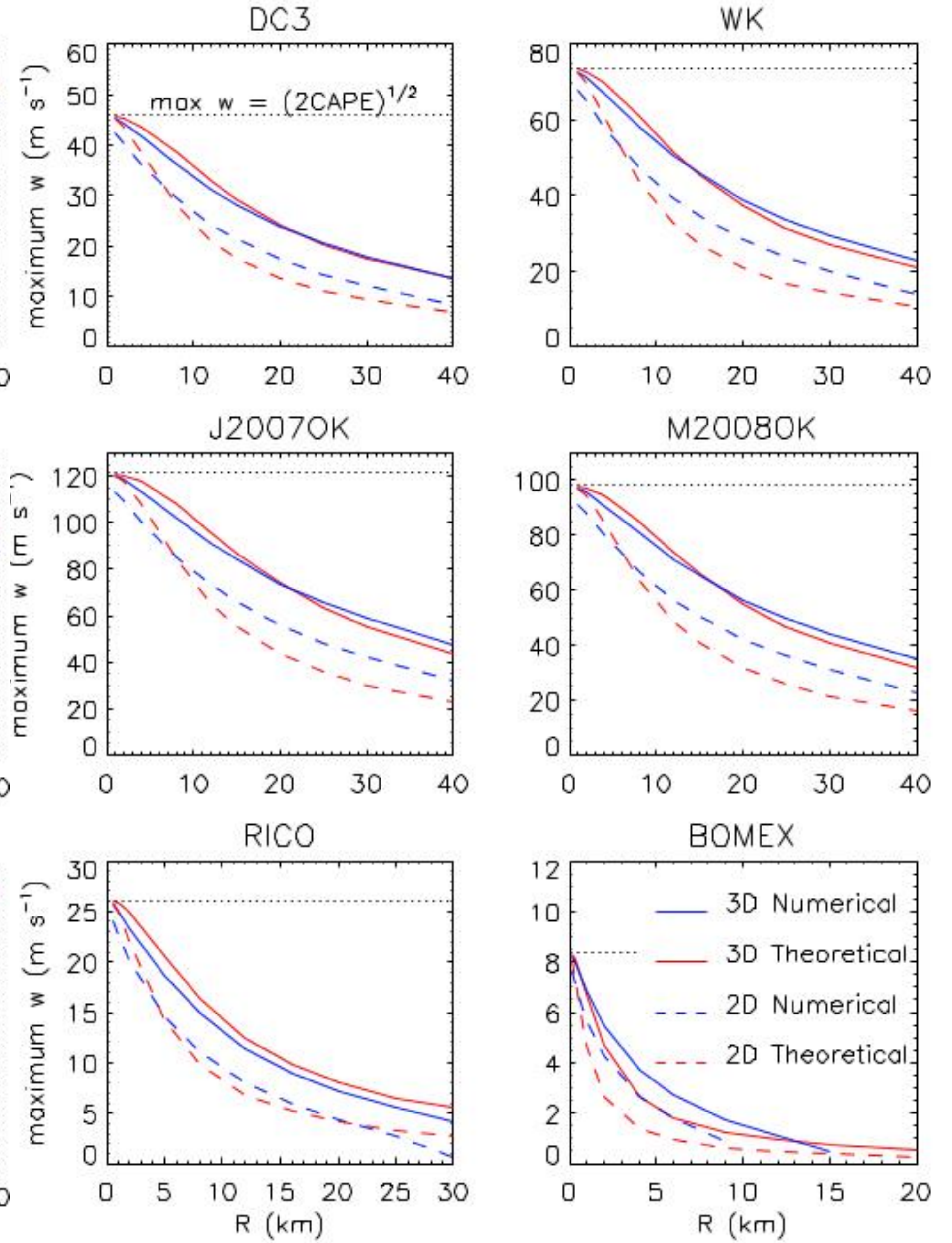
Hydrostatic regime ($\alpha R/H$)^2 >> 1:

$$\Delta p \approx \Delta p_h = \rho_0 CAPE$$

Comparison of Δp_B

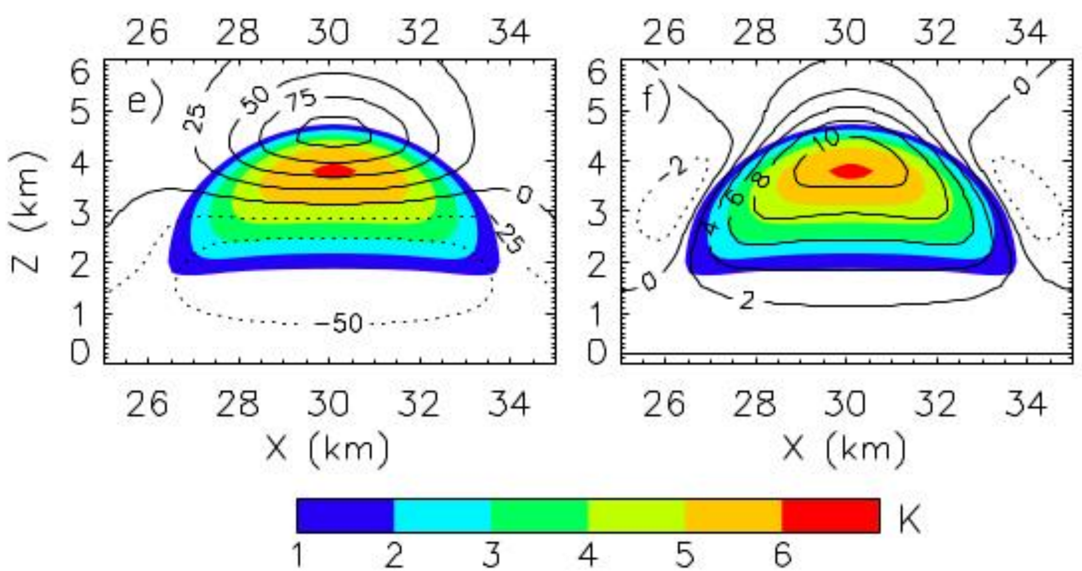


Comparison of w_{max}



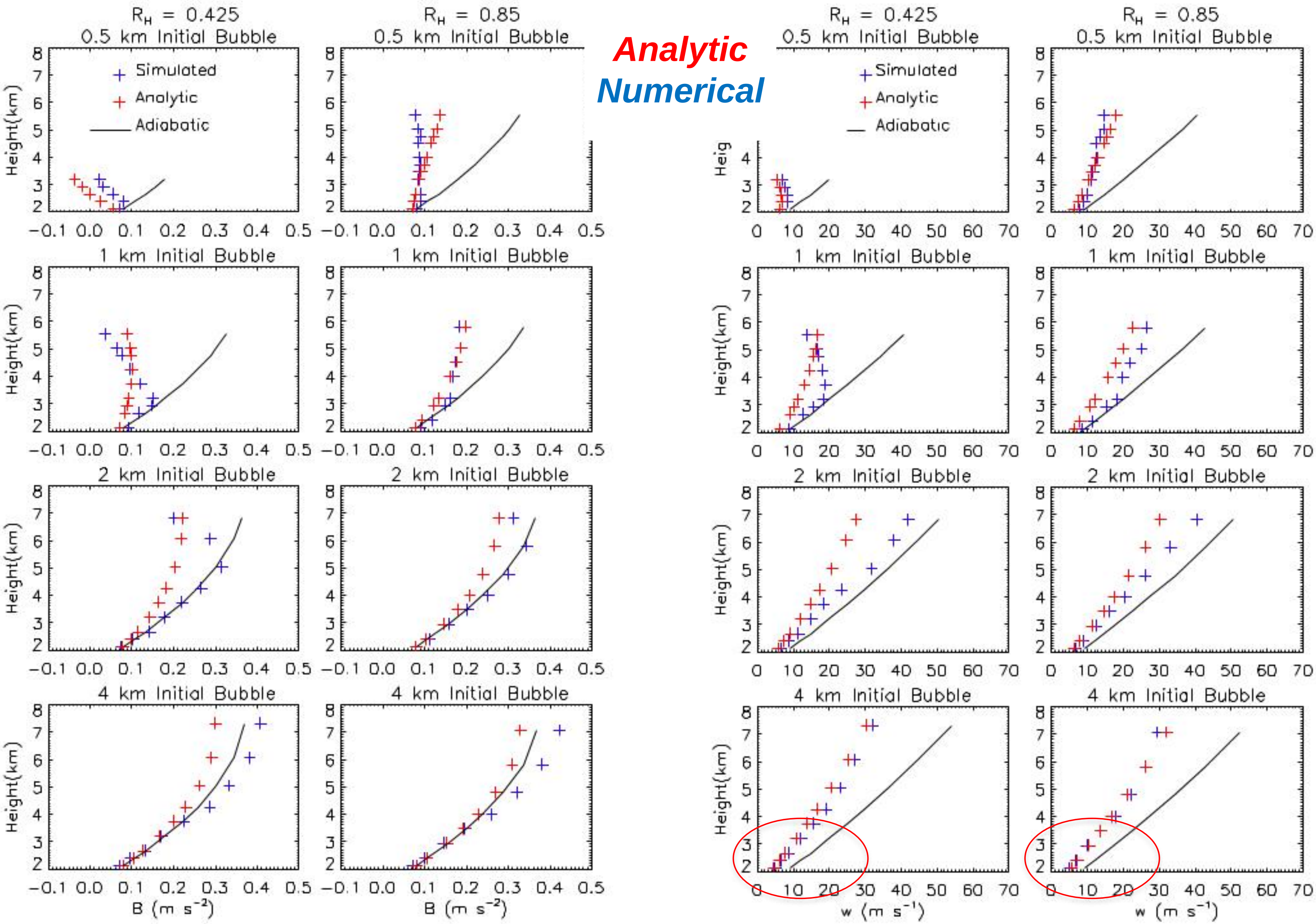
Comparison with fully dynamical "updraft" simulations

Initial warm-bubble radius (km)	R (km)	H (km)	CAPE (J kg^{-1})		α	Δp (hPa) SIM	Δp (hPa) TH	Max w (m s^{-1}) SIM	Max w (m s^{-1}) TH
1.5	1.5	2.4	241	3D	0.78	76	66	18.1	18.8
3	2.5	2.6	261	3D	0.82	123	125	14.4	15.7
10	5.7	2.8	290	3D	0.77	179	207	11.0	9.8
1.5	2.2	2.4	224	2D	0.47	112	118	16.4	13.3
3	3.3	2.6	250	2D	0.48	153	162	11.7	11.2
10	5.4	2.6	260	2D	0.57	184	205	9.0	6.6
1.5	1.8	7.4	905	3D	0.38	9	10	42.6	42.2
3	2.8	8.8	1250	3D	0.48	170	37	56.9	48.9
10	4.6	8.8	1730	3D	0.75	387	252	47.0	51.4
1.5	3.2	5.4	258	2D	0.54	160	72	23.7	16.8
3	3.0	7.0	504	2D	0.58	296	120	21.7	26.0
10	3.6	6.4	1076	2D	0.66	487	394	38.7	32.5



Results are calculated from 400-1080 sec

Buoyancy and *w* at the center of the first thermal



The ***plume*** and ***thermal*** models originated from applying dimensional analysis to idealized flows*:

- ***plumes*** → buoyant jet in which buoyancy is supplied from a steady point source
- ***thermal*** → discrete rising buoyant bubble generated from a pulse source of buoyancy

***See Emanuel (1994)**

The *plume* and *thermal* models originated from applying dimensional analysis to idealized flows*:

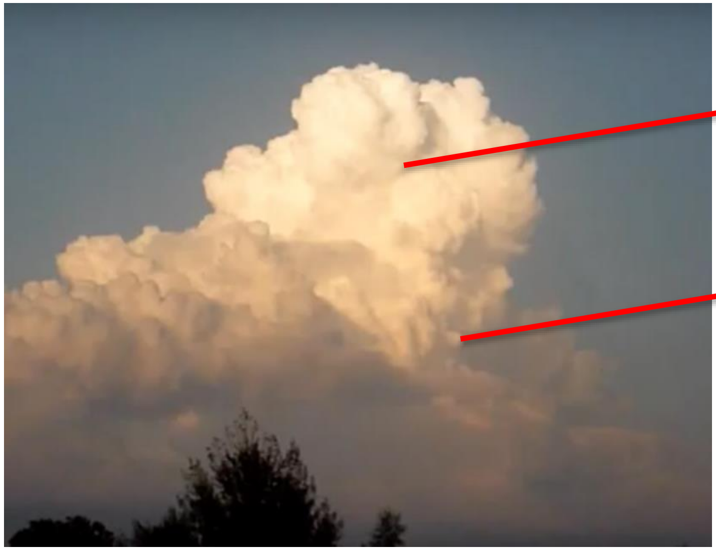
- ***plumes*** → buoyant jet in which buoyancy is supplied from a steady point source
- ***thermal*** → discrete rising buoyant bubble generated from a pulse source of buoyancy

• Theoretical scalings are well-supported by lab studies, e.g. entrainment ~ 0.2/*R* in steady plumes.

