Substructuring a Nose Landing Gear – Fuselage System

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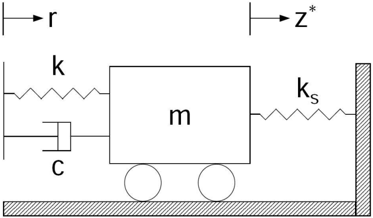
27 March 2014, Exeter

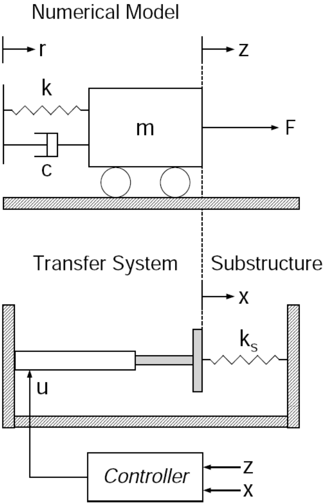
Motivation

- Under certain circumstances pilots can feel vibrations in the cockpit during take off and landing
- The nose gear, which is known to vibrate under certain circumstances, is underneath the cockpit
- The interaction between the nose gear and the fuselage affects the dynamics of both the gear and the fuselage

[NASA test] [Birmingham landing]

Real-time dynamic substructuring (RTDS)





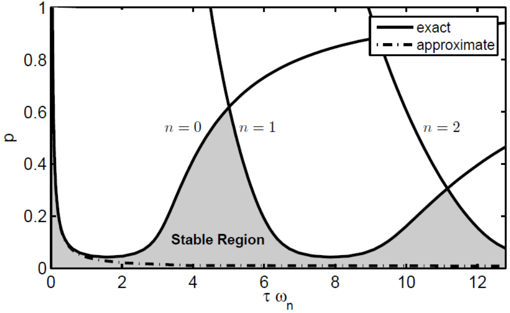
Wallace, M. I., et al. "Stability analysis of realtime dynamic substructuring using delay differential equation models." *Earthquake engineering and structural dynamics* 34.15 (2005): 1817-1832

Real-time dynamic substructuring (RTDS)

$$m\ddot{y} + c\dot{y} + k\,y + k_s y(t-\tau) = 0,$$

$$\ddot{z} + 2\zeta\dot{z} + 1 + pz(t-\tau) = 0,$$

$$\lambda^2 + 2\zeta\lambda + 1 + pe^{-\lambda\tau} = 0$$



Wallace, M. I., et al. "Stability analysis of realtime dynamic substructuring using delay differential equation models."

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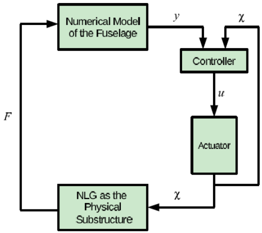
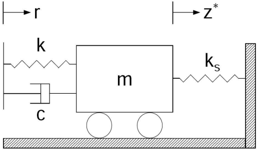
Fuselage model

Substructured landing gear





Motions with lateral component at the nose gear attachment point

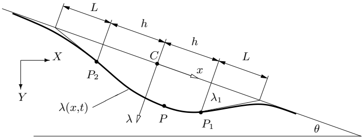
Landing gear model

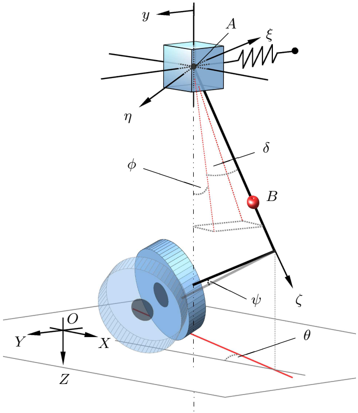
3 degrees of freedom

1. lateral bending angle δ
2. torsional angle ψ
3. lateral fuselage displacement y

2 constrained states

1. vertical fusleage displacement z
2. lateral tyre displacement λ

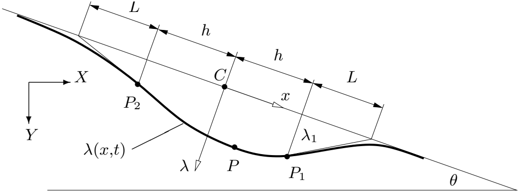




N. Terkovics, S. Neild, M. Lowenberg and B. Krauskopf; Bifurcation Analysis of a Coupled Nose Landing Gear-Fuselage System,

