Bifurcation Analysis of Periodic Movement Coordination

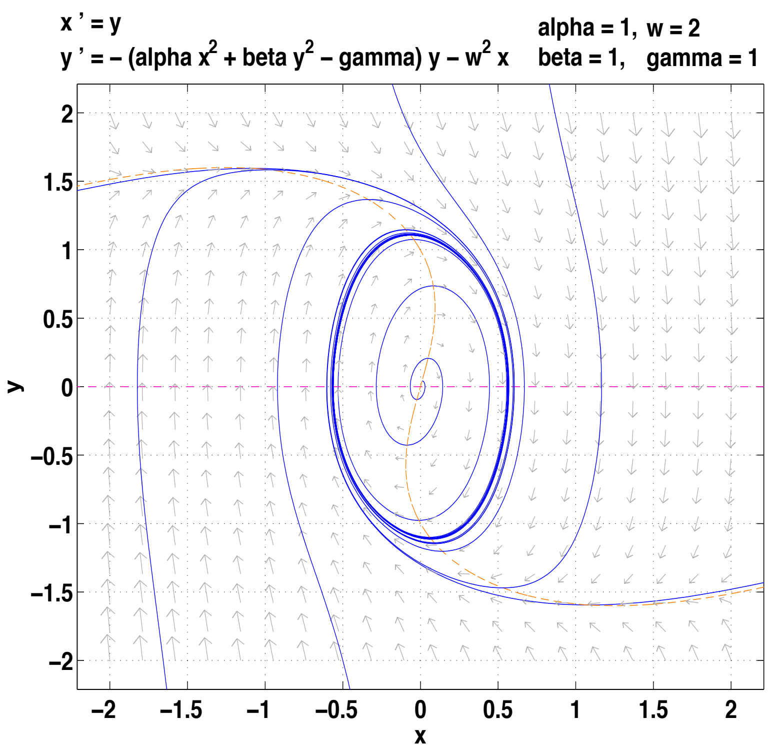
Ed Rooke
University of Bristol, United Kingdom
Benoit Bardy
Université Montpellier-1, France
Krasimira Tsaneva-Atanasova
University of Bristol, United Kingdom

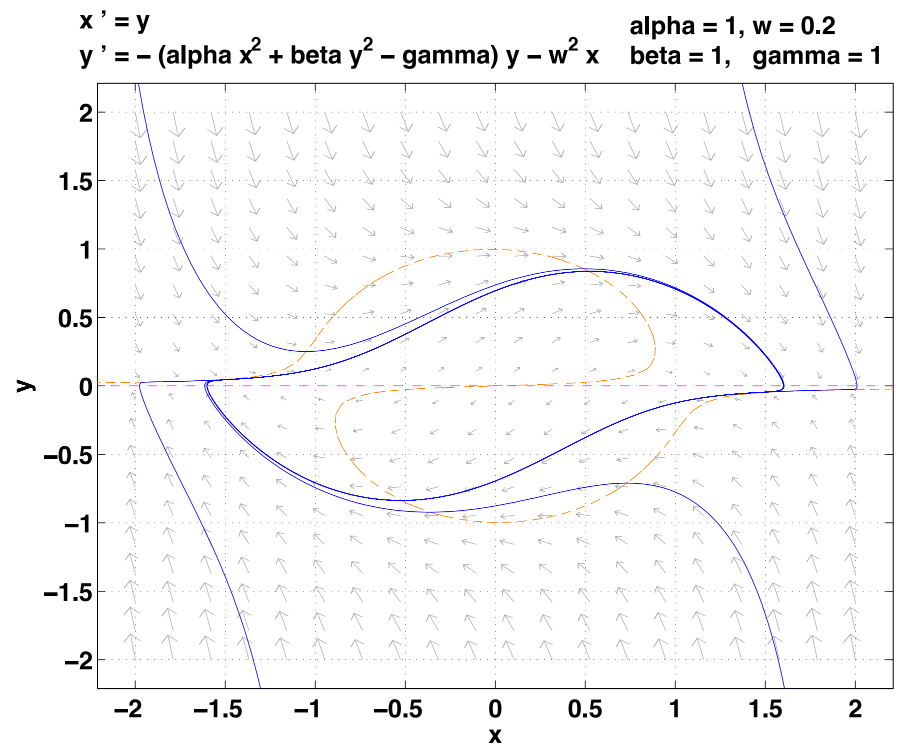
HKB Oscillator

$$\ddot{x} + (\alpha x^2 + \beta \dot{x}^2 - \gamma)\dot{x} + \omega^2 x = 0$$

$$\dot{x} = y$$

$$\dot{y} = -(\alpha x^2 + \beta y^2 - \gamma)y - \omega^2 x$$





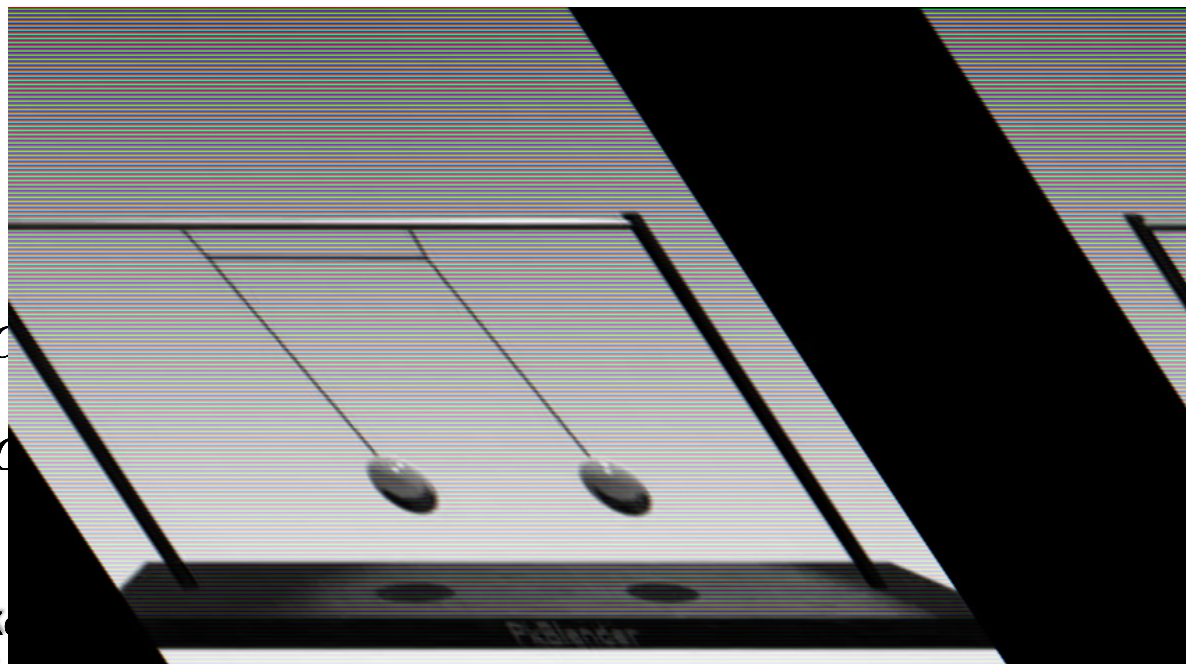
Haken, H., Kelso, J. A., & Bunz, H. (1985). A theoretical model of phase transitions in human hand movements. Biological cybernetics, 51(5), 347-56.

HKB Model

$$\ddot{x}_1 + (\alpha_1 x_1^2 + \beta_1 \dot{x}_1^2 - \gamma_1)\dot{x}_1 + \omega_1^2 x_1 = I_{12},$$
$$\ddot{x}_2 + (\alpha_2 x_2^2 + \beta_2 \dot{x}_2^2 - \gamma_2)\dot{x}_2 + \omega_2^2 x_2 = I_{21},$$

$$I_{12} = (a_1 + b_1(x_1 - x_2)^2)(\dot{x}_1 - \dot{x}_2),$$
$$I_{21} = (a_2 + b_2(x_2 - x_1)^2)(\dot{x}_2 - \dot{x}_1),$$

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = x_4$$
$$\dot{x}_3 = -(\alpha \; \ldots \; x_3 - x_4)$$
$$\dot{x}_4 = -(\alpha \; \ldots \;)(x_4 - x_3)$$



Haken, H., K... an hand movements. ...

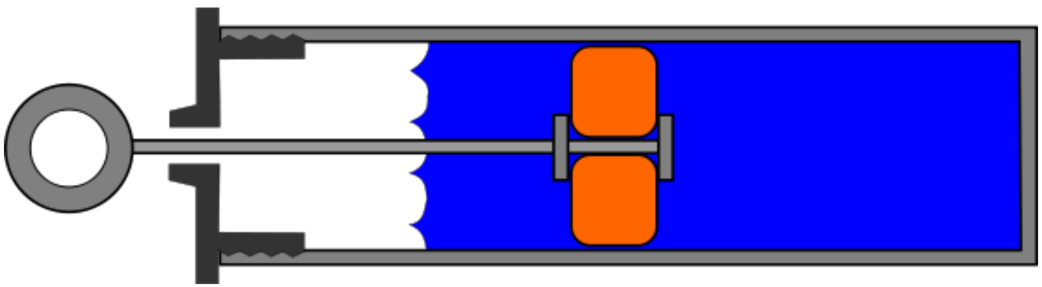
HKB

$$I_{12} = a\,(\dot{x}_1 - \dot{x}_2)\text{-}\ x_2)^2)(\dot{x}_1 - \dot{x}_2)$$

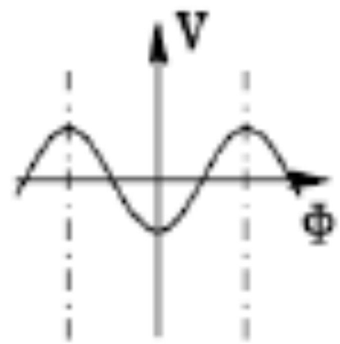
Linear Daspot

if $b = 0$

$$acceleration = \ddot{x} = a(\dot{x}_1 - \dot{x}_2)$$



$$V(\phi) = \frac{a(r_1^2 + r_2^2)}{2r_1 r_2}\cos\phi$$



[Kuramoto, Yoshiki (1975), *Lecture Notes in Physics, International Symposium on Mathematical Problems in Theoretical Physics*, 39, Springer-Verlag, New York, p. 42.]