• Large differences in flow characteristics between plumes and thermals (e.g. entrainment, vertical velocity structure)

***See Emanuel (1994)**

• Two critical features for *moist* convection not accounted for by these models:



Increase of buoyancy from condensation and latent heating aloft

Decrease of buoyancy from entrainment and evaporation

• Modification of traditional models to account for these challenges, e.g.,

- separation of <u>*dynamic*</u> and smaller-scale <u>*turbulent*</u> entrainment
- buoyancy sorting

• The earliest convection parameterizations and most current ones are based on the *steady state plume* framework

- straightforward to implement

- unlike the simple R^{-1} scaling of entrainment in the traditional plume model, current schemes use a wide variety of methods to formulate entrainment (and detrainment) rates (functions of w, z, RH, etc.)

• The earliest convection parameterizations and most current ones are based on the *steady state plume* framework

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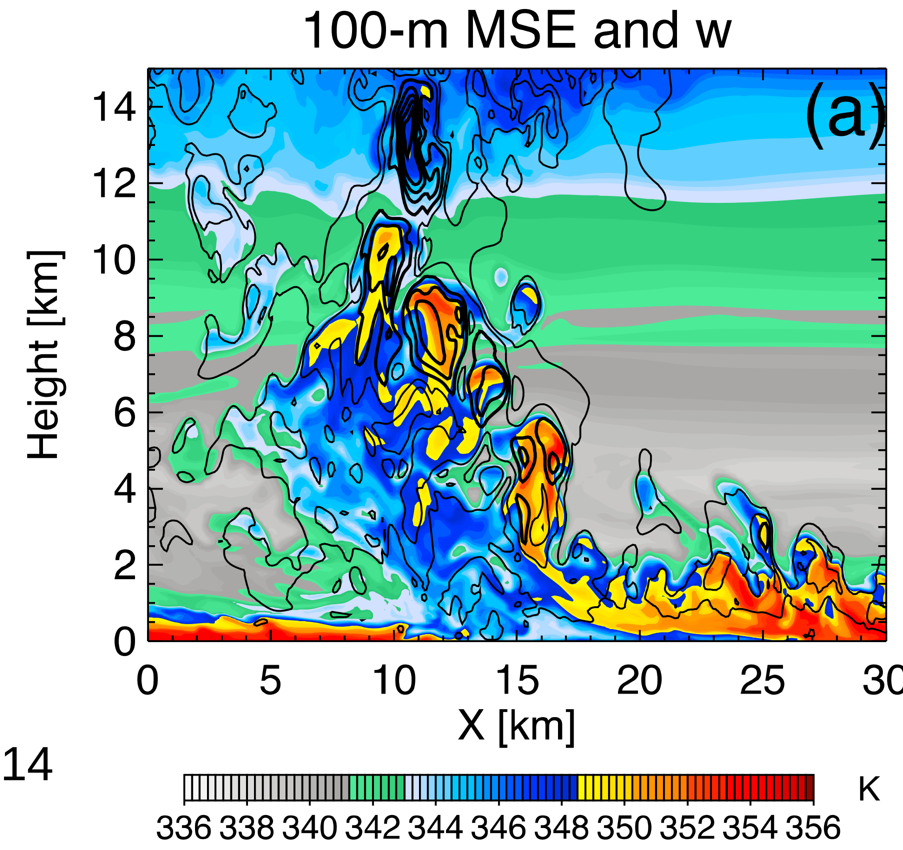
- unlike the simple R^{-1} scaling of entrainment in the traditional plume model, current schemes use a wide variety of methods to formulate entrainment (and detrainment) rates (functions of w, z, RH, etc.)

→ ***no consistent scaling relationship has been found for entrainment rate in moist convection!***

• Observations of moist updrafts comprised of "bubbles" (thermal-like structures) go back at least to Scorer and Ludlam in the 1950's.

• More recent examples: Blyth et al. (1988), Damiani et al. (2006)

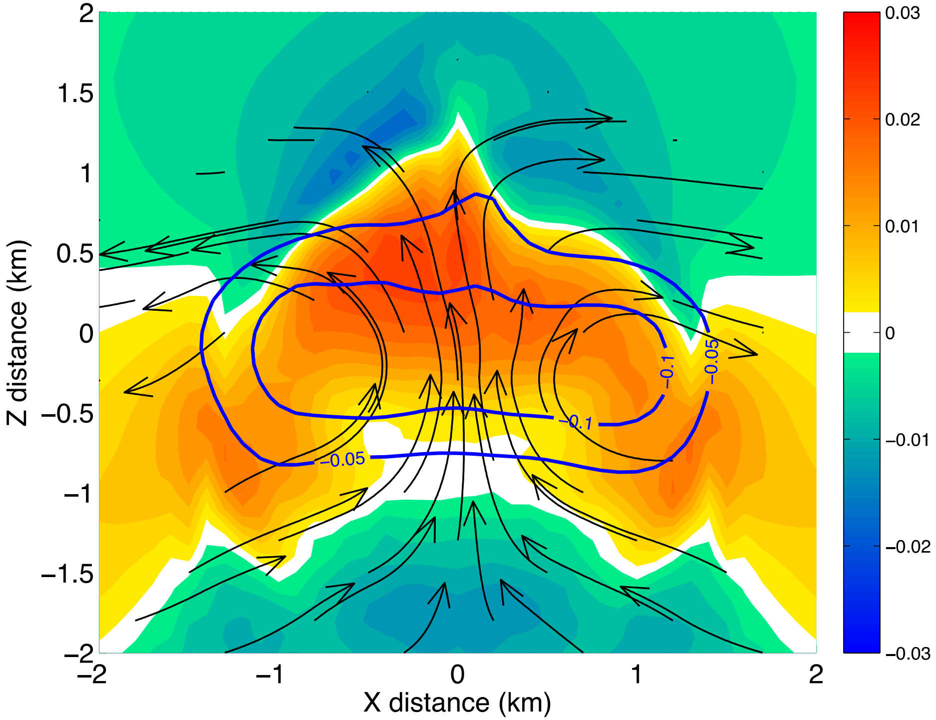
• Recent LES ($\Delta x \sim 100$ m) support a thermal-like view, even for deep convection…



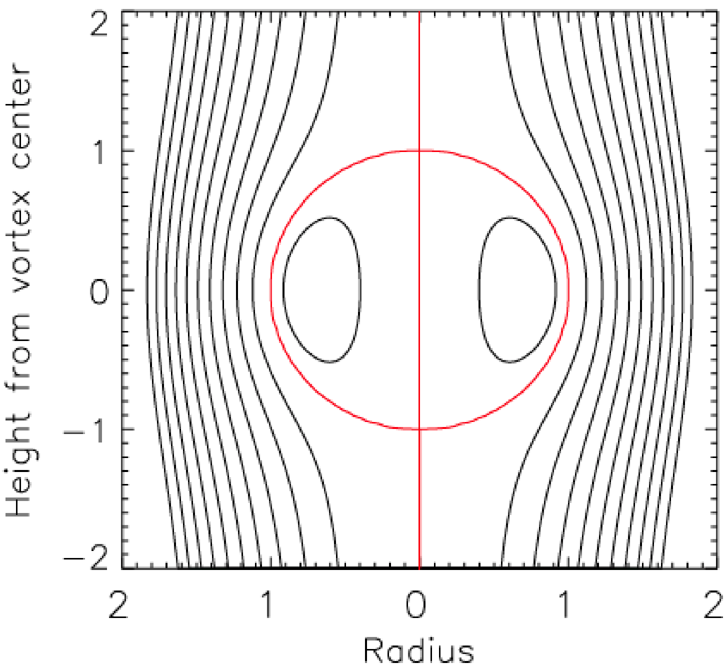
Varble et al. 2014

- The flow within individual thermals in high-resolution LES resembles *Hill's analytic spherical vortex* (Sherwood et al. 2013; Romps and Charn 2015):

CM1 simulations (Δx = 100 m) with no environmental shear



Streamlines; Hill's vortex



- Hill's vortex is non-buoyant, extensions including the effects of buoyancy were derived by Morrison and Peters (2018), *JAS*.

- The thermal structure of updrafts challenges the steady-state plume framework assumed by most parameterizations

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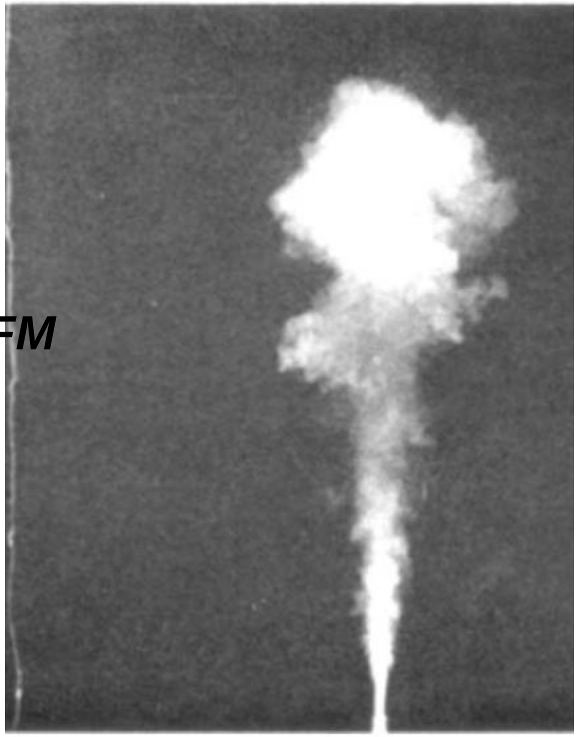
A combination of the two:

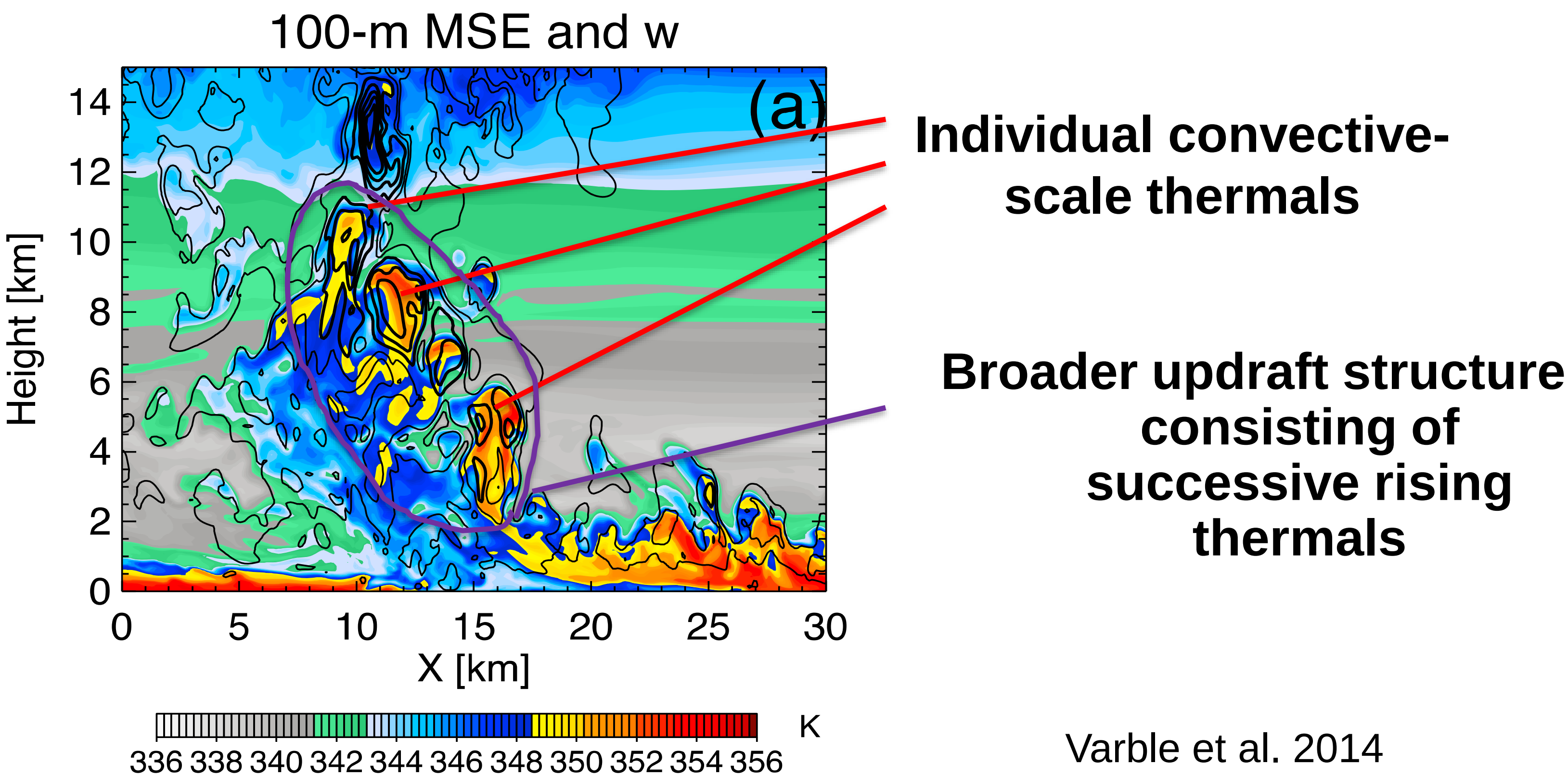
the *starting plume*…

Turner 1962, *JFM*



ing plume in which the supply of dye has been

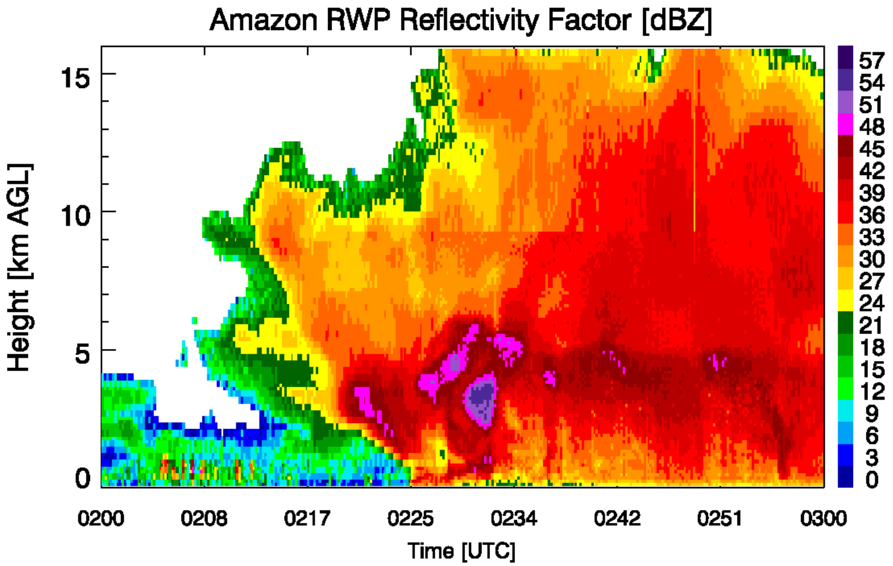
However… the <u>starting plume</u> model is not very satisfying because the wake behind the thermal head often itself consists of thermal-like structures. On the other hand, the rising <u>thermal</u> model is also unsatisfactory because the thermals that comprise a convective cloud are typically <u>well organized</u>.



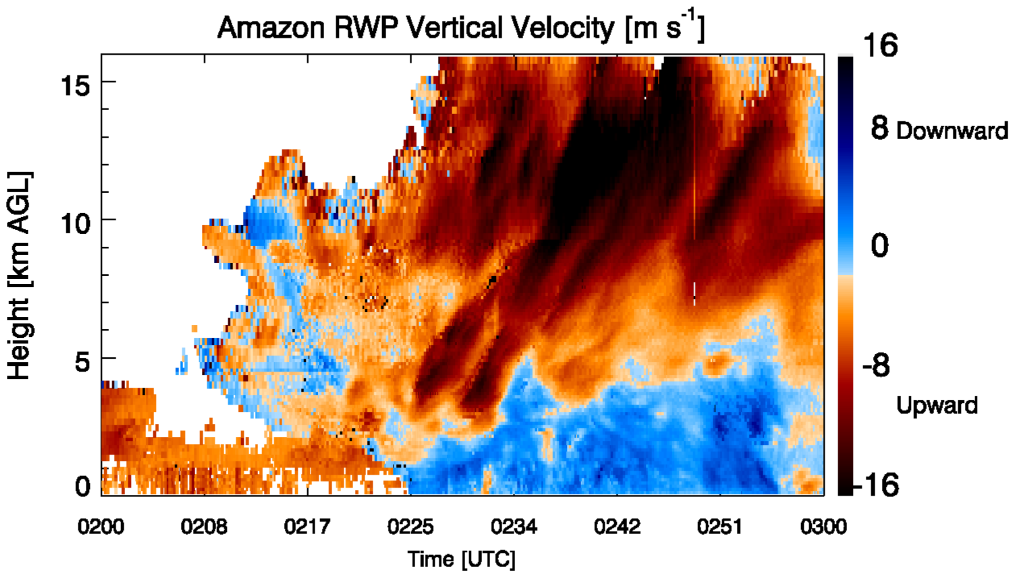
Varble et al. 2014

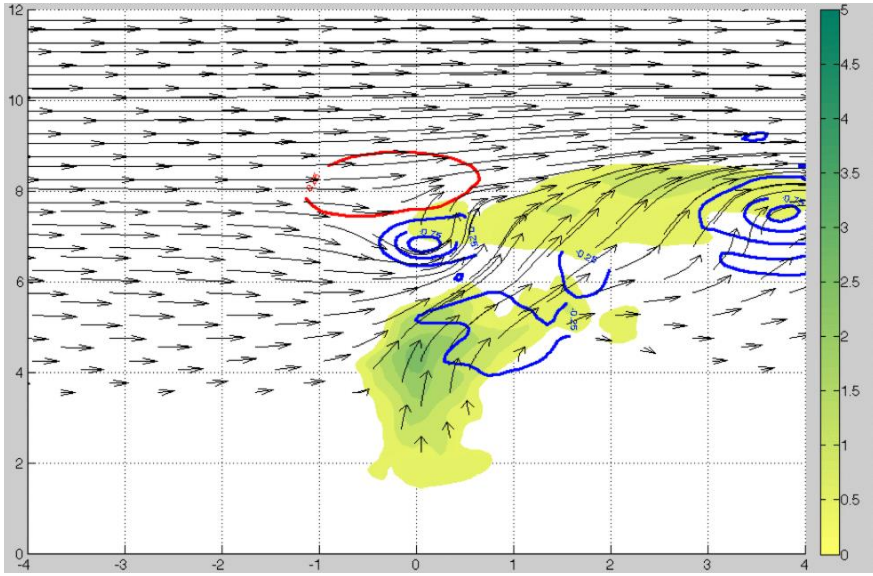
Play timelapse movie

Vertical velocity retrievals from radar 1290 mHz profiler, Manaus, Brazil, Nov. 22 2014



see Giangrande et al. (2016) JGR for retrieval/deployment details





The succession of rising thermals is potentially a key aspect:

- Implications for entrainment/detrainment

- Implications for perturbation pressure structure

- Microphysics-dynamics interactions (Moser and Lasher-Trapp 2017)

To briefly summarize…

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Observations and LES suggest moist convection often occurs as a succession of rising thermals → "*<u>thermal chain</u>*"

Neither *thermal*, *plume*, nor *starting plume* models adequately describe thermal chains…

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Observations and LES suggest moist convection often occurs as a succession of rising thermals → "*__thermal chain__*"

Neither *thermal*, *plume*, nor *starting plume* models adequately describe thermal chains…

1. *Why* does this structure occur, and *what* are the driving mechanisms?
2. Can the behavior be described by simple scalings?
3. Are there linkages to the traditional plume or thermal models?

A theory for thermal chains*

*Morrison (2017), *JAS*
Morrison, Peters, Varble, Giangrande, Hannah, in prep.

Governing equations in axisymmetric coordinates (*r*, *z*)*

$$\frac{\partial \vec{u}}{\partial t} = -u\frac{\partial \vec{u}}{\partial r} - w\frac{\partial \vec{u}}{\partial z} - \rho_0^{-1}\left(\frac{\partial p}{\partial r}\hat{\imath} + \frac{\partial p}{\partial z}\hat{k}\right) + B\hat{k} \qquad (1)$$

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial \theta}{\partial t} = -u\frac{\partial \theta}{\partial r} - w\frac{\partial \theta}{\partial z} + \frac{L_v}{c_p}CE \qquad (3)$$

$$\frac{\partial q_v}{\partial t} = -u\frac{\partial q_v}{\partial r} - w\frac{\partial q_v}{\partial z} - CE \qquad (4)$$

$$CE = \frac{\partial q_v}{\partial t} - \frac{\partial \theta}{\partial t}\frac{dq_s}{d\theta} \text{ if } q_v = q_s;\ CE = 0 \text{ otherwise} \qquad (5)$$

***Boussinesq, inviscid**

Analytic approximation

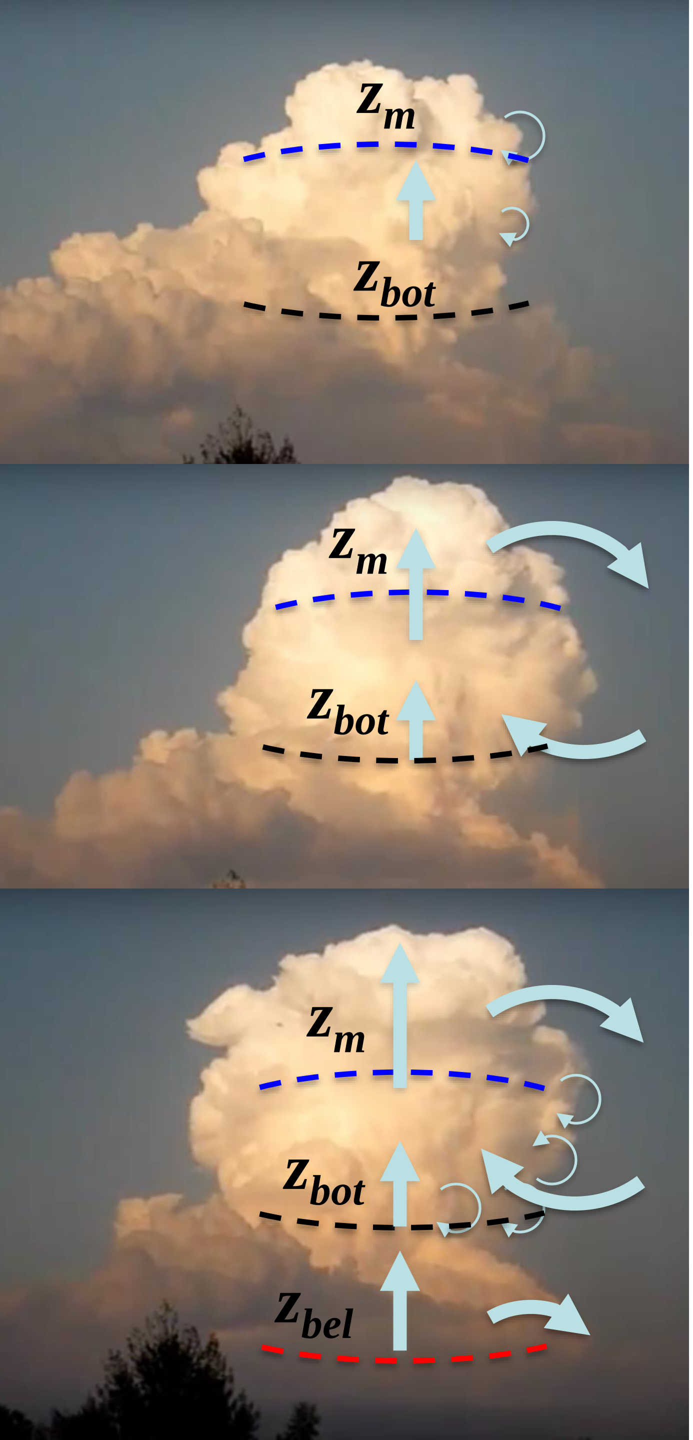
• Reynolds averaging applied, lateral turbulent fluxes represented by first-order Smagorinsky-type approach.