Submitted to the Journal of Aircraft, 2013

Tyre model



$$\mathbf{v}_P = \dot{\mathbf{r}}_{OP} = \begin{bmatrix} V_x + \dot{r}_{AC}^X + \dot{x}\cos\theta - x\,\dot{\theta}\sin\theta - \frac{d}{dt}\lambda(x,t)\sin\theta - \lambda(x,t)\,\dot{\theta}\cos\theta \\ \dot{y} + \dot{r}_{AC}^Y + \dot{x}\sin\theta + x\,\dot{\theta}\cos\theta + \frac{d}{dt}\lambda(x,t)\cos\theta - \lambda(x,t)\,\dot{\theta}\sin\theta \\ \dot{z} + \dot{r}_{AC}^Z \end{bmatrix},$$

$$\frac{d}{dt}\lambda(x,t) = \dot{\lambda}(x,t) + \lambda'(x,t)\,\dot{x}.$$

$$\text{Boundary condition} \;\rightarrow\; \lambda'(x,t)\Big|_{x=h} = -\frac{\lambda_1(t)}{L}.$$

$$\dot{\lambda}(x,t)\Big|_{x=h} = \dot{\lambda}_1 = \left(V_x + \dot{r}_{AC}^X\right)\left(\sin\theta - \frac{\lambda_1}{L}\cos\theta\right) - \left(\dot{y} + \dot{r}_{AC}^Y\right)\left(\cos\theta + \frac{\lambda_1}{L}\sin\theta\right) - \left(h - \frac{{\lambda_1}^2}{L}\right)\dot{\theta}.$$

Equations of motion

Equations of motion (linearized)

$$\boxed{(\mu+m)\,\ddot{y}+2\,\mu\,qf\,\dot{y}+\mu\,f^2y-ml\ddot{\delta}+\frac{J_w\,V\cos(\phi)\,\dot{\psi}}{R^2}-7\,\frac{(M+m)\,g\,k_\lambda\,\lambda}{L}=0}$$

$$\left(ml^2+Jyy\right)\ddot{\delta}-ml\ddot{y}+c_\delta\,\dot{\delta}-\frac{J_w\,LR\,\cos(\phi)\,V\dot{\psi}}{R^2}-\cos(\phi)\,E\,(M+m)\,g\psi$$
$$-\left((LR\,(M+m)-ml)\,g-k_\delta\right)\delta+\left(7\,\frac{LR\,k_\lambda}{L}-\frac{\sin(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda=0$$

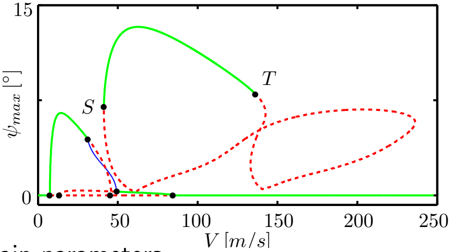
$$Jzz\,\ddot{\psi}+cPsi\,\dot{\psi}+\frac{J_w\,LR\,\cos(\phi)\,V\dot{\delta}}{R^2}-\frac{J_w\,V\cos(\phi)\,\dot{y}}{R^2}+\left(-\sin(\phi)\,E\,(M+m)\,g+k_\psi\right.$$
$$\left.+\frac{J_w\,V^2\,(\cos(\phi))^2}{R^2}\right)\psi-\cos(\phi)\,E\,(M+m)\,g\delta+\left(7\,\frac{E\,k_\lambda}{L}+\frac{\cos(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda=0$$

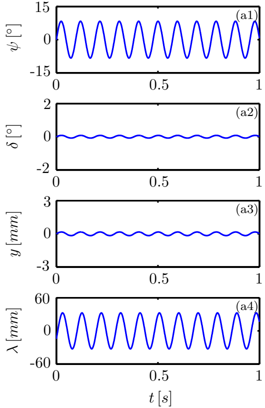
Kinematic constraint of tyre displacement λ

$$(E-h\cos(\phi))\dot{\psi}-\dot{y}+LR\dot{\delta}+V\cos(\phi)\,\psi-\frac{V\lambda}{L}$$

One-parameter bifurcation diagrams

Torsional angle ψ vs velocity V for fixed values of M, μ and f_n



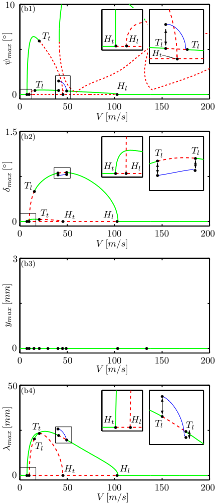


Main parameters

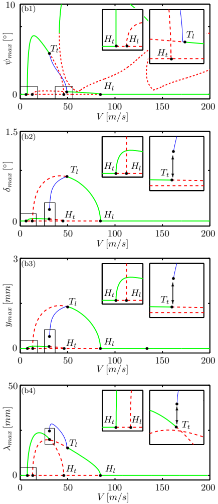
1. Vertical load M
2. Fuselage modal mass μ
3. Natural frequency of the fuselage mode f_n
4. Forward velocity V