[Kijima A, Kadota K, Yokoyama K, Okumura M, Suzuki H, et al. (2012) Switching Dynamics in an Interpersonal Competition Brings about "Deadlock" Synchronization of Players. PLoS ONE 7(11): e47911. doi:10.1371/journal.pone.0047911]

HKB

$$I_{12} = (a + b(x_1 - x_2)^2)(\dot{x}_1 - \dot{x}_2)$$

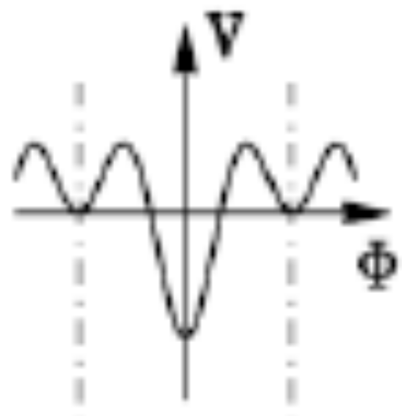
if $b \neq 0$

Linear Damper

Van de Pol Term

$$\dot{x}x^2$$

$$V(\phi) = \frac{r_1^2 + r_2^2}{8}\left(\frac{4a + b(r_1^2 + r_2^2)}{r_1 r_2}\cos\phi - \frac{b}{2}\cos 2\phi\right)$$



The HKB model

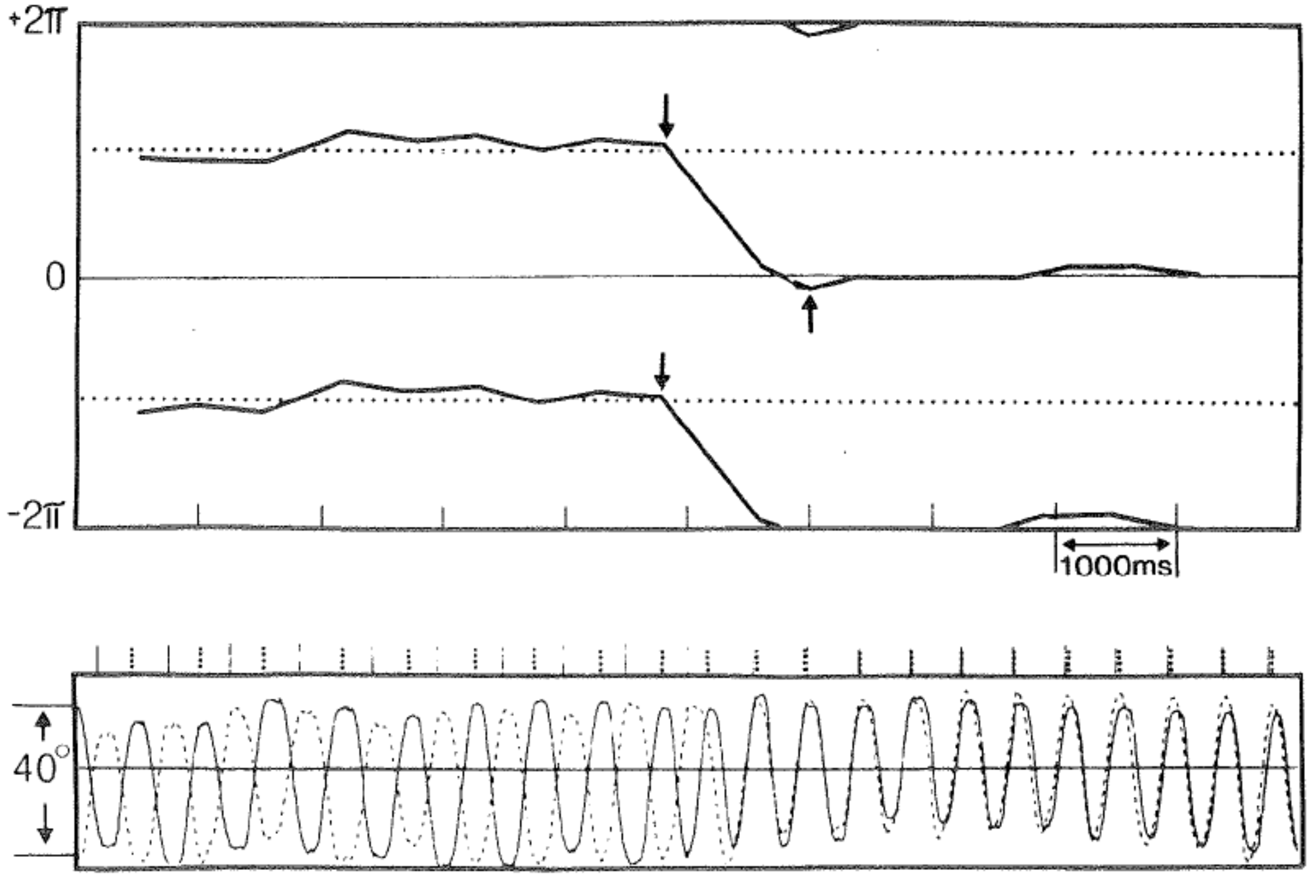
Two patterns of behaviour

- in phase
- antiphase

Haken, H., Kelso, J. A., & Bunz, H. (1985). A theoretical model of phase transitions in human hand movements. Biological cybernetics, 51(5), 347-56.

Phase Transition

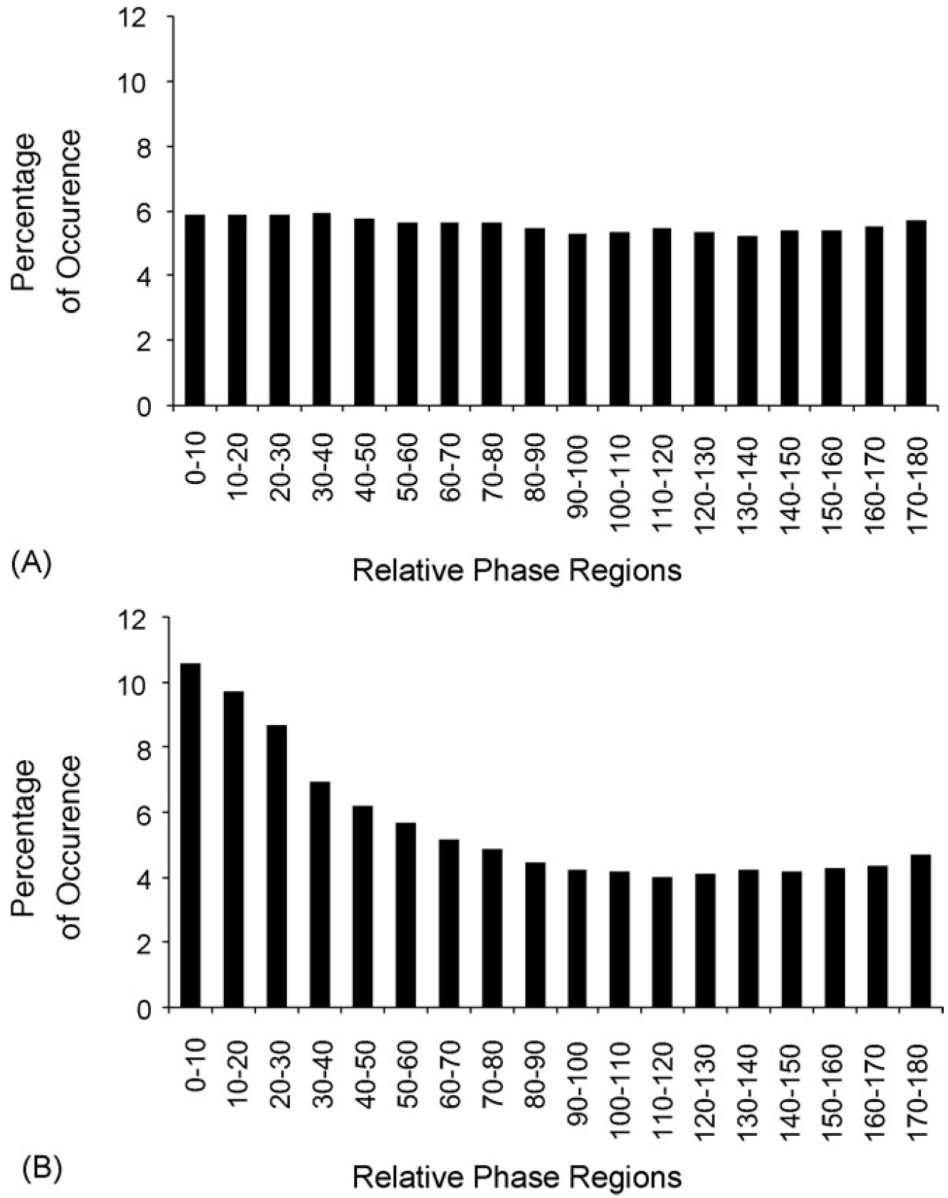
$$\dot{\phi} = -A\sin\phi - 2B\sin 2\phi$$



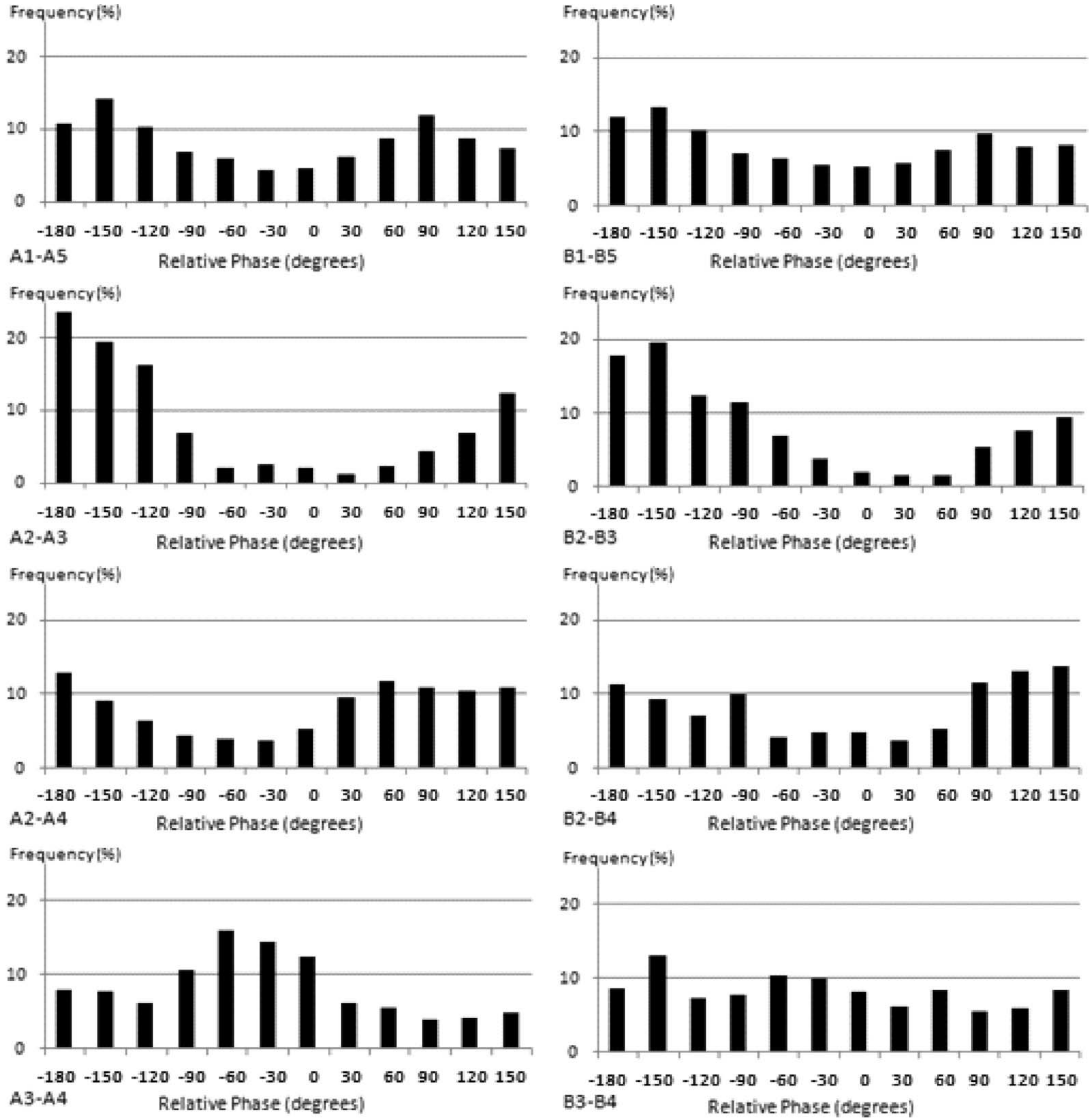
Haken, H., Kelso, J. A., & Bunz, H. (1985). A theoretical model of phase transitions in human hand movements. Biological cybernetics, 51(5), 347-56.