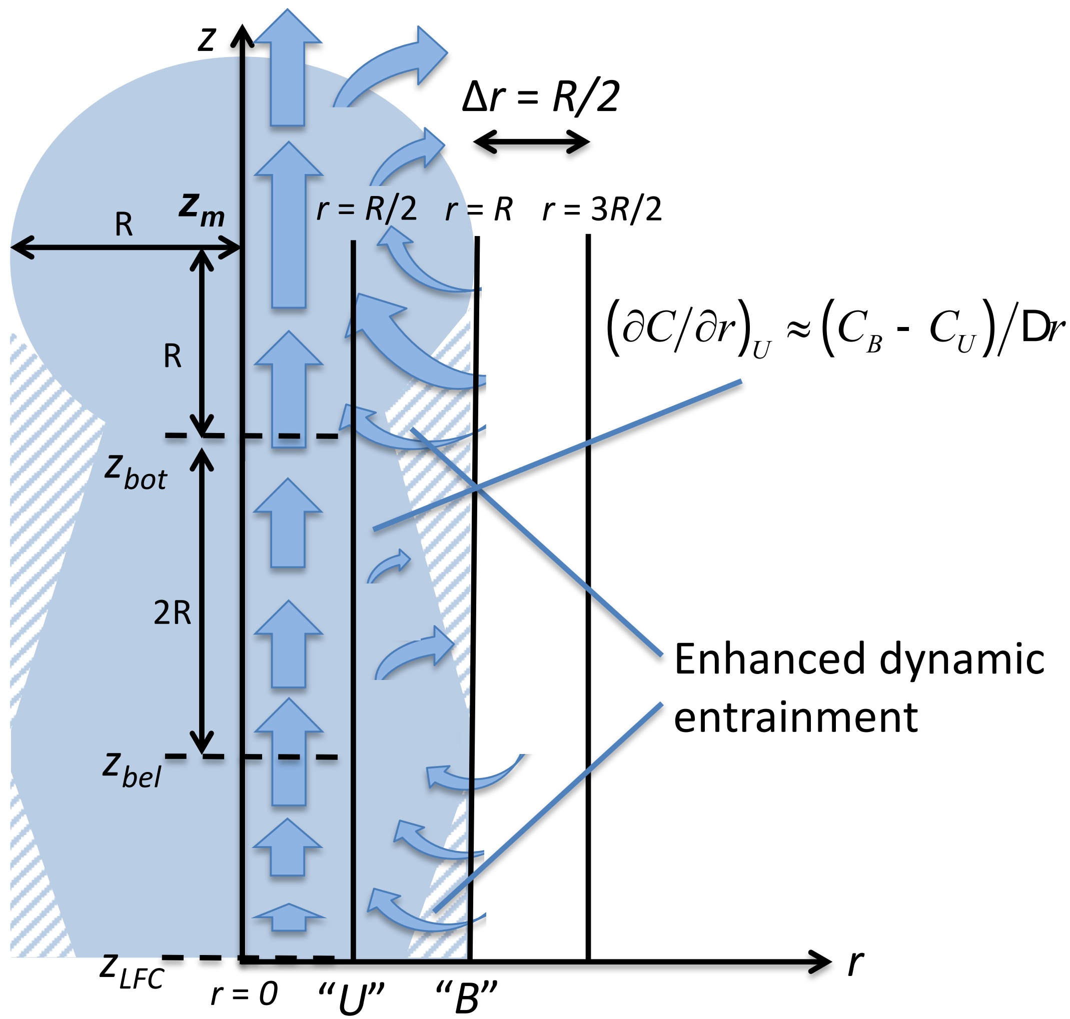
• Vertical turbulent fluxes are neglected.

• Condensate loading neglected in buoyancy, and within cloudy updrafts it is assumed that $q_v = q_s$ and sufficient cloud water is always available to retain saturated conditions when evaporation occurs.

• Solutions are first obtained for a scalar C, B, and w at $r = 0$ and height of maximum w, z_m, for the primary ascending thermal, next at the thermal bottom, z_{bot}, then for additional thermals below in the chain.

Schematic of the analytic thermal chain model





Idealized WRF simulations (NOTE: *not* LES)

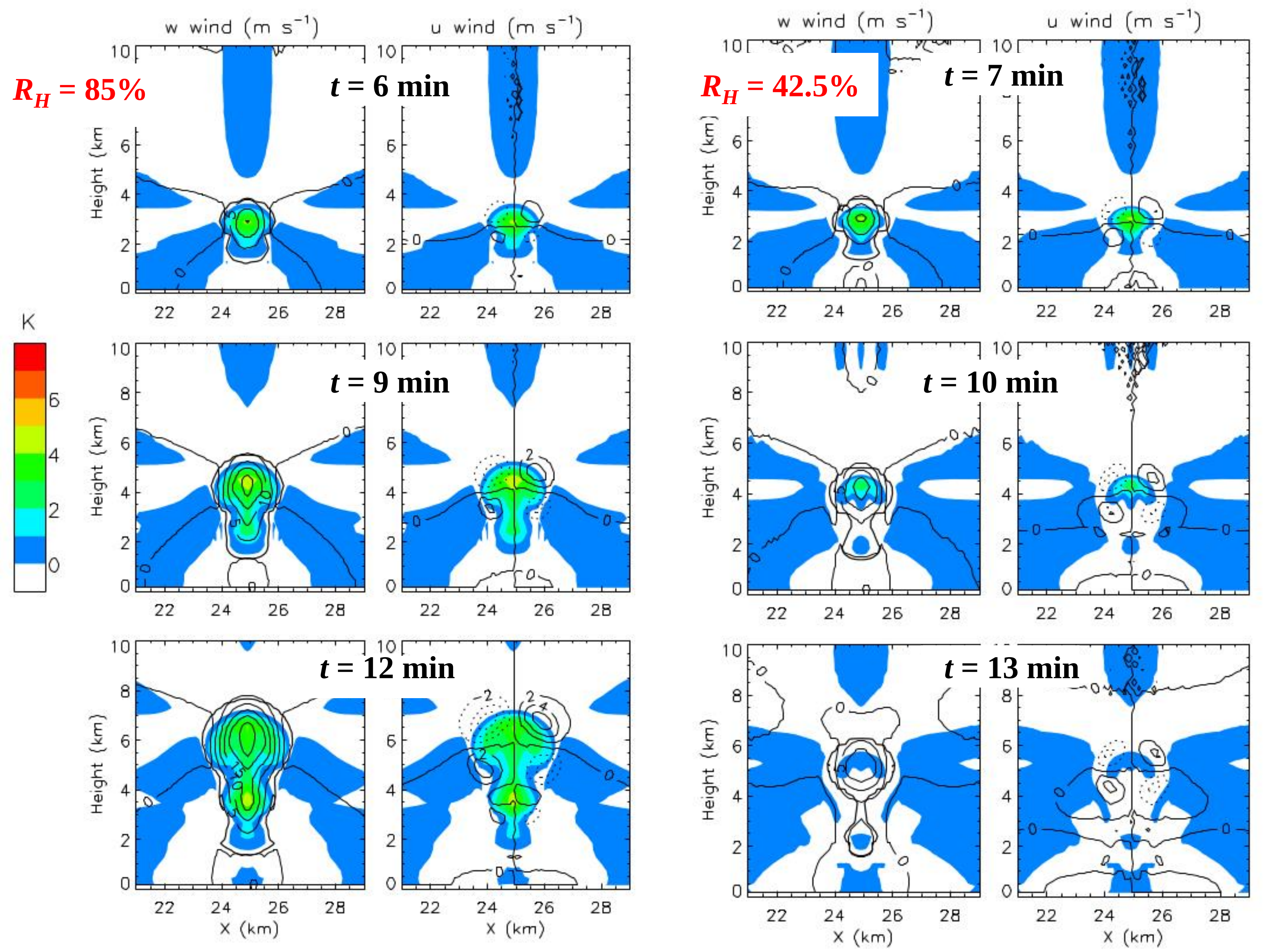
- **100 m horizontal grid spacing, enhanced Smagorinsky-type sub-grid scale mixing (mixing length *L* = 500 m)**

- **No microphysics except cloud condensation/evaporation, no condensate loading for simplicity**

- **Weisman-Klemp sounding, modified to have constant *RH* above the level of free convection, unsheared environment**

- **Passive tracer added (held fixed at 1 below the LFC, initial values are zero above LFC)**

***Initial bubble width*: 0.5, 1, 2, 4 km**
***Initial environmental* R_H: 42.5 and 85%**



w wind (m s^{-1})
u wind (m s^{-1})
R_H = 85%
t = 6 min
t = 9 min
t = 12 min
R_H = 42.5%
t = 7 min
t = 10 min
t = 13 min
Height (km)
X (km)
K

Evolution of a passive scalar at z_m

The analytic model:

- Organized horizontal advection is zero at updraft center by symmetry (u = 0 at updraft center r = 0)

- Horizontal convergence across updraft is 0 at height of maximum w ($z = z_m$)