The effect of fuselage oscillations on the coupled system

Inactive fuselage



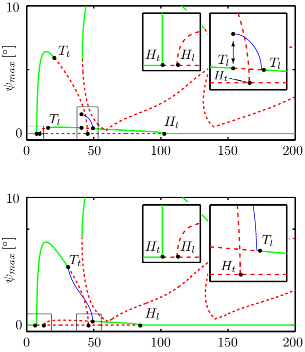
Active fuselage



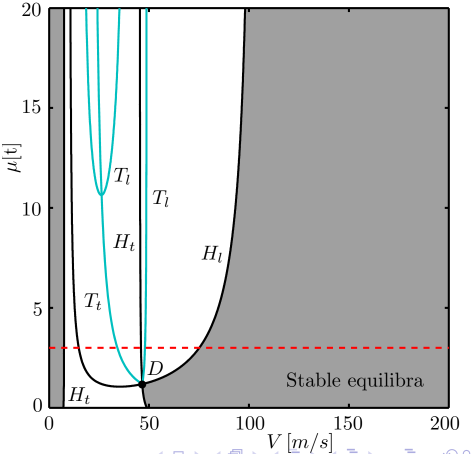
In the active case:

- Activated fuselage oscillations
- Larger velocity range for stable rolling
- Different types of predicted shimmy oscillations for certain velocity ranges

Two-parameter bifurcation diagrams

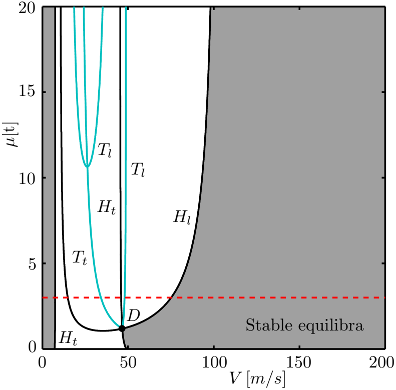


Modal mass μ vs velocity V for fixed values of M and f_n



The influence of vertical load

Modal mass μ vs velocity V
for fixed values of M and f_n



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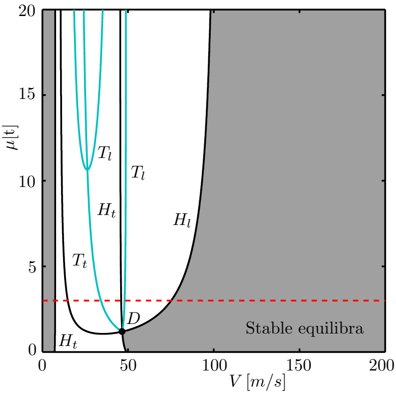
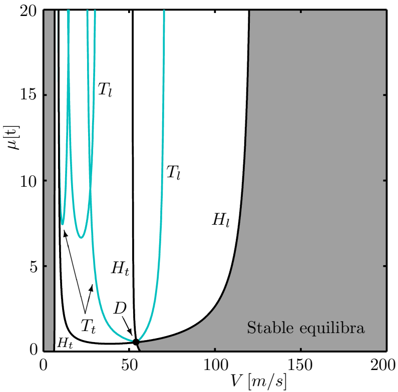
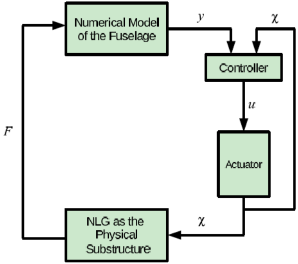


For higher load



- It has been shown that there is significant interaction between the fuselage and the nose landing gear
- The feasibility study of RTDS is, therefore, relevant

Substructuring of the coupled system



Physical substructure:	Landing gear
Numerical model:	Fuselage
Transfer system:	Hydraulic actuator

Modelling the actuator

1. time delay $\rightarrow \chi = y(t - \tau)$
2. time lag $\rightarrow \dot{\chi} = \frac{1}{T}(y - \chi)$