Goals and Questions

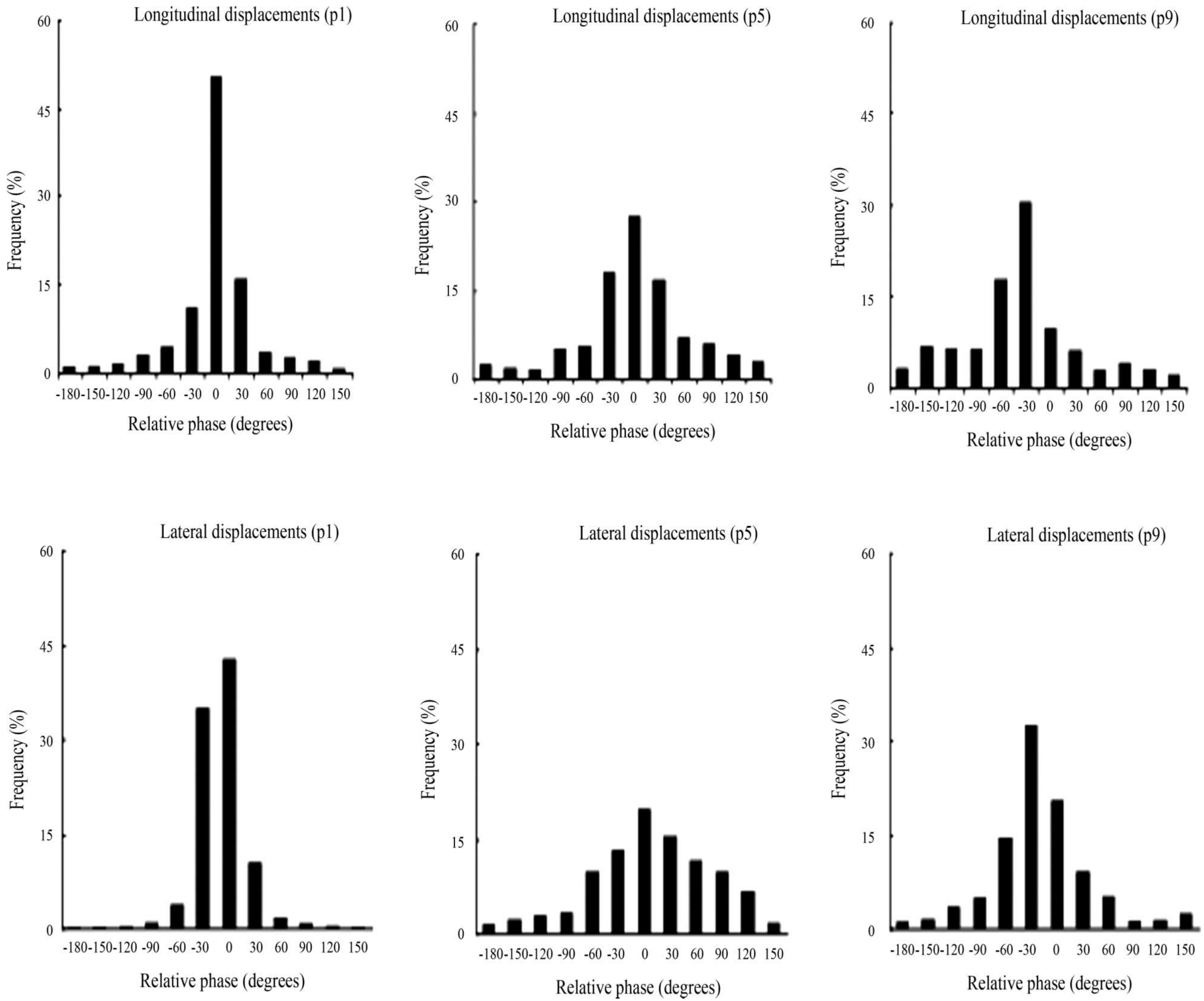
- ✓ **To characterize the dynamical regimes of the HKB model in terms of parameter dependence, stability and transitions between possible states.**
- ✓ **Why do we care about that?**
- ✓ **Why is this important?**



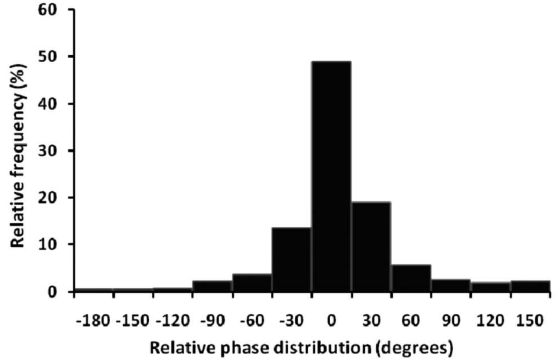
Issartel J, Marin L, Cadopi M. Unintended interpersonal co-ordination: "can we march to the beat of our own drum?", Neurosci Lett. 2007 Jan 16;411(3):174-9.

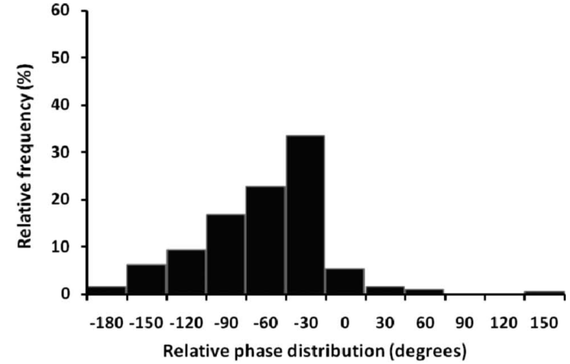


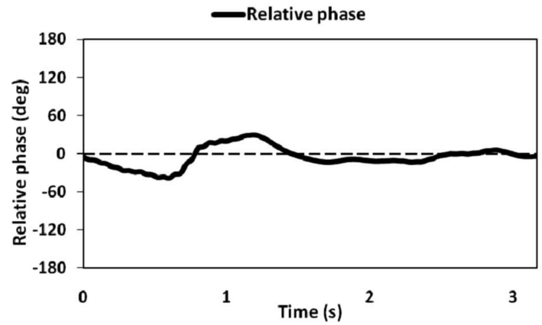
Bourbousson J, Sève C, McGarry T., Space-time coordination dynamics in basketball: Part 1. Intra- and inter-couplings among player dyads, J Sports Sci. 2010 Feb;28(3):339-47.

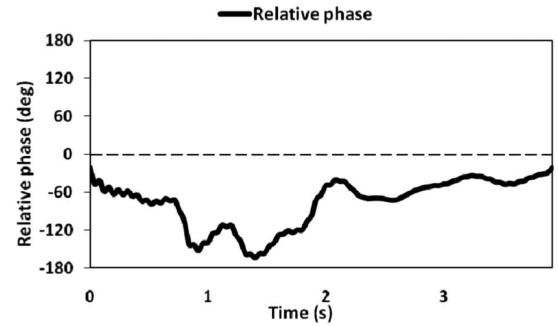


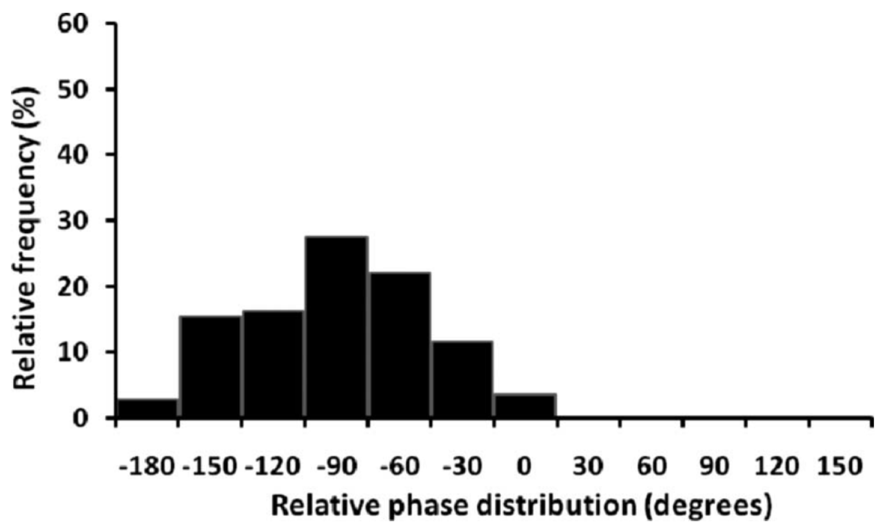
Esteves PT, Araújo D, Davids K, Vilar L, Travassos B, Esteves C., Interpersonal dynamics and relative positioning to scoring target of performers in 1 vs. 1 sub-phases of team sports, J Sports Sci. 2012;30(12):1285-93.