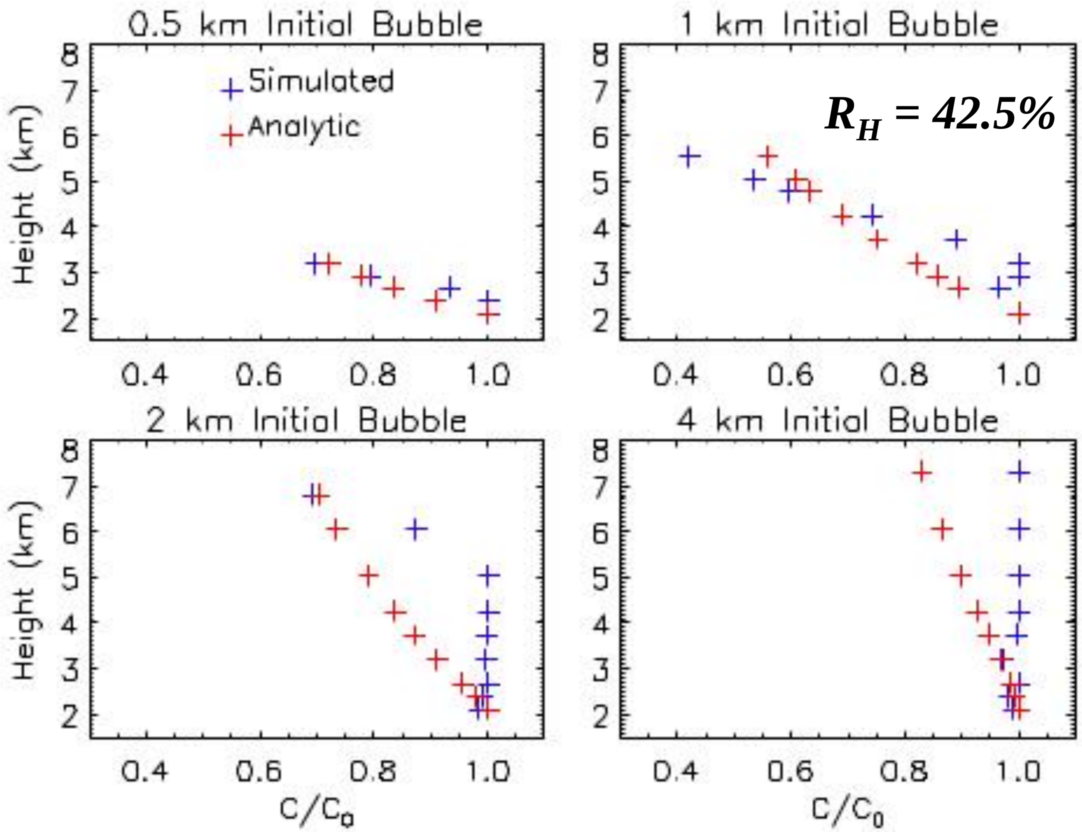
- Assume C = 0 in the environment

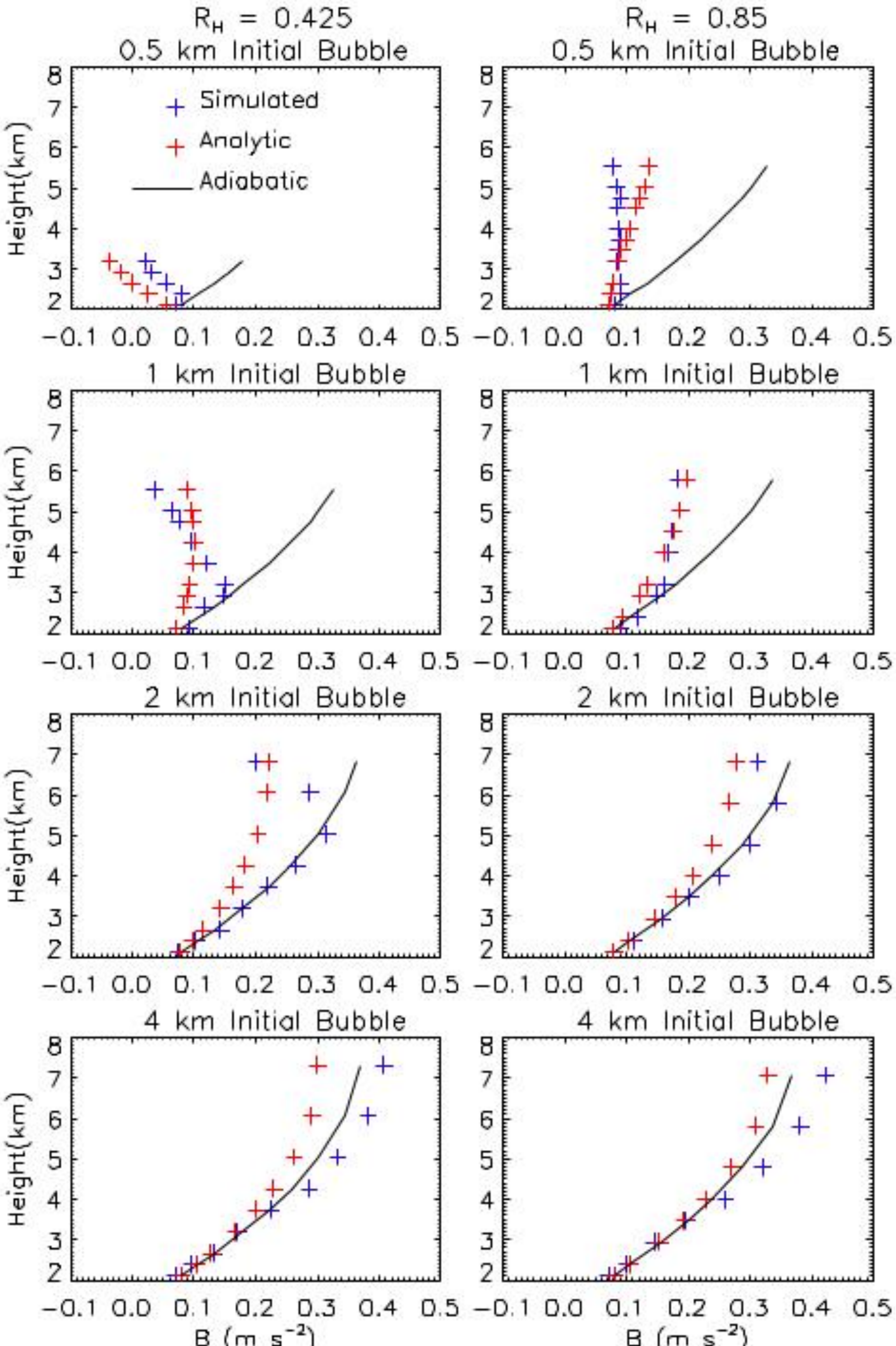
- R is calculated directly from the simulations

Comparison of analytic and numerical solutions

$$C_z = C_0 e^{\frac{-2k^2 Lz}{P_r R^2}} \approx \frac{\left(1 - k^2 Lz / \left(P_r R^2\right)\right)}{\left(1 + k^2 Lz / \left(P_r R^2\right)\right)} C_0$$

Analytic solution





Buoyancy at z_m

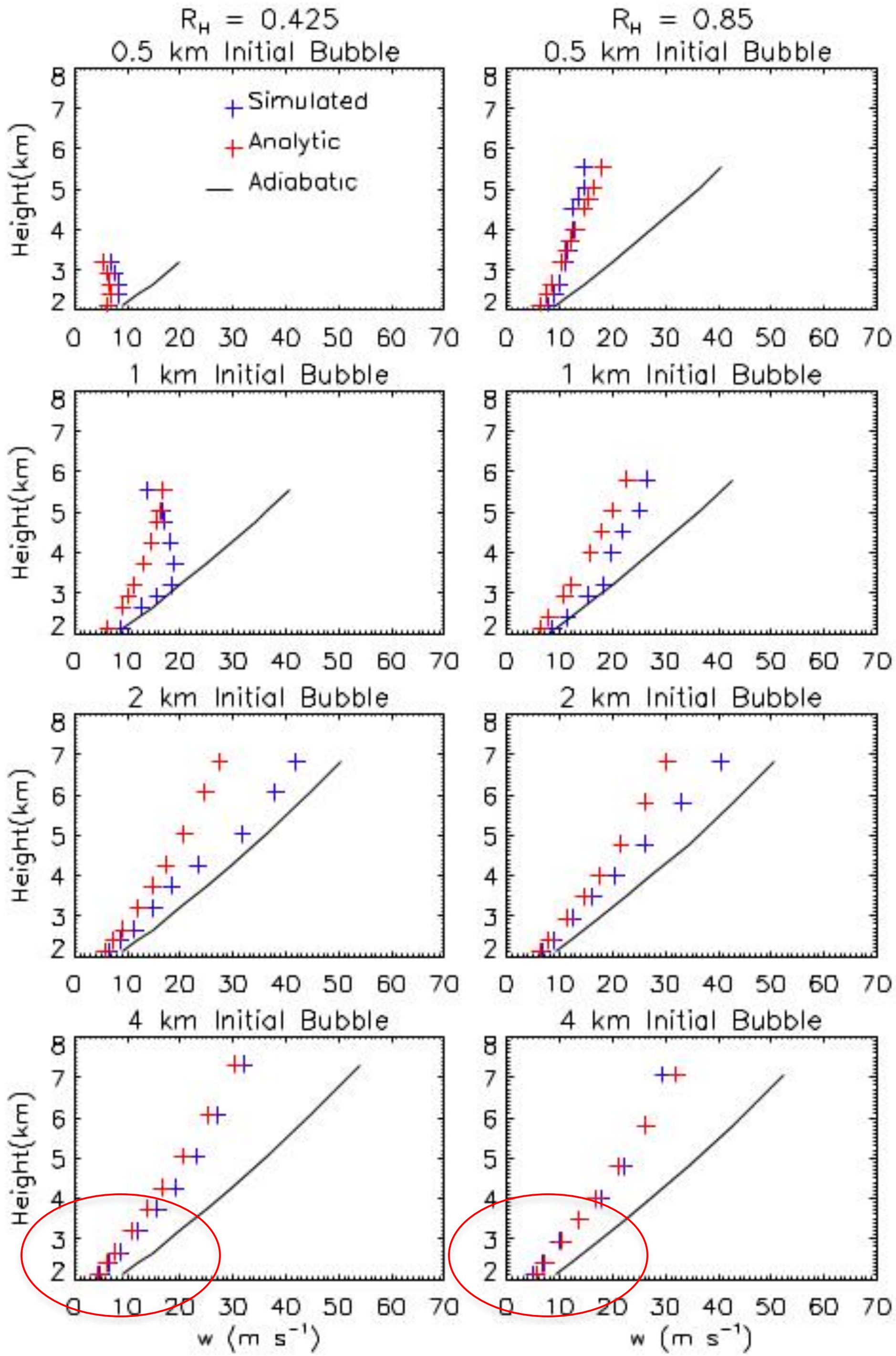
Similar procedure as scalar mixing, but accounts for $\frac{\partial q_v}{\partial t}$ and $\frac{\partial \theta}{\partial t}$:

1) mixing of buoyancy itself (assuming B = 0 in the environment)

2) mixing of dry air from the environment

$$B = \frac{B_{AD}}{1 + \dfrac{k^2 L z}{P_r R^2}} - \frac{2 L_v g k^2 L \Phi}{c_p \left(\dfrac{P_r R^2}{z} + k^2 L \right)}$$

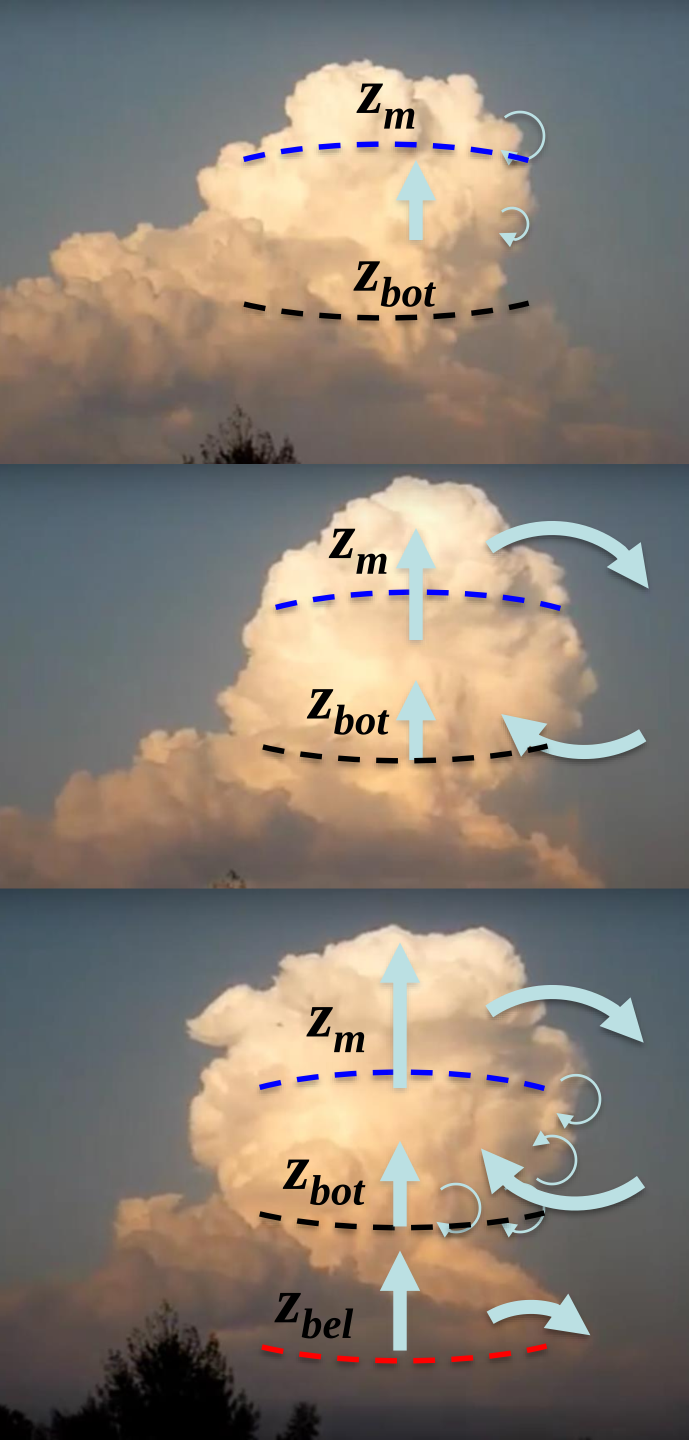
$$\Phi = \frac{1}{z} \int_{LFC}^{z} \frac{q_{sE} \left(1 - R_H\right)}{\Gamma T_E} dz$$