Time delay model

Substructuring of the coupled system

Time delay model – Substructured equations (linearized)

$$\boxed{(\mu+m)\,\ddot{y}+2\,\mu\,qf\,\dot{y}+\mu\,f^2y-ml\ddot{\delta}+\frac{J_w\;V\cos(\phi)\,\dot{\psi}}{R^2}-7\,\frac{(M+m)\,g\,k_\lambda\,\lambda}{L}=0}$$

$$\boxed{\begin{aligned}&\left(ml^2+Jyy\right)\ddot{\delta}-ml\ddot{y}+c_\delta\,\dot{\delta}-\frac{J_w\,LR\,\cos(\phi)\,V\dot{\psi}}{R^2}-\cos(\phi)\,E\,(M+m)\,g\psi\\&-((LR\,(M+m)-ml)\,g-k_\delta)\,\delta+\left(7\,\frac{LR\,k_\lambda}{L}-\frac{\sin(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda=0\end{aligned}}$$

$$\boxed{\begin{aligned}&Jzz\,\ddot{\psi}+cPsi\,\dot{\psi}+\frac{J_w\,LR\,\cos(\phi)\,V\dot{\delta}}{R^2}-\frac{J_w\,V\cos(\phi)\,\dot{y}}{R^2}+\Bigg(-\sin(\phi)\,E\,(M+m)\,g+k_\psi\\&+\frac{J_w\,V^2\,(\cos(\phi))^2}{R^2}\Bigg)\psi-\cos(\phi)\,E\,(M+m)\,g\delta+\left(7\,\frac{E\,k_\lambda}{L}+\frac{\cos(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda=0\end{aligned}}$$

$$\boxed{(E-h\cos(\phi))\dot{\psi}-\dot{y}+LR\dot{\delta}+V\cos(\phi)\,\psi-\frac{V\lambda}{L}}$$

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Time delay model – Substructured equations (linearized)

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Substructuring of the coupled system

Time delay model – Substructured equations (linearized)

$$\boxed{\underline{(\mu + m)\,\ddot{y} + 2\,\mu\,qf\,\dot{y} + \mu\,f^2 y} - ml\ddot{\delta} + \frac{J_w\,V\cos(\phi)\,\dot{\psi}}{R^2} - 7\,\frac{(M+m)\,g\,k_\lambda\,\lambda}{L} = 0}$$

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$$\boxed{\begin{aligned}&Jzz\,\ddot{\psi} + cPsi\,\dot{\psi} + \frac{J_w\,LR\,\cos(\phi)\,V\dot{\delta}}{R^2} - \frac{J_w\,V\cos(\phi)\,\dot{y}}{R^2} + \Bigg(-\sin(\phi)\,E\,(M+m)\,g + k_\psi \\ &+ \frac{J_w\,V^2\,(\cos(\phi))^2}{R^2}\Bigg)\psi - \cos(\phi)\,E\,(M+m)\,g\delta + \left(7\,\frac{E\,k_\lambda}{L} + \frac{\cos(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda = 0\end{aligned}}$$

$$\boxed{(E - h\cos(\phi))\dot{\psi} - \dot{y} + LR\dot{\delta} + V\cos(\phi)\,\psi - \frac{V\lambda}{L}}$$

Substructuring of the coupled system

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Substructuring of the coupled system

Substructured equations (linearized)

$$\boxed{(\mu + m)\,\ddot{y} + 2\,\mu\, qf\,\dot{y} + \mu\, f^2 y - ml\ddot{\delta} + \frac{J_w\, V \cos(\phi)\,\dot{\psi}}{R^2} - 7\,\frac{(M+m)\, g\, k_\lambda\, \lambda}{L} = 0}$$

$$\left(ml^2 + Jyy\right)\ddot{\delta} - ml\ddot{y}(t-\tau) + c_\delta\,\dot{\delta} - \frac{J_w\, LR\, \cos(\phi)\, V\dot{\psi}}{R^2} - \cos(\phi)\, E\,(M+m)\, g\psi$$
$$- ((LR\,(M+m) - ml)\, g - k_\delta)\,\delta + \left(7\,\frac{LR\, k_\lambda}{L} - \frac{\sin(\phi)\, k_\alpha}{L}\right)(M+m)\, g\lambda = 0$$

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$$\left.+\frac{J_w\, V^2\,(\cos(\phi))^2}{R^2}\right)\psi - \cos(\phi)\, E\,(M+m)\, g\delta + \left(7\,\frac{E k_\lambda}{L} + \frac{\cos(\phi)\, k_\alpha}{L}\right)(M+m)\, g\lambda = 0$$

$$\boxed{(E - h\cos(\phi))\dot{\psi} - \dot{y}(t-\tau) + LR\dot{\delta} + V\cos(\phi)\,\psi - \frac{V\lambda}{L}}$$