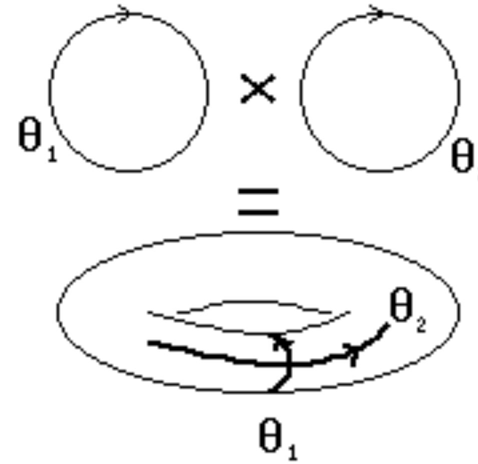


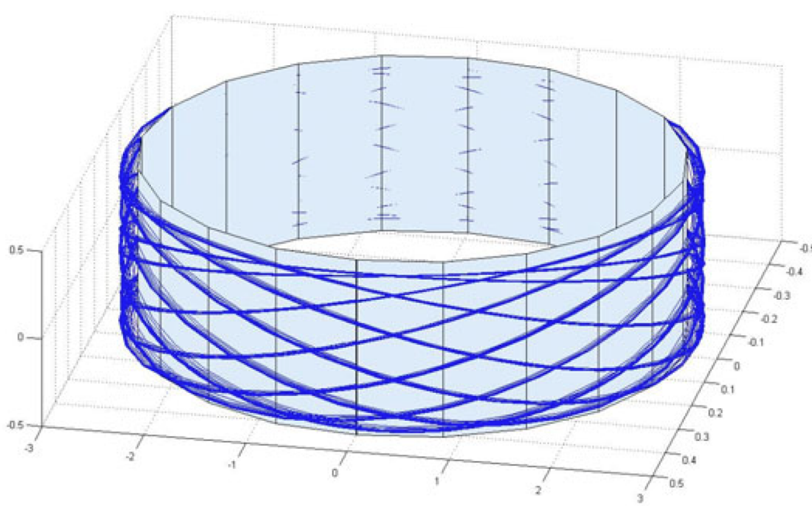


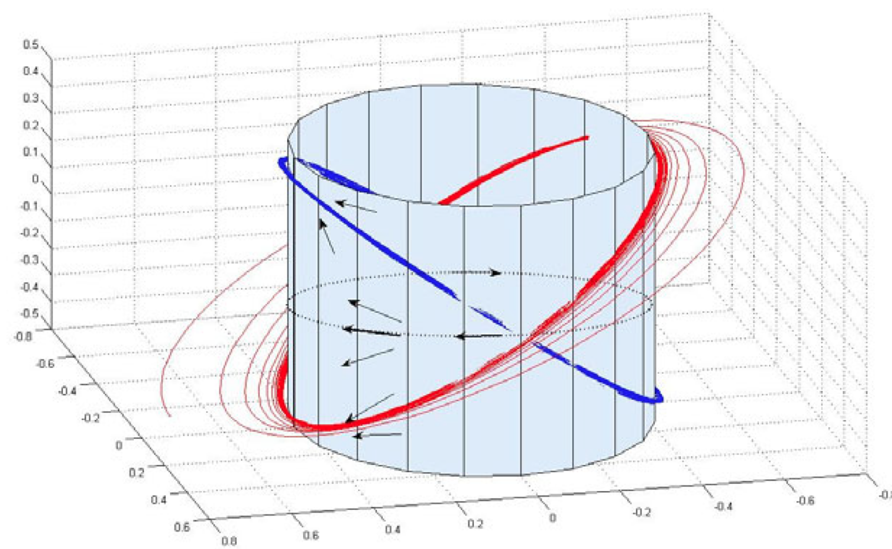


Duarte R, Araújo D, Davids K, Travassos B, Gazimba V, Sampaio J., Interpersonal coordination tendencies shape 1-vs-1 sub-phase performance outcomes in youth soccer, J Sports Sci. 2012 May;30(9):871-7.

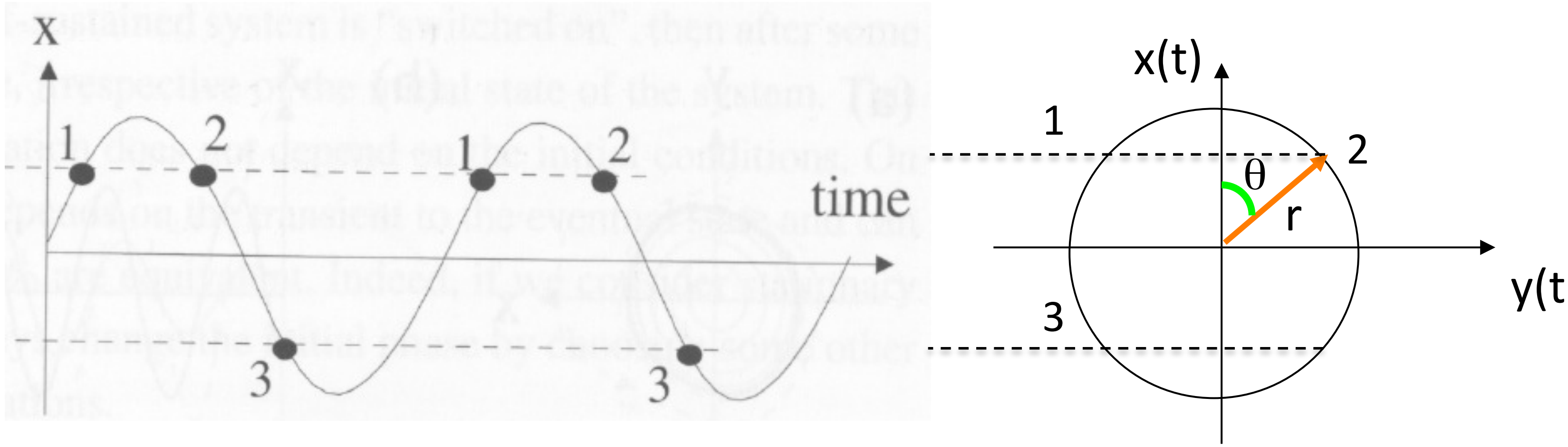
- Solution of oscillator is a circle (S^1)
  Solution of two coupled oscillators is on a torus ($S^1XS^1=T^2$)







Calvin S, Jirsa V: Perspectives on the Dynamic Nature of Coupling in Human Coordination. In: Nonlinear Dynamics in Human Behavior. Volume 328, edn. Edited by Huys R, Jirsa V: Springer Berlin Heidelberg; 2011: 91-114.



Assume $x_k(t) = r_k(t)\cos\left(\omega t + \theta_k(t)\right)$

and $\dot{x}_k(t) = -r_k(t)\omega\sin\left(\omega t + \theta_k(t)\right)$

Requires $-r_k(t)\dot{\theta}_k(t)\sin\left(\omega t + \theta_k(t)\right) + \dot{r}_k(t)\cos\left(\omega t + \theta_k(t)\right) = 0$

(where $k = 1, 2$)

Substitute in the HKB equations and solve for $\dot{r}_1$, $\dot{r}_2$, $\dot{\theta}_1$,and $\dot{\theta}_2$

Integrate each of these over a period $\dfrac{2\pi}{\omega}$

To find the fixed points, set equal to zero and solve for r_1, r_2, and $\phi = \theta_1 - \theta_2$

HKB model in polar coordinates

(1)

$$\dot{r}_1 = \frac{1}{8}\left(r_1(4(a+\gamma)+r_1^2(b-\alpha-3\beta\omega^2)+2br_2^2)-r_2(4a+b(3r_1^2+r_2^2))\cos\phi+br_1r_2^2\cos 2\phi\right)$$

$$\dot{r}_2 = \frac{1}{8}\left(r_2(4(a+\gamma)+r_2^2(b-\alpha-3\beta\omega^2)+2br_1^2)-r_1(4a+b(3r_2^2+r_1^2))\cos\phi+br_2r_1^2\cos 2\phi\right)$$

$$\dot{\phi} = \frac{(r_1^2+r_2^2)}{8}\left(\frac{4a+b(r_1^2+r_2^2)}{r_1r_2}\sin\phi-b\sin 2\phi\right)$$

Assuming that $r_1 = r_2 = r \Rightarrow$

(2)

$$\dot{r} = \frac{1}{8}\left(4r(a+\gamma)-4r(a+br^2)\cos\phi+r^3(3b-\alpha-3\beta\omega^2+b\cos 2\phi)\right)$$

$$\dot{\phi} = \left(a+\frac{br^2}{2}\right)\sin\phi-\frac{br^2}{4}\sin 2\phi$$

Assuming that $r_1 = r_2 = r = const. \Rightarrow$

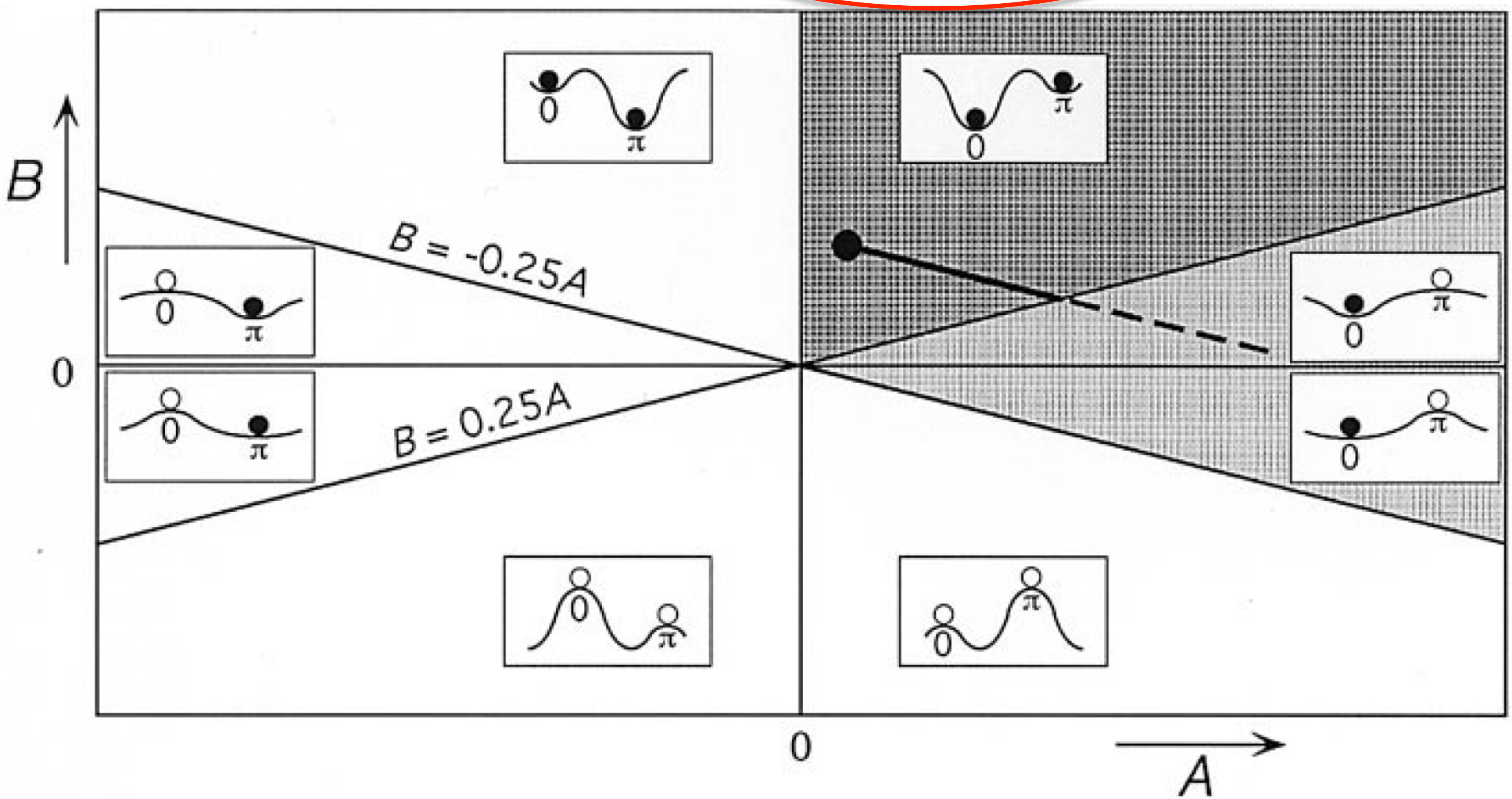
(3)

$$\dot{\phi} = \left(a+\frac{br^2}{2}\right)\sin\phi-\frac{br^2}{4}\sin 2\phi$$

(3)

$$\dot{\phi} = \left(a + \frac{br^2}{2}\right)\sin\phi - \frac{br^2}{4}\sin 2\phi = 0 \Rightarrow$$

$$\phi_1 = 0,\ \phi_2 = \pi,\ \phi_3 = \pm\arccos\left(\frac{2a + br^2}{br^2}\right)$$



Beek PJ, Peper CE, Daffertshofer A. Modeling rhythmic interlimb coordination: beyond the Haken-Kelso-Bunz model. Brain Cogn. 2002 Feb;48(1):149-65.