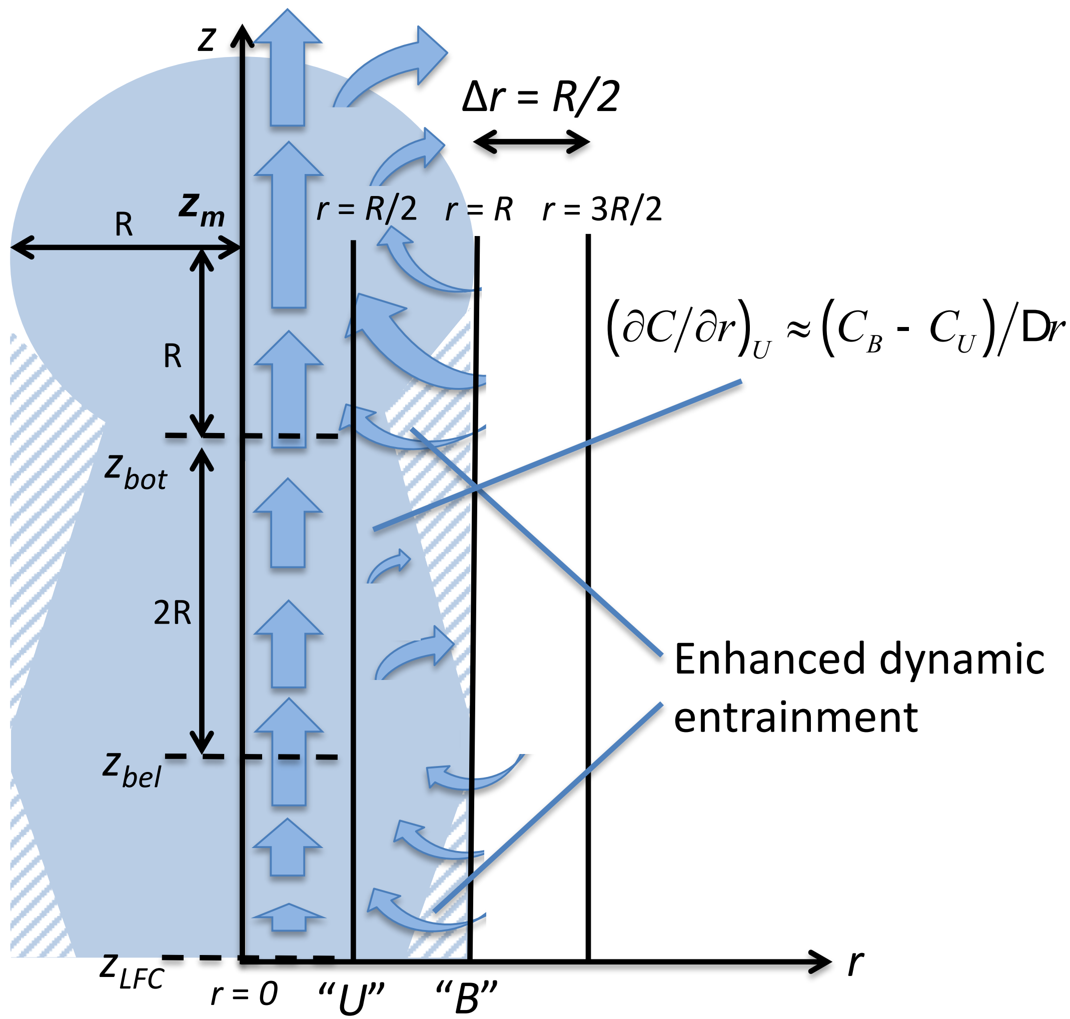
Vertical velocity at z_m

- **Similar procedure compared to scalar mixing, but accounts for:**
  - **1) dilution of buoyancy**
  - **2) dilution of momentum**
  - **3) buoyant perturbation pressure effects**

$$\frac{\left(\frac{\ldots gk^2 L F}{\ldots R^2 CAPE}\right)\left(\frac{P_r R^2 z}{k^2 L} - \frac{P_r^2 R^4}{k^4 L^2}\ln\left(\frac{k^2 L z}{P_r R^2}+1\right)\right)}{\left(1+\frac{a^2 R^2}{H^2}+\frac{4k^2 L z}{3R^2}\right)}$$


z m
z bot
z m
z bot
z m
z bot
z bel

Now for z_{bot}…


z
Δr = R/2
R
z m
r = R/2
r = R
r = 3R/2
R
$\left(\partial C/\partial r\right)_U \approx \left(C_B - C_U\right)/\Delta r$
z bot
2R
z bel
Enhanced dynamic entrainment
z LFC
r = 0
"U"
"B"
r

Assuming a linear horizontal profile of u such that $u_U \sim u_B/2$ (since $u = 0$ at $r = 0$), $C_B = 0$, combining with the horizontally-averaged mass continuity equation, approximating vertical derivatives as $\frac{\partial \varphi}{\partial z} \approx \frac{\varphi_z - \varphi_{LFC}}{z - z_{LFC}}$, and with boundary conditions $w_U = 0$ and $C_U = C_0$ at the LFC gives

$$\boxed{C_U \approx \frac{2}{3} C_0}$$

All else equal, this suggests an increase in horizontal gradients between $r = 0$ and $r = R/2$ at the thermal bottom by a factor of 3/2 that is self-similar, i.e. does not depend on R, w, z – *this increases lateral mixing of environmental and updraft air at $r = 0$ and z_{bot}*

Assuming a similar scaling applies to *w* (though not a passive scalar) gives an increase in the lateral turbulent mixing of 9/4 at *r* = 0, since mixing is proportional to $\frac{\partial C}{\partial r}\left|\frac{\partial w}{\partial r}\right|$.

Similar approach is applied to give B_{bot} and w_{bot}…

$$C = \frac{1 - 9k^2 Lz/(4P_r R^2)}{1 + 9k^2 Lz/(4P_r R^2)} C_0$$

$$B = \frac{B_{AD}}{1 + \dfrac{9k^2 Lz}{4P_r R_{HMB}^2}} - \frac{9L_v g k^2 L\mathrm{F}}{2c_p\left(\dfrac{P_r R_{HMB}^2}{z} + \dfrac{9k^2 L}{4}\right)}$$

$$w = \sqrt{\frac{2CAPE\left(\left(\dfrac{1}{z^2} - \dfrac{9L_v g k^2 L\mathrm{F}}{4c_p P_r R_{HMB}^2 CAPE}\right)\left(\dfrac{4P_r R_{HMB}^2 z}{9k^2 L} - \dfrac{16P_r^2 R_{HMB}^4}{81k^4 L^2}\ln\left(\dfrac{9k^2 Lz}{4P_r R_{HMB}^2} + 1\right)\right)\right)}{\left(1 + \dfrac{a^2 R_{HMB}^2}{H^2} + \dfrac{3k^2 Lz}{R_{HMB}^2}\right)}}$$

Enhanced mixing and reduced B and w at the thermal bottom in turn leads to reduced (or even negative) $\partial w/\partial z$ below the thermal. With approximation, this is captured with a similar finite differencing to give B and w at z_{bel}:

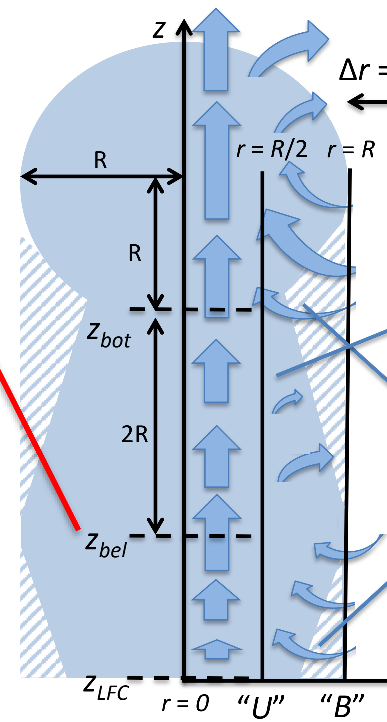
$$C = \frac{1 - \xi k^2 Lz/(P_r R^2)}{1 + \xi k^2 Lz/(P_r R^2)} C_0$$

at the $\left[\left(\frac{w_L}{z_{bel}}\right)^{-1}\left(\frac{w_L}{z_{bel}} + \frac{w_{bot}}{z_{bot}}\right)\right]^2$

$$B_{bel} = \frac{B_{AD}}{1 + \xi k^2 Lz/(P_r R^2)} - \frac{2\xi L_v g k^2 L\Phi}{c_p(P_r R^2/z + \xi k^2 L)}$$

$$B_{bel} = \frac{B_{AD}}{1 + \xi k^2 Lz/(P_r R^2)} - \frac{2\xi L_v g k}{c_p(P_r R^2/z}$$

17



Method is repeated for multiple thermals…

Comparison with numerical modeling (axisymmetric CM1)

- **100 m grid spacing (vertical and horizontal), enhanced Smagorinsky-type sub-grid scale mixing (*horizontal* mixing length *L* = 500 m)**

- **Convection initiated with applying warm bubbles of varying radius (600 to 2000 m)**

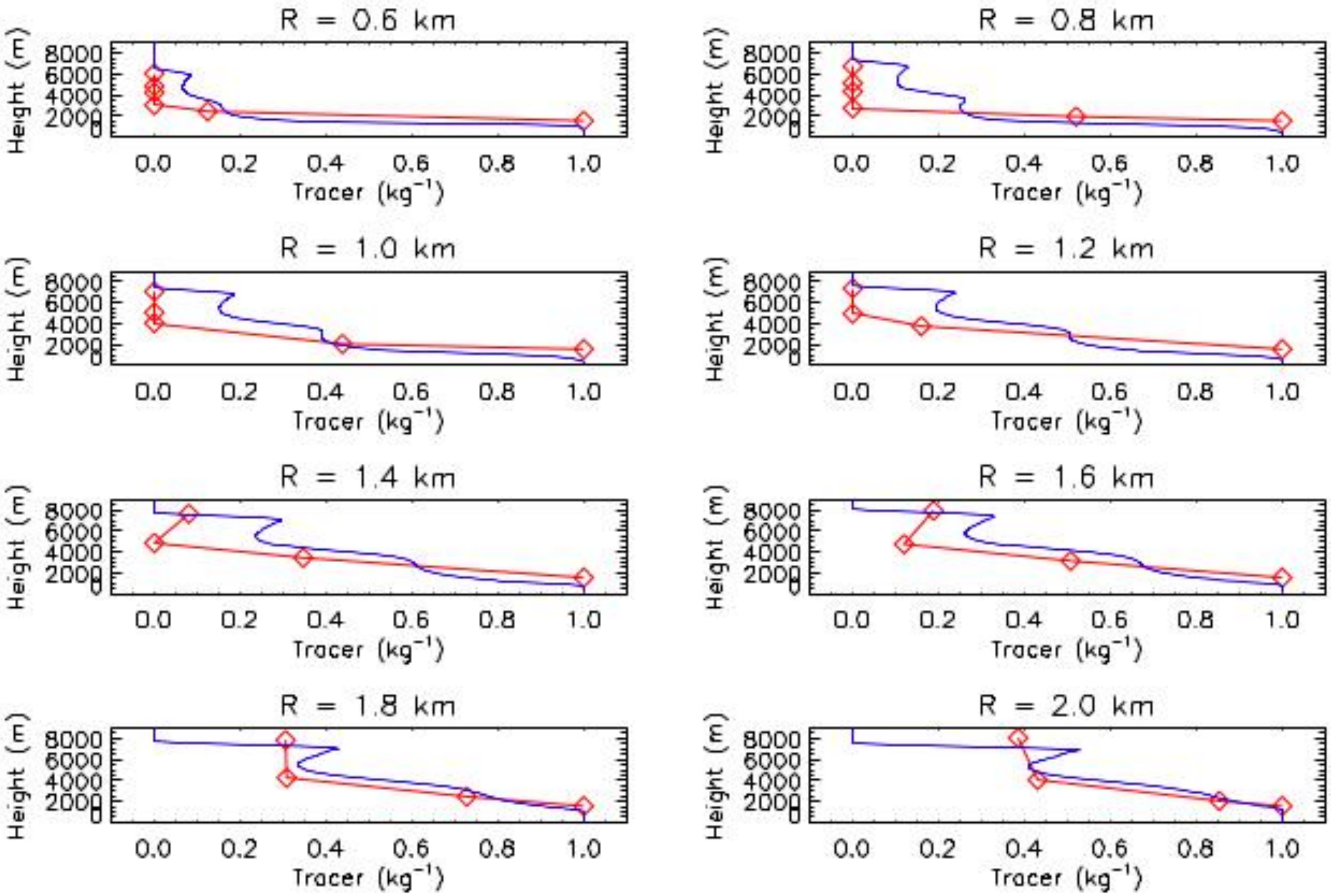
- **Environmental (above level of free convection) *RH* of 42.5% or 85%**