Critical delay

A neutral functional differential equation:

$$\dot{x}(t) = A_0 x(t) + A_1 \dot{x}(t-\tau) + A_2 x(t-\tau)$$

Critical delay

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Characteristic equation

$$det(-\lambda I + A_0 + (\lambda A_1 + A_2)\, e^{-\lambda\tau}) = 0$$

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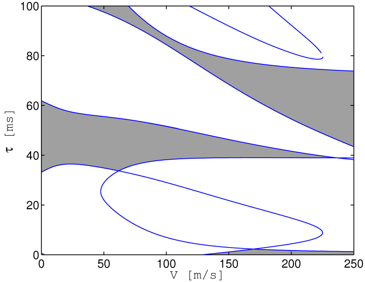

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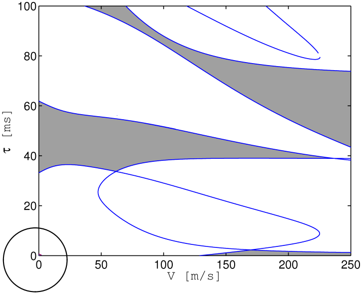

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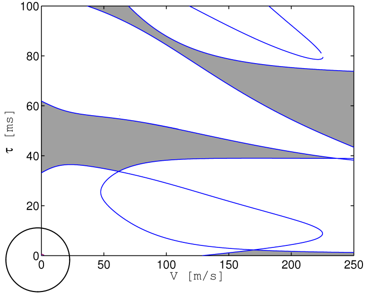

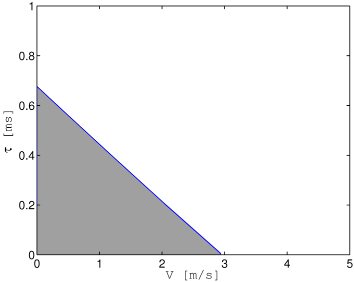

Time lag model

Substructuring of the coupled system

Time lag model – Substructured equations (linearized)

$$\boxed{(\mu + m)\,\ddot{y} + 2\,\mu\,qf\,\dot{y} + \mu\,f^2 y - ml\ddot{\delta} + \frac{J_w\,V\cos(\phi)\,\dot{\psi}}{R^2} - 7\,\frac{(M+m)\,g\,k_\lambda\,\lambda}{L} = 0}$$

$$\left(ml^2 + Jyy\right)\ddot{\delta} - ml\ddot{y} + c_\delta\,\dot{\delta} - \frac{J_w\,LR\,\cos(\phi)\,V\dot{\psi}}{R^2} - \cos(\phi)\,E\,(M+m)\,g\psi$$
$$-\left((LR\,(M+m) - ml)\,g - k_\delta\right)\delta + \left(7\,\frac{LR\,k_\lambda}{L} - \frac{\sin(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda = 0$$

$$Jzz\,\ddot{\psi} + cPsi\,\dot{\psi} + \frac{J_w\,LR\,\cos(\phi)\,V\dot{\delta}}{R^2} - \frac{J_w\,V\cos(\phi)\,\dot{y}}{R^2} + \Bigg(-\sin(\phi)\,E\,(M+m)\,g + k_\psi$$
$$+\frac{J_w\,V^2\,(\cos(\phi))^2}{R^2}\Bigg)\psi - \cos(\phi)\,E\,(M+m)\,g\delta + \left(7\,\frac{E\,k_\lambda}{L} + \frac{\cos(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda = 0$$

$$\boxed{(E - h\cos(\phi))\dot{\psi} + \dot{y} + LR\dot{\delta} + V\cos(\phi)\,\psi - \frac{V\lambda}{L}}$$

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$$\boxed{\begin{gathered}\left(ml^2+Jyy\right)\ddot{\delta}-ml\ddot{\chi}+c_\delta\,\dot{\delta}-\frac{J_w\,LR\,\cos(\phi)\,V\dot{\psi}}{R^2}-\cos(\phi)\,E\,(M+m)\,g\psi\\-\left((LR\,(M+m)-ml)\,g-k_\delta\right)\delta+\left(7\,\frac{LR\,k_\lambda}{L}-\frac{\sin(\phi)\,k_\alpha}{L}\right)(M+m)\,g\lambda=0\end{gathered}}$$

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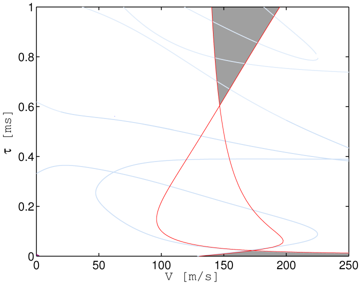
$$\boxed{(E-h\cos(\phi))\dot{\psi}-\dot{\chi}+LR\dot{\delta}+V\cos(\phi)