(2)

$$\dot{r} = \frac{1}{8}\left(4r(a+\gamma) - 4r(a+br^2)\cos\phi + r^3(3b - \alpha - 3\beta\omega^2 + b\cos 2\phi)\right)$$

$$\dot{\phi} = \left(a + \frac{br^2}{2}\right)\sin\phi - \frac{br^2}{4}\sin 2\phi$$

Fixed points:

$\phi = 0,\ r = 0$ - stable if $a < 0$ and $a < -\gamma$

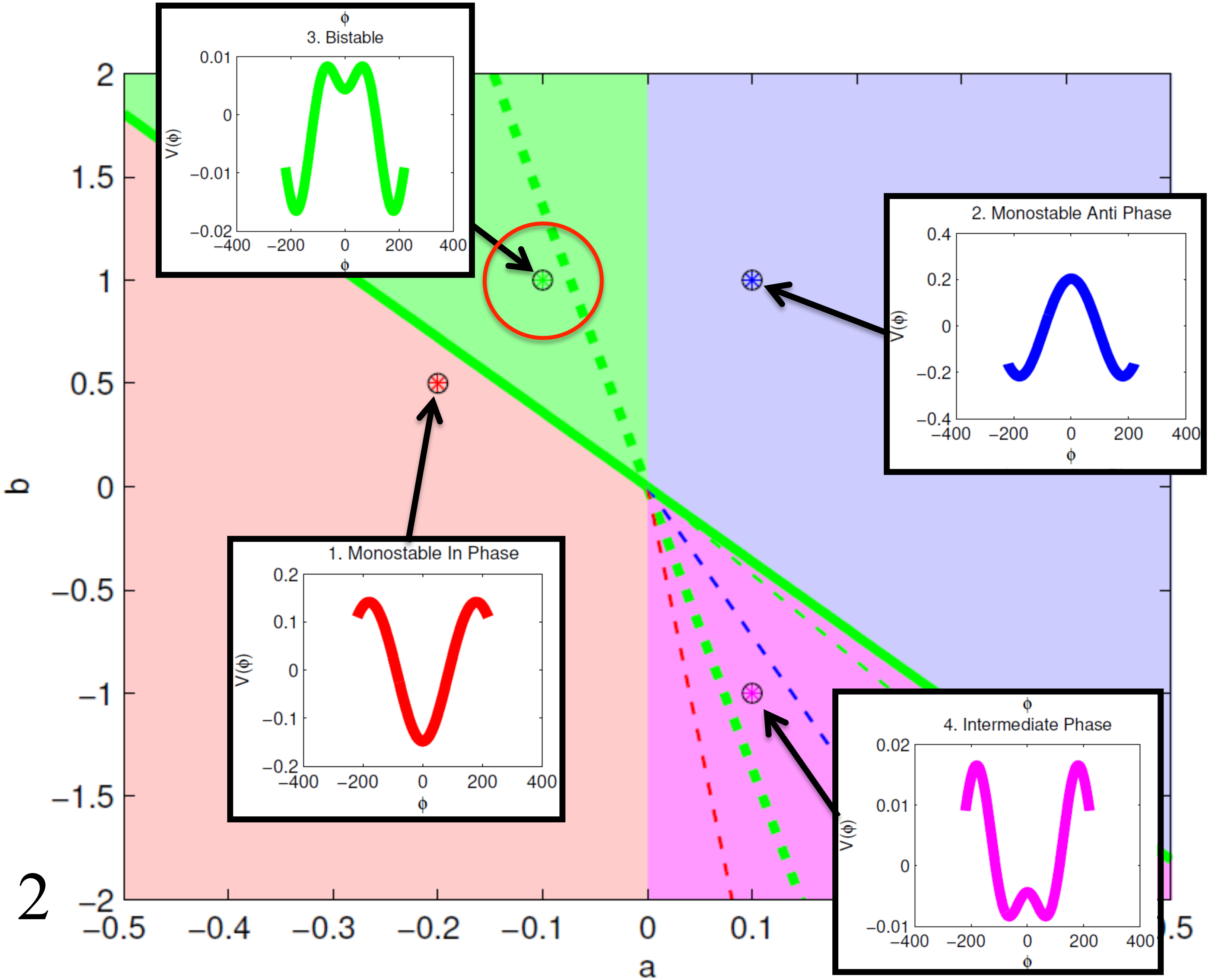
$\phi = 0,\ r = \pm\sqrt{\frac{4\gamma}{\alpha + 3\beta\omega^2}}$ - stable if $a < 0$ and $\frac{4\gamma}{\alpha + 3\beta\omega^2} > 0$

$\phi = \pi,\ r = 0$ - stable if $a > 0$ and $a < \frac{\gamma}{2}$

$\phi = \pi,\ r = \pm\sqrt{\frac{8a + 4\gamma}{\alpha + 3\beta\omega^2 - 8b}}$ - stable if $b > -a\frac{\alpha + 3\beta\omega^2}{4\gamma}$ and $b < \frac{\alpha + 3\beta\omega^2}{8}$

$\phi = \pm\arccos\left(\frac{2a + br^2}{br^2}\right),\ r = 0$ - stable if $-br^2 < a < 0$ and $a < -\gamma$

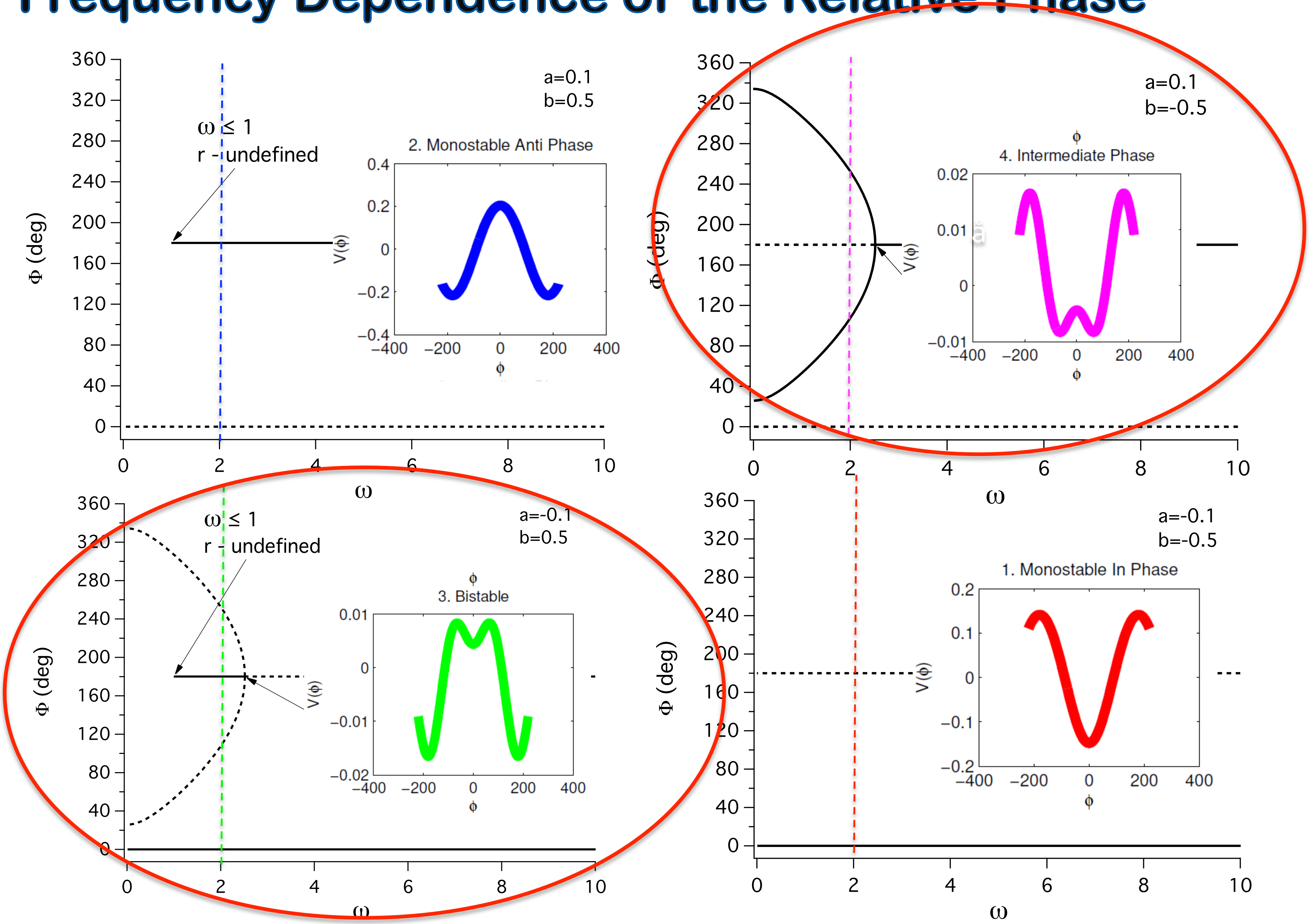
$\phi = \pm\arccos\left(\frac{2a + br^2}{br^2}\right),\ r = \pm\sqrt{\frac{4\gamma}{\alpha + 3\beta\omega^2}}$ - stable if $-br^2 < a < 0$ and $\frac{4\gamma}{\alpha + 3\beta\omega^2} > 0$



3. Bistable
2. Monostable Anti Phase
1. Monostable In Phase
4. Intermediate Phase
V(ϕ)
ϕ
b
a

$\omega = 2$

Frequency Dependence of the Relative Phase



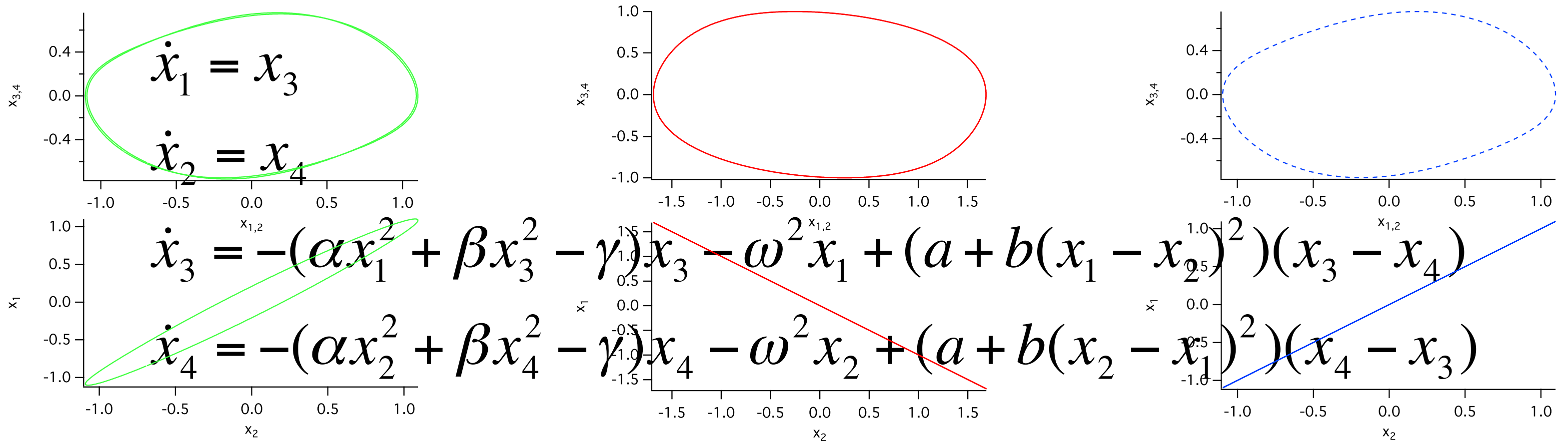
Frequency Dependence of the Relative Phase