- **Weisman-Klemp initial sounding (except RH changes), no environmental shear**

- **Passive tracer set to 1 in the lowest 1.5 km**

Comparison of numerical and analytic solutions

Analytic
Numerical



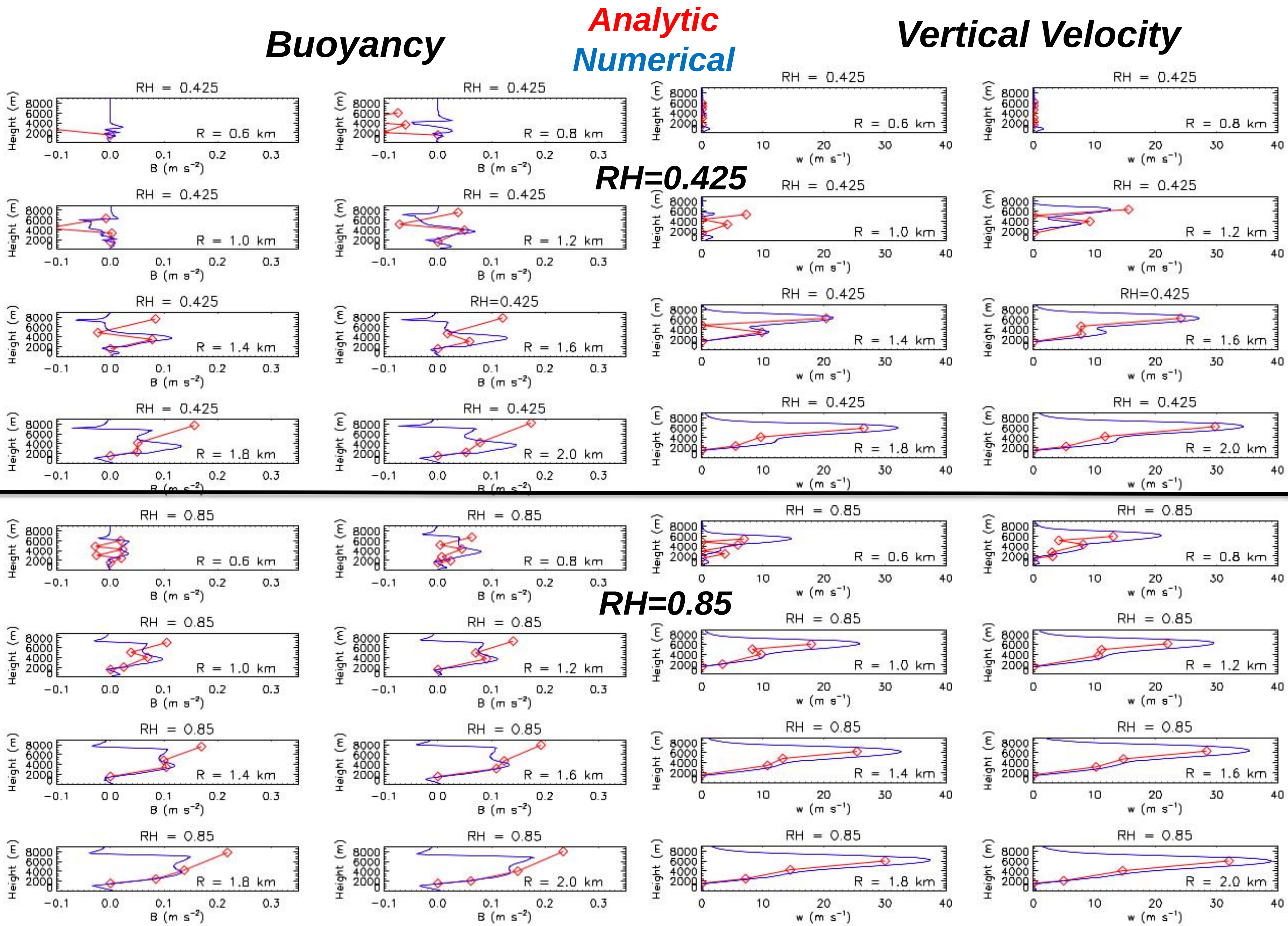
Passive Tracer

RH=0.85

(but similar results for RH=0.425)

Results for each panel are when z_m ~ 6 km.

Comparison of numerical and analytic solutions



Main findings:

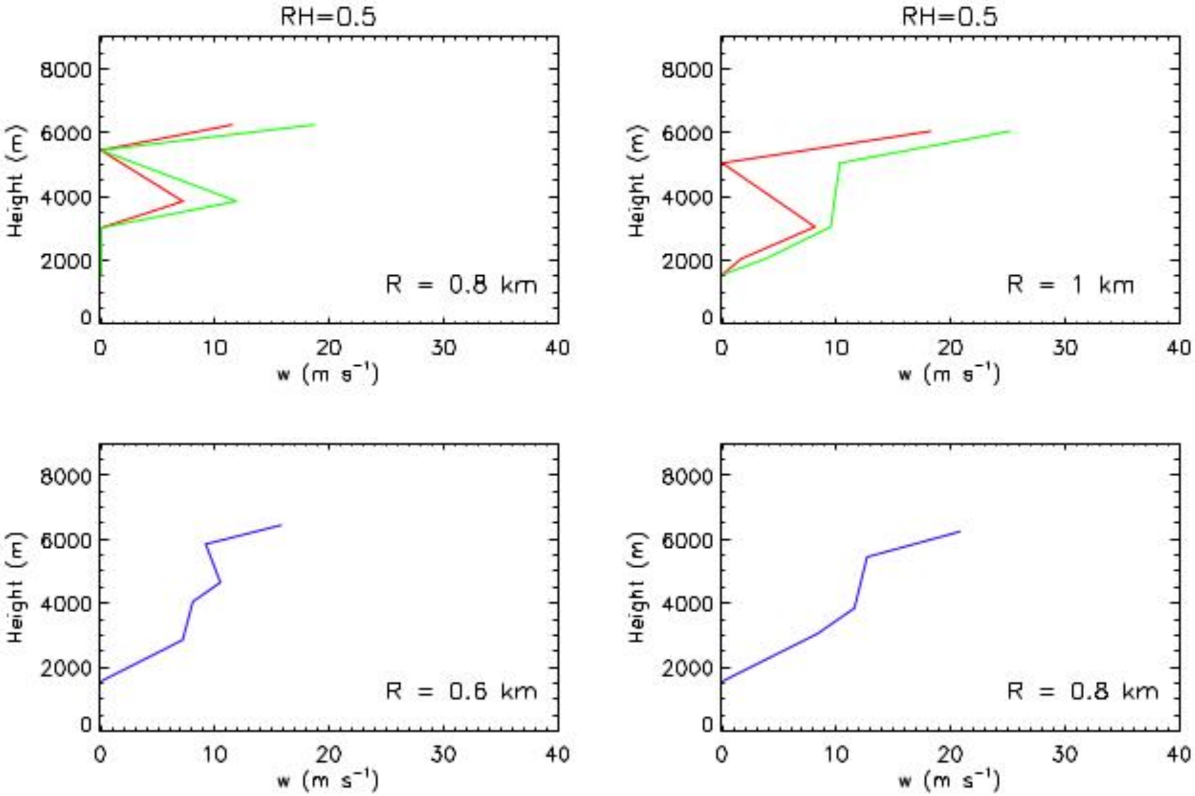
1. Analytic and numerical models show a continuum behavior between *thermal* and *plume* structures, determined by updraft zL/R^2, environmental *RH*, and *CAPE*

2. The ***thermal chain*** is a transitional regime between *plumes* and *thermals*, has features of both but also unique characteristics. Occurs over a wide range of conditions.

3. The thermal chain structure arises directly from solutions of the governing equations for *moist* convection. Results suggest interactions between entrainment, evaporation, buoyancy, and flow structure are critical for thermal chains.

EXTRA

Vertical Velocity



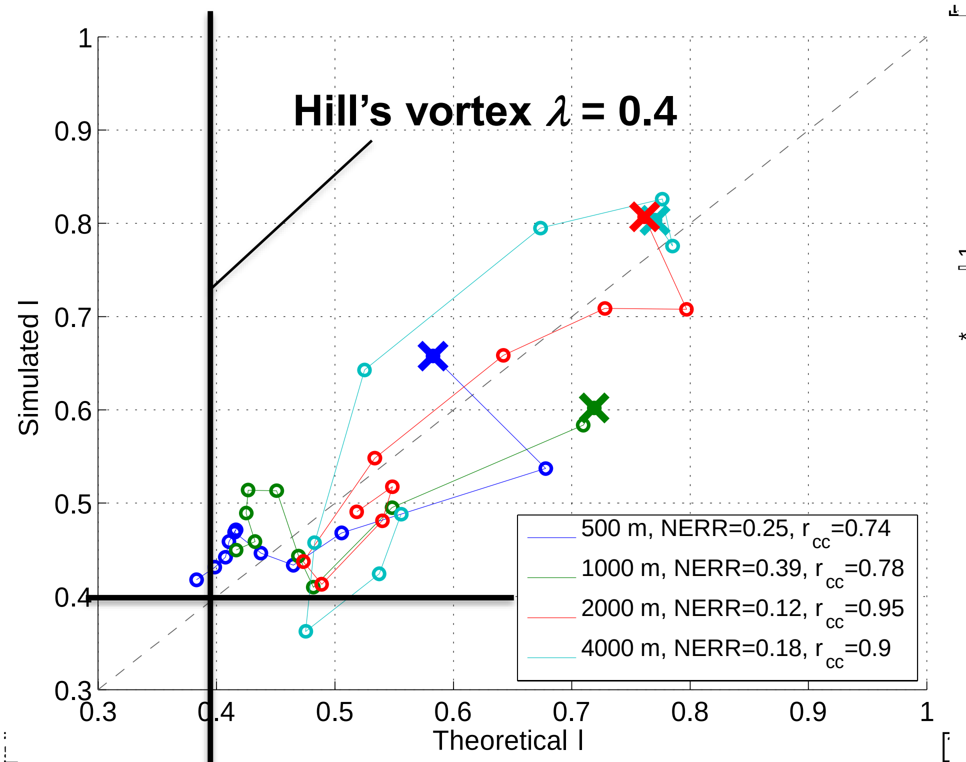
Standard WK
WK CAPE x 1.5
No evaporation

- **Increasing *CAPE* → transition from <u>isolated thermal</u> to <u>thermal chain</u> to <u>plume</u> occurs at smaller *R*.**

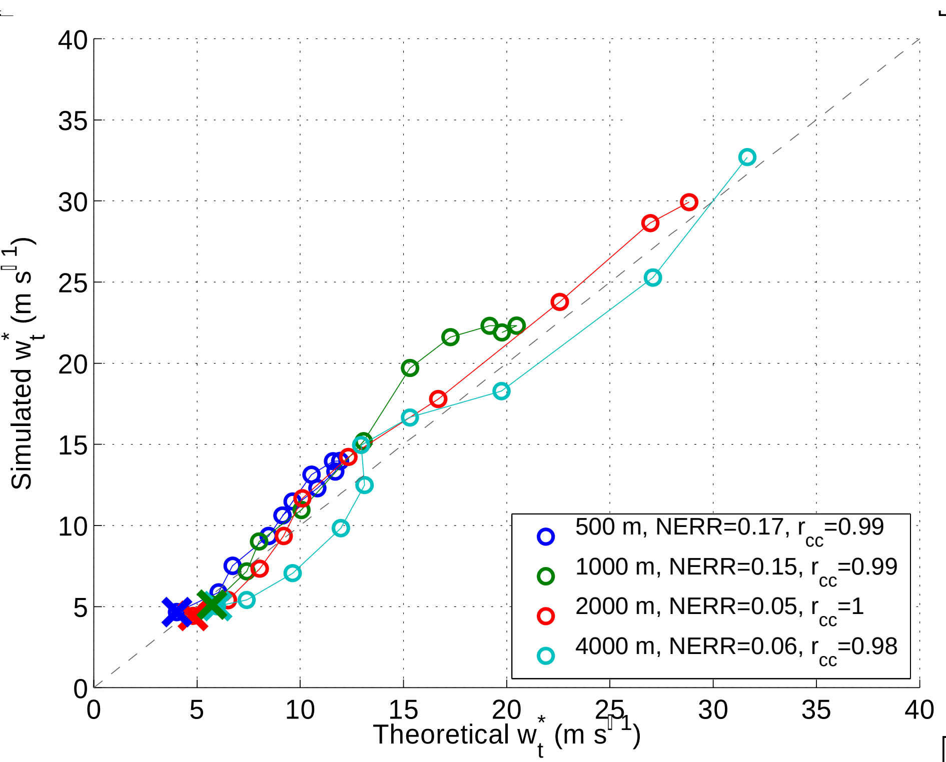
- **Interactions between entrainment, evaporation, buoyancy, and flow are *critical* for thermal chain structure.**

- By extending Hill's vortex solution to include buoyancy, we derive an analytic expression for the *ratio of thermal ascent rate and maximum vertical velocity, λ,* that is a quadratic function of two non-dimensional buoyancy parameters (Morrison and Peters 2018, *JAS*)

Numerically simulated vs. theoretical *ratio* of *thermal ascent rate* and *maximum vertical velocity* within the thermal