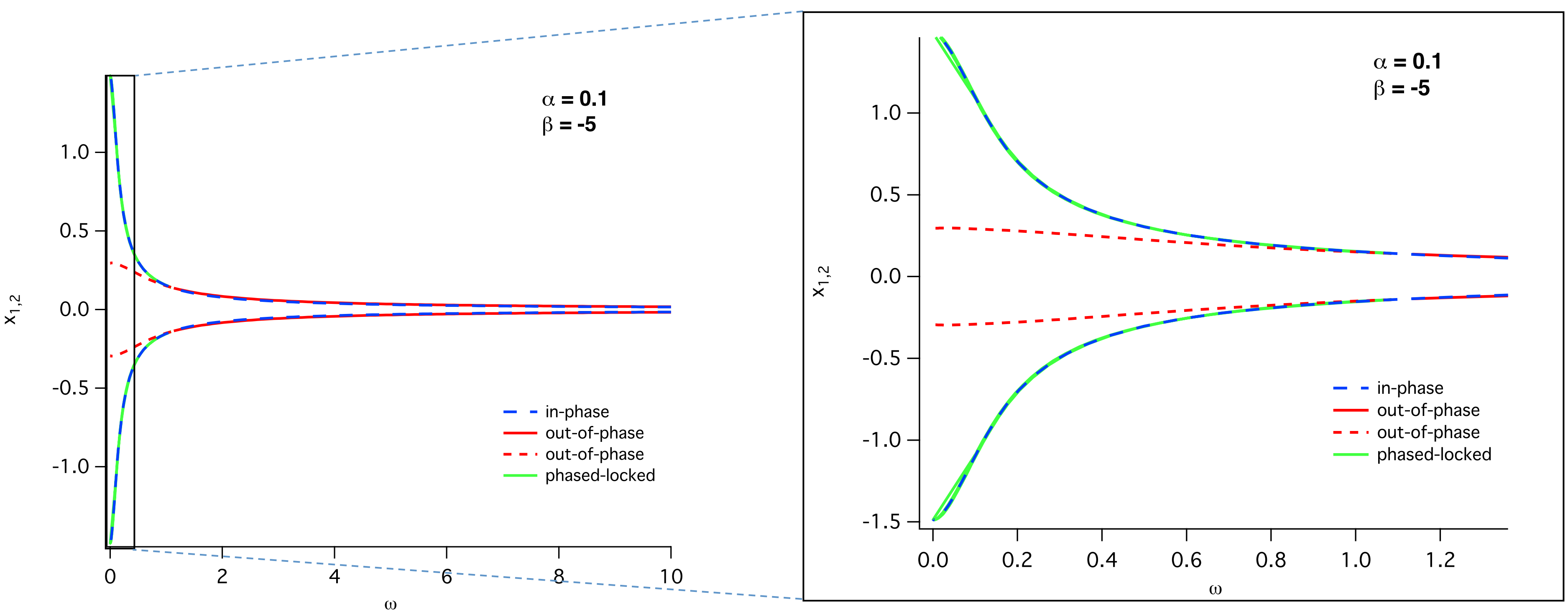
$$\dot{x}_1 = x_3$$

$$\dot{x}_2 = x_4$$

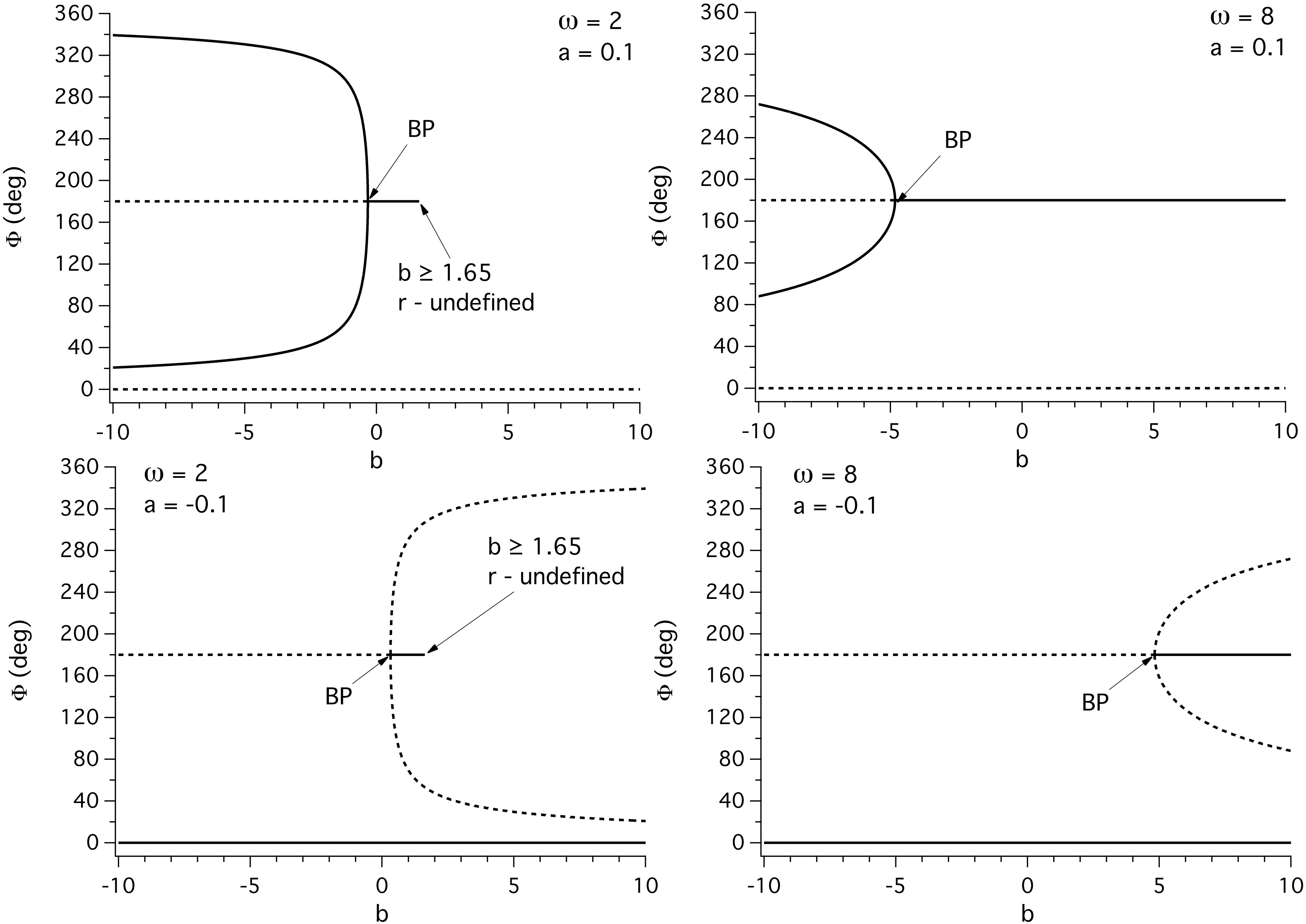
$$\dot{x}_3 = -(\alpha x_1^2 + \beta x_3^2 - \gamma)x_3 - \omega^2 x_1 + (a + b(x_1 - x_2)^2)(x_3 - x_4)$$

$$\dot{x}_4 = -(\alpha x_2^2 + \beta x_4^2 - \gamma)x_4 - \omega^2 x_2 + (a + b(x_2 - x_1)^2)(x_4 - x_3)$$

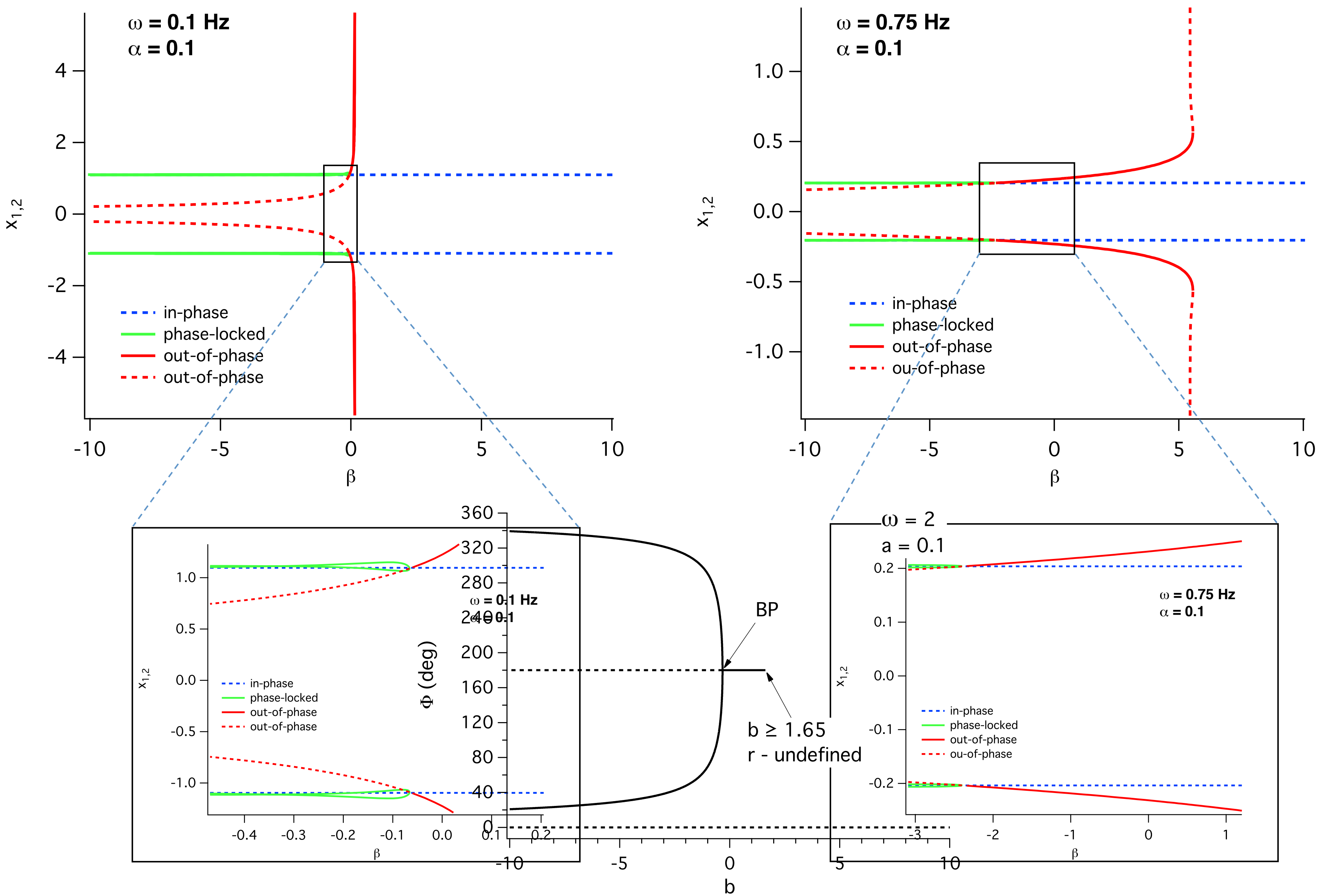




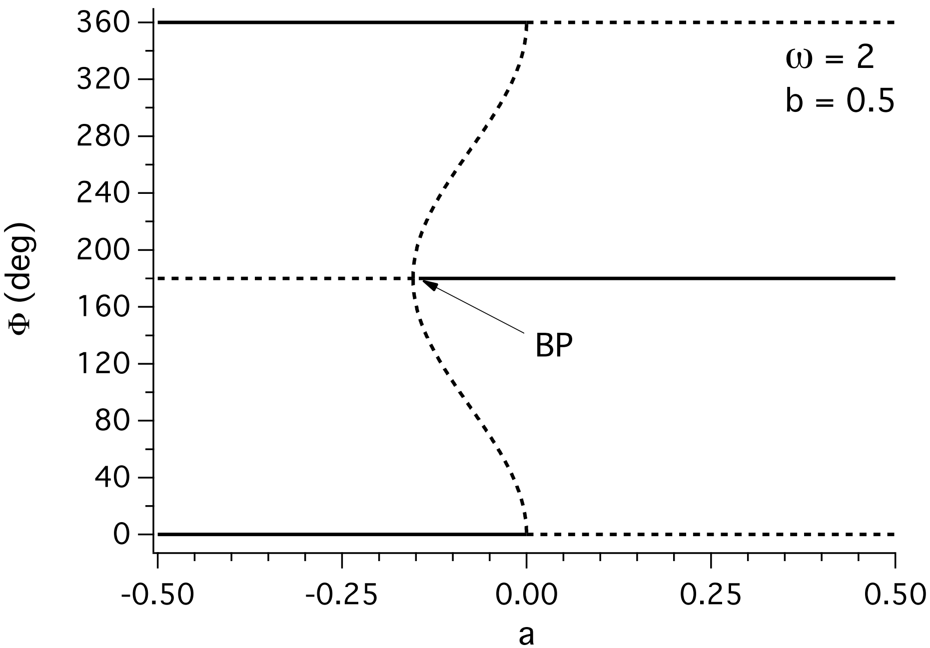
Dependence of the Relative Phase on *b*

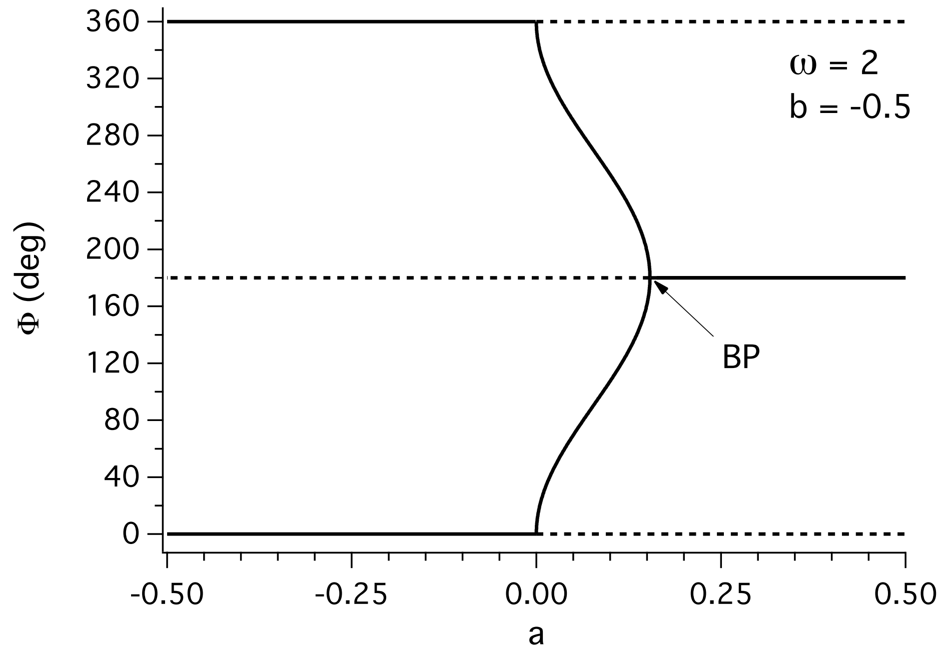


Dependence of the Relative Phase on *b*

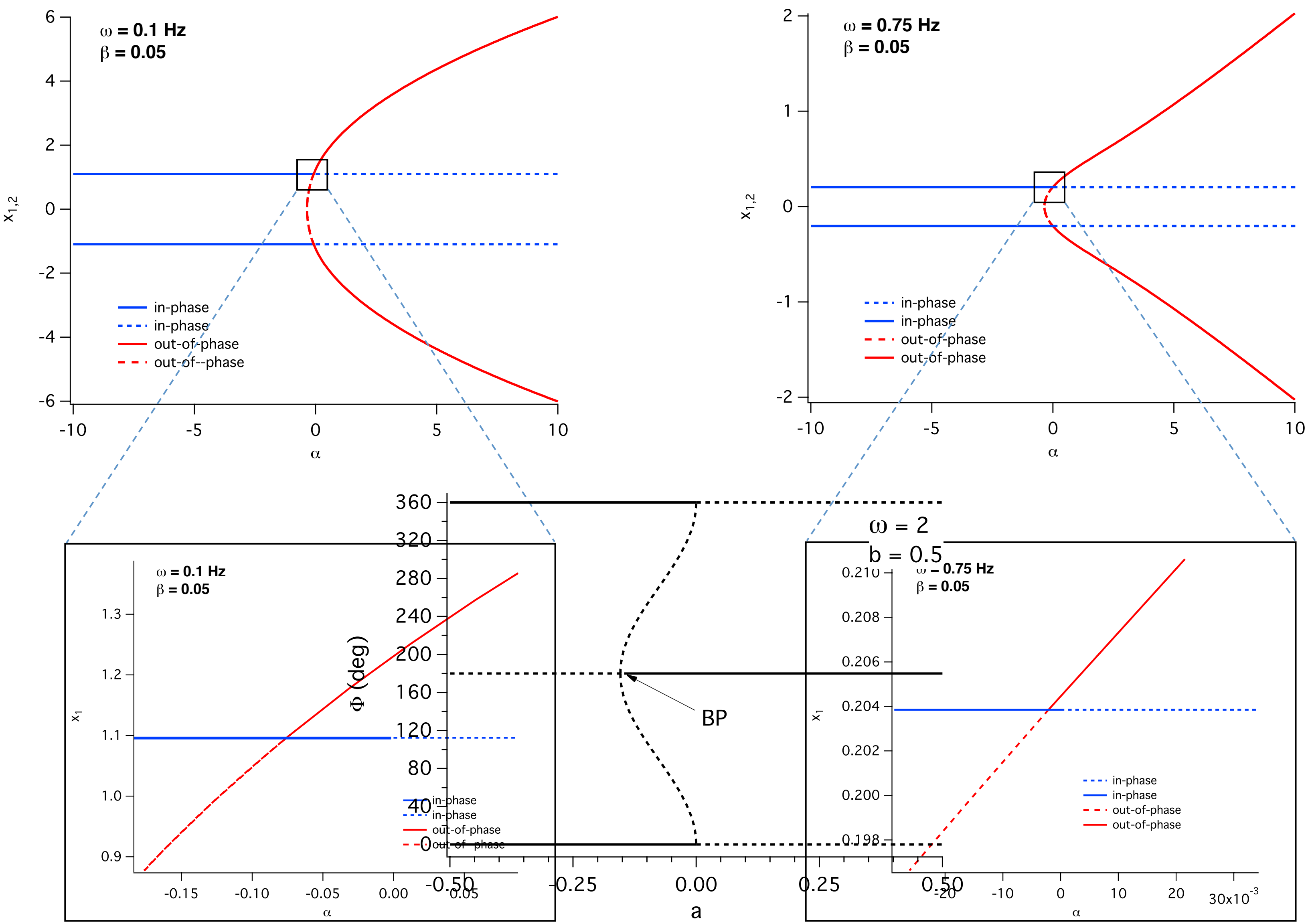


Dependence of the Relative Phase on *a*

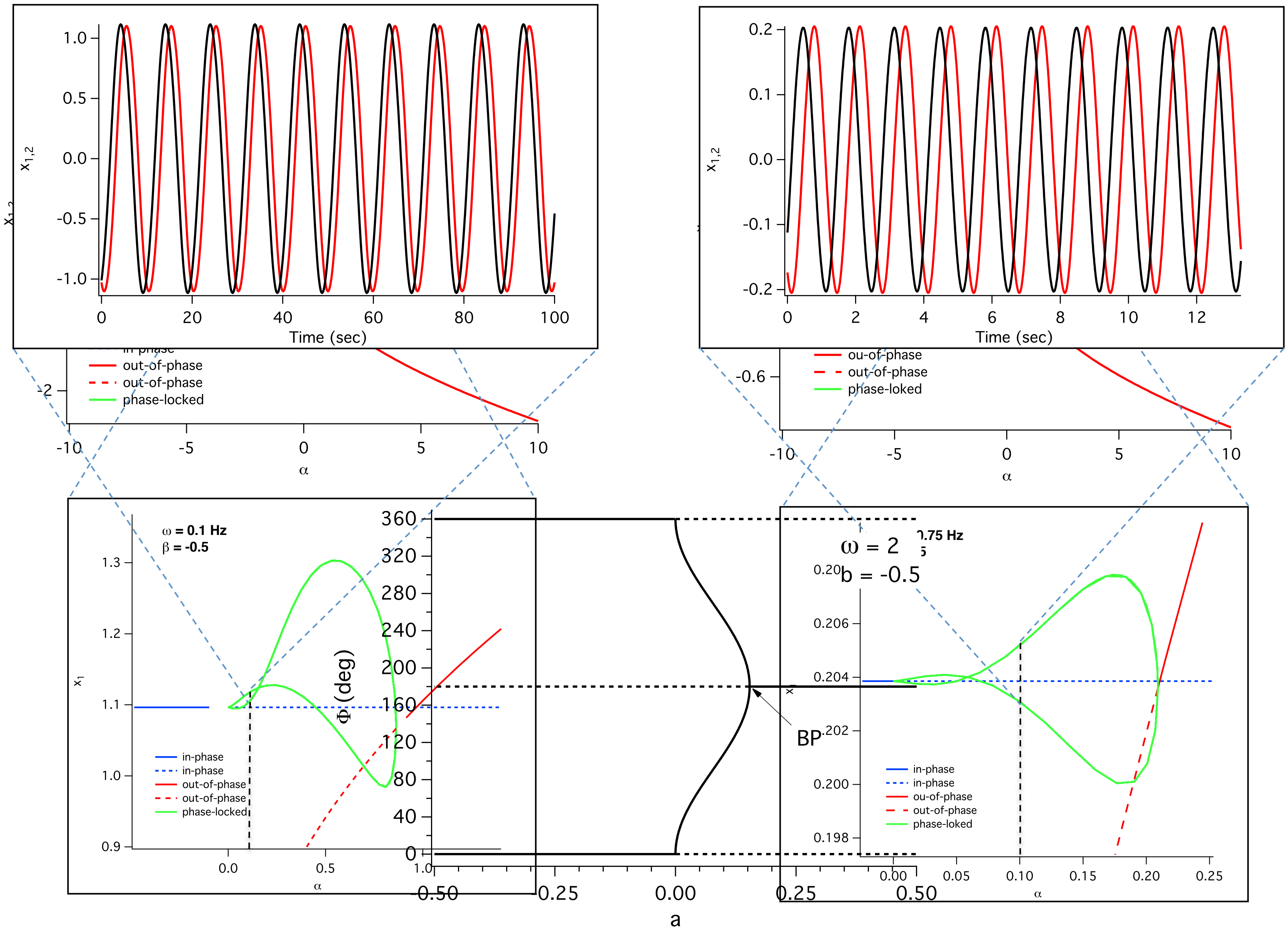


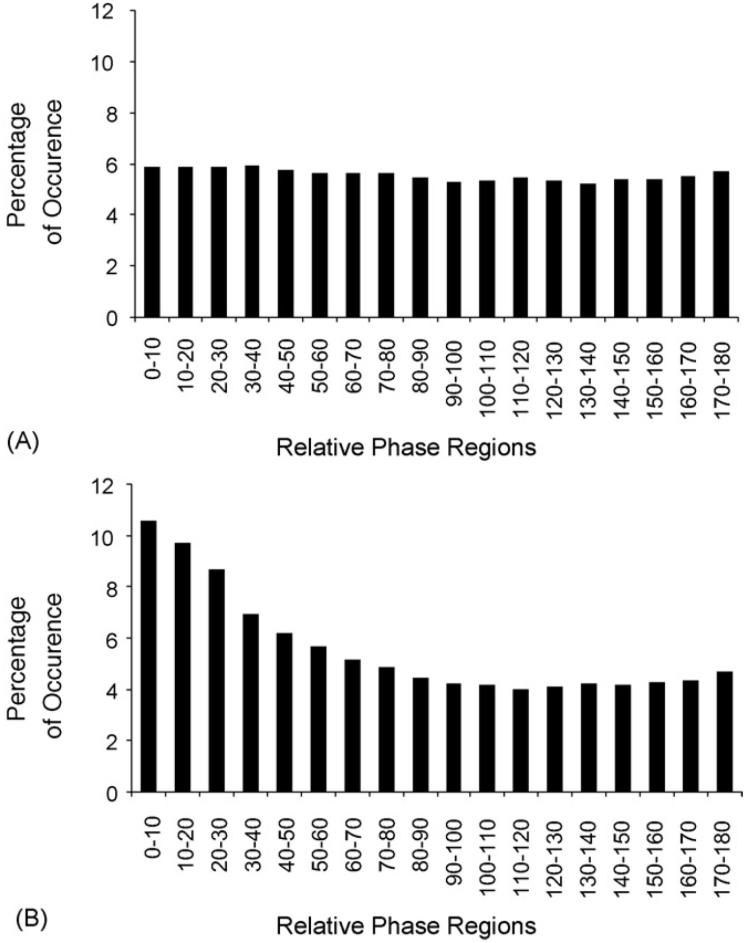


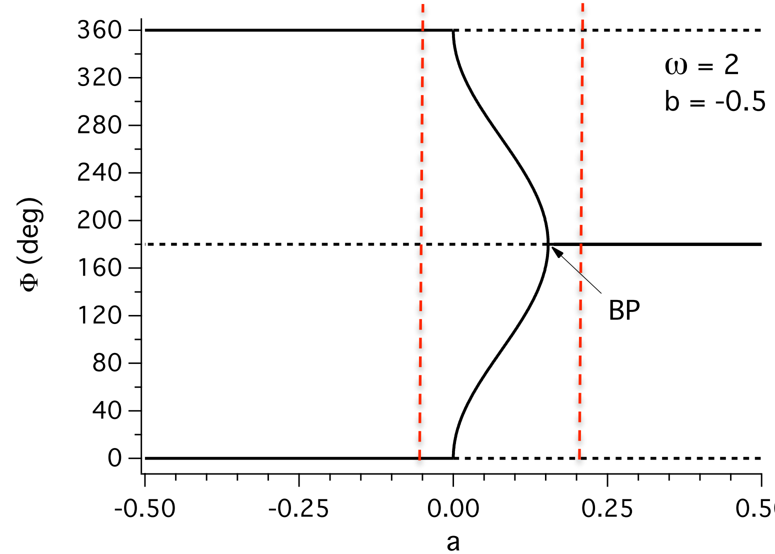
Dependence of the Relative Phase on *a*

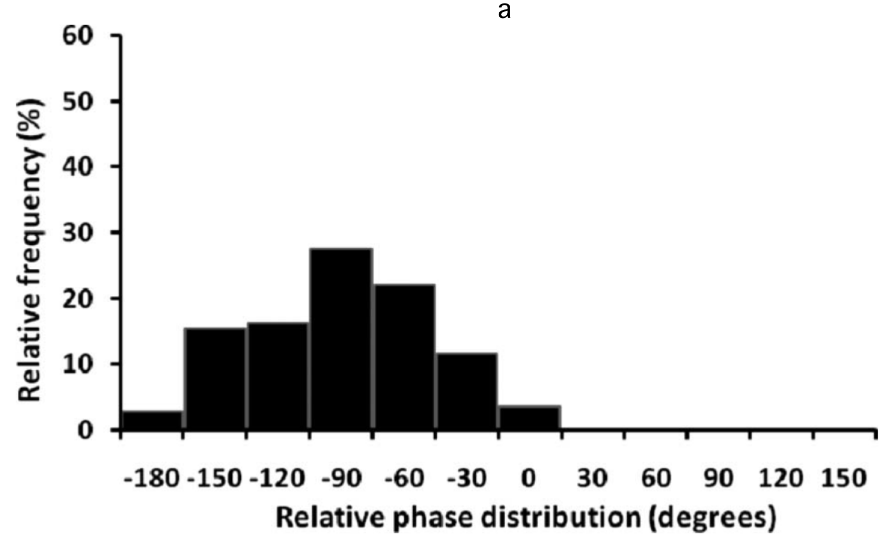


Dependence of the Relative Phase on *a*





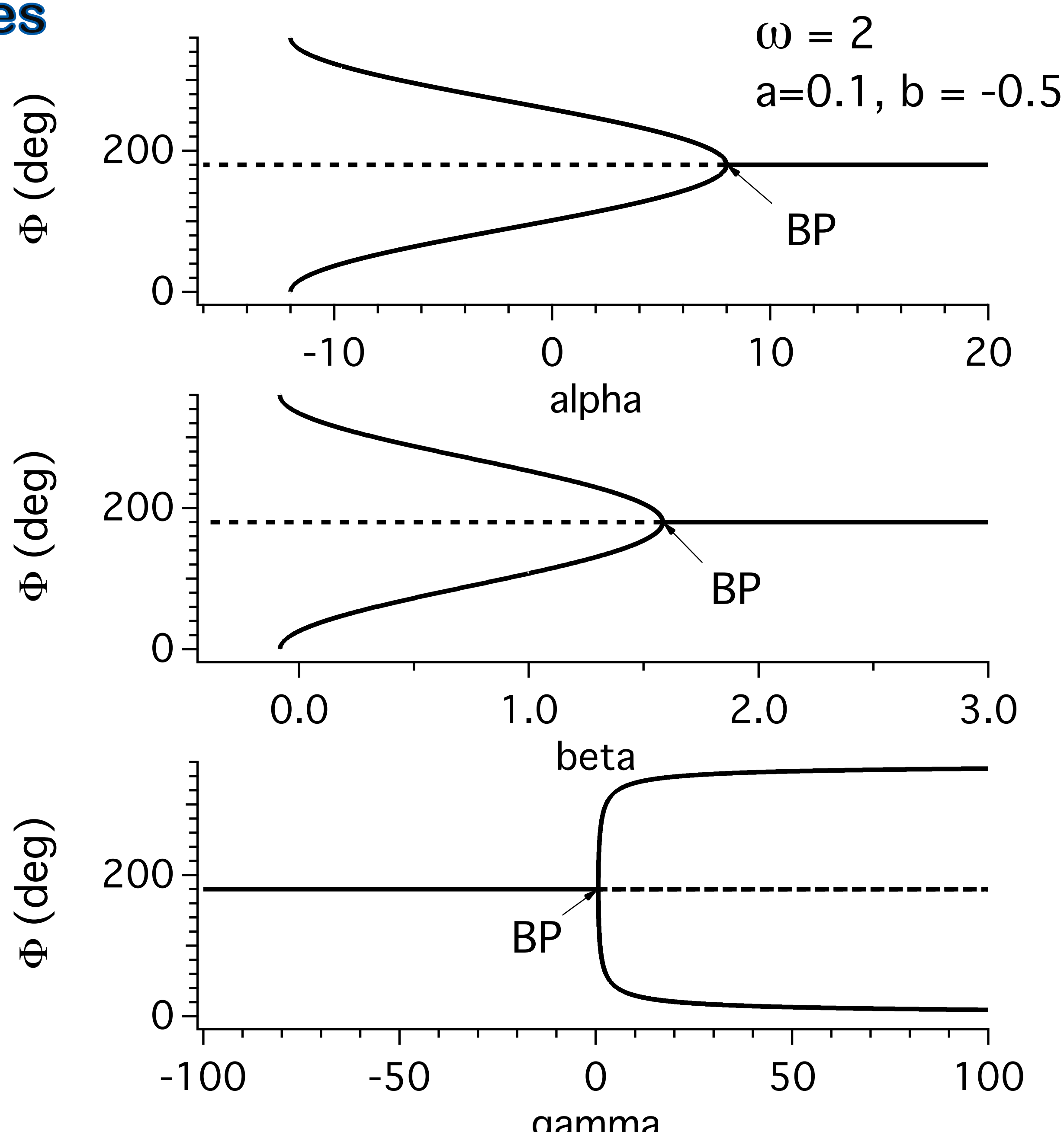




Issartel J, Marin L, Cadopi M. Unintended interpersonal co-ordination: "can we march to the beat of our own drum?", Neurosci Lett. 2007 Jan 16;411(3):174-9.

Duarte R, Araújo D, Davids K, Travassos B, Gazimba V, Sampaio J., Interpersonal coordination tendencies shape 1-vs-1 sub-phase performance outcomes in youth soccer, J Sports Sci. 2012 May;30(9):871-7.