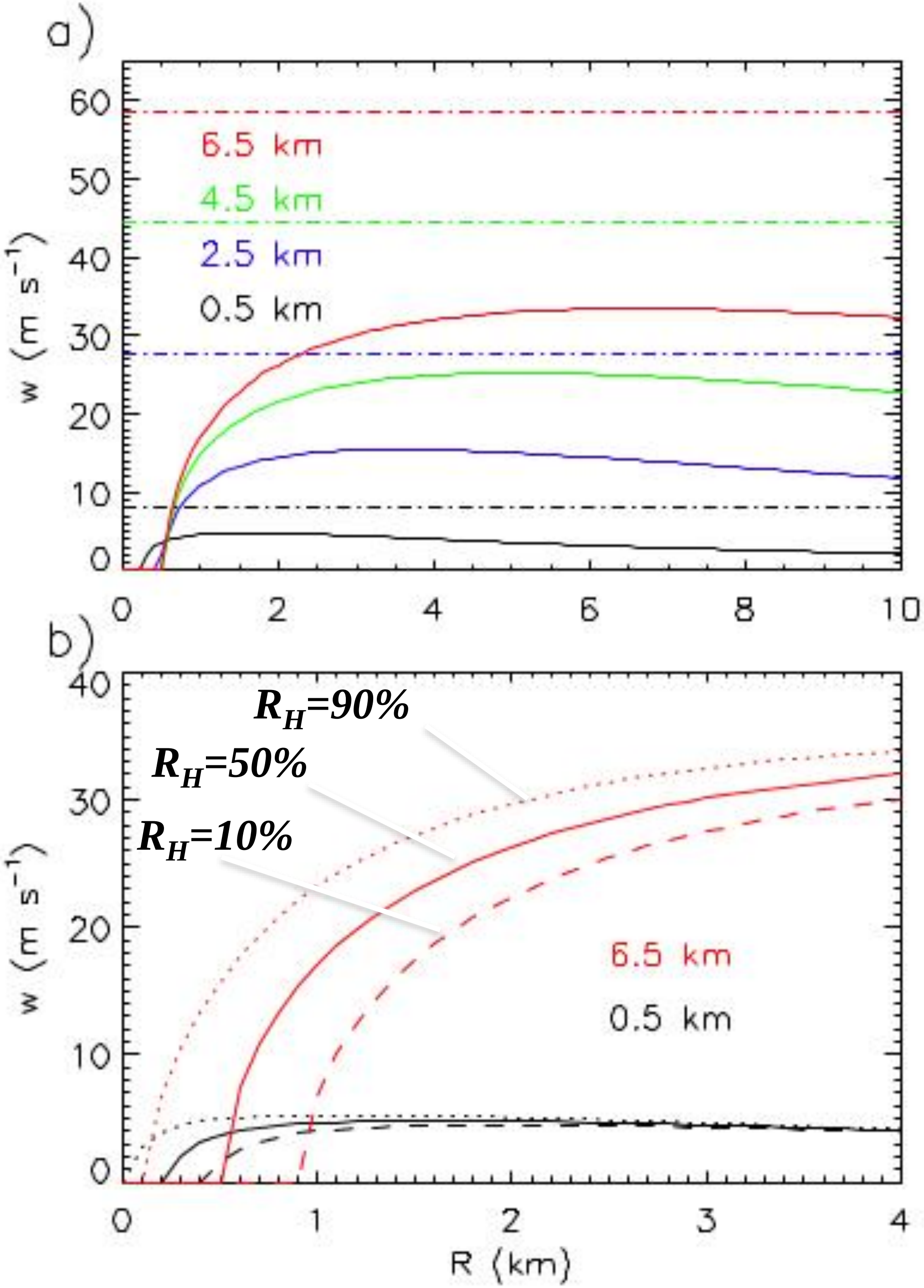


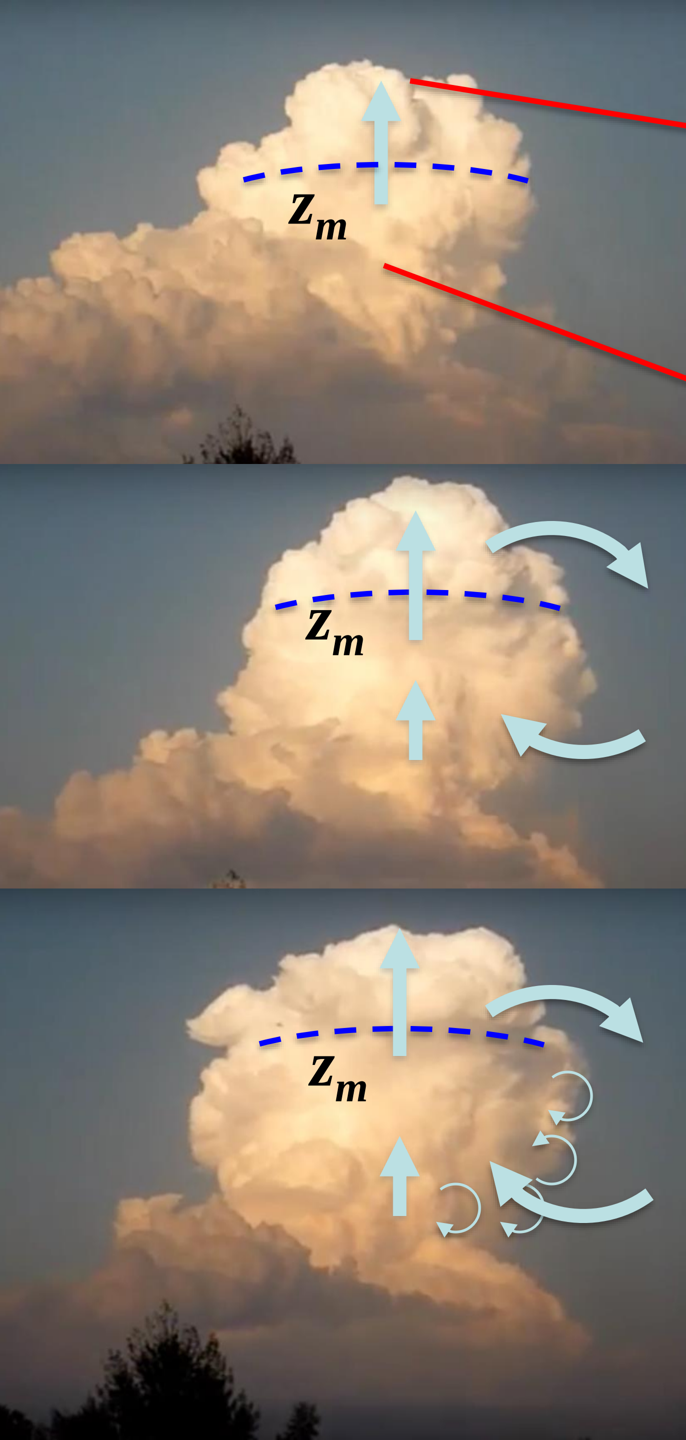
Numerically simulated vs. theoretical *thermal ascent rate*

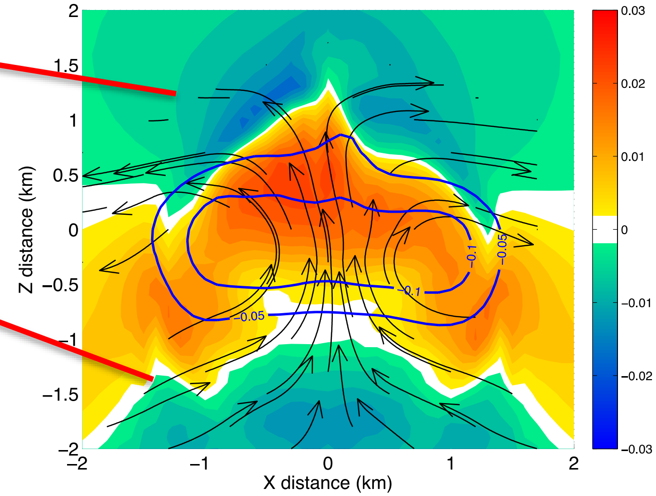


Analytic vertical velocity as a function of *R*

(R_H = 50%, $L \sim R$)



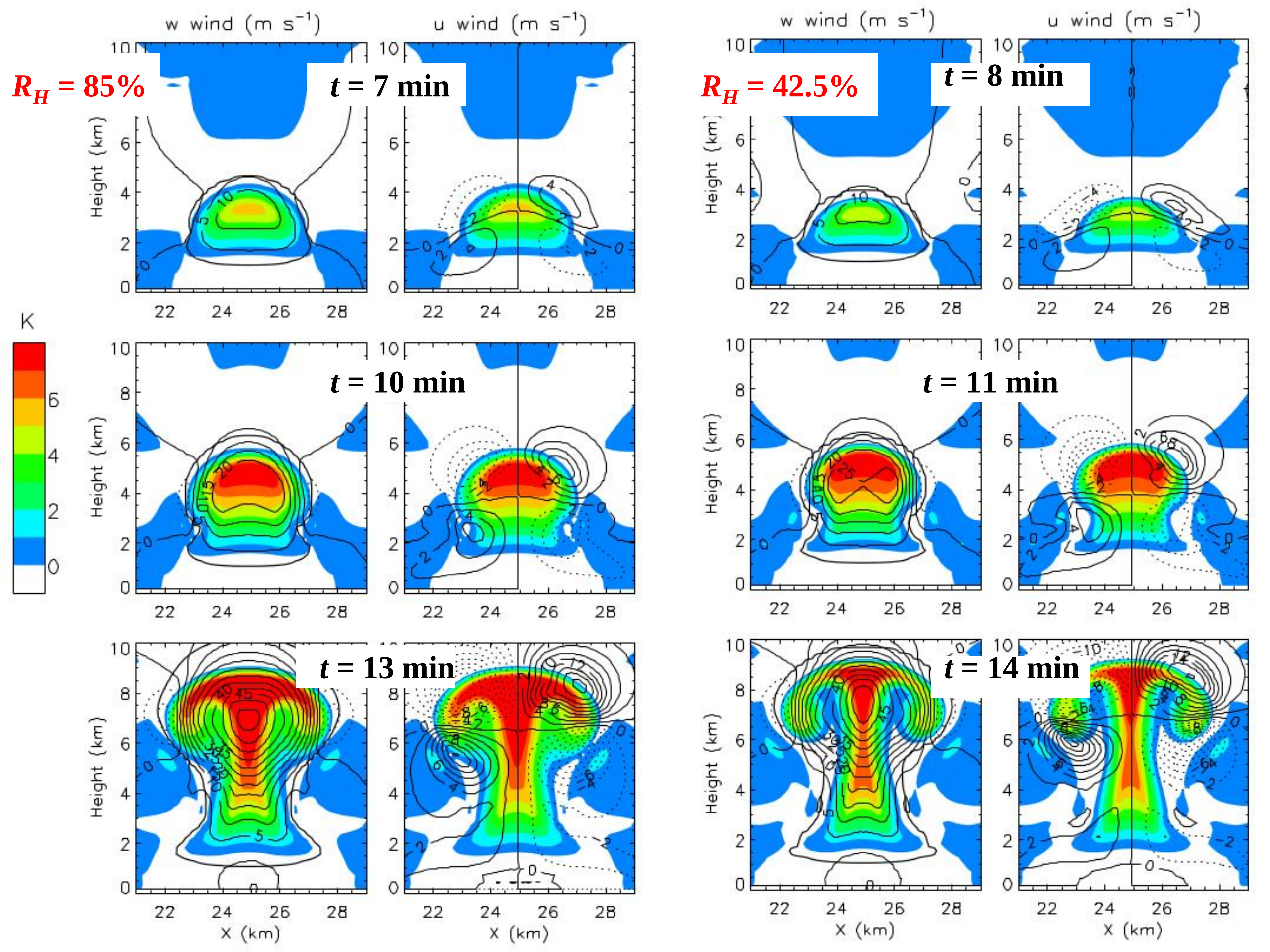




Hill's vortex-like flow in the upper turret with maximum *w* in the vortex center at height z_m.

u ~ 0 near z_m → limited impact of dynamic entrainment from organized convective-scale flow at this height.

Below z_m dynamic entrainment is important → inflow of environmental air leads to engulfment and mixing, decreasing *R* and sharpening horizontal gradients – this in turn increases lateral turbulent mixing at the updraft center.



w wind (m s^{-1})
u wind (m s^{-1})
R_H = 85%
t = 7 min
t = 10 min
t = 13 min
R_H = 42.5%
t = 8 min
t = 11 min
t = 14 min
Height (km)
X (km)
K