Dependence of the Relative Phase on intrinsic oscillator properties



$$\dot{r}_1 = \frac{1}{8}\Big(r_1(4(a+\gamma) + r_1^2(b-\alpha-3\beta\omega^2) + 2br_2^2) - r_2(4a + b(3r_1^2 + r_2^2))\cos\phi + br_1r_2^2\cos 2\phi \Big)$$

(1)
$$\dot{r}_2 = \frac{1}{8}\Big(r_2(4(a+\gamma) + r_2^2(b-\alpha-3\beta\omega^2) + 2br_1^2) - r_1(4a + b(3r_2^2 + r_1^2))\cos\phi + br_2r_1^2\cos 2\phi \Big)$$

$$\dot{\phi} = \frac{(r_1^2 + r_2^2)}{8}\left(\frac{4a + b(r_1^2 + r_2^2)}{r_1r_2}\sin\phi - b\sin 2\phi \right)$$

Fixed points:

$$\phi = 0,\ r_{1,2} = 0,\ r_1 = r_2,\ \ r_1 = -r_2$$

$$\phi = \pi,\ r_{1,2} = 0,\ r_1 = r_2,\ \ r_1 = -r_2$$

$$\phi = \pm\arccos\left(\frac{2a + br^2}{br^2} \right),\ r_{1,2} = 0,\ r_1 = r_2,\ \ r_1 = -r_2$$

Frequency Dependence of the Relative Phase (cont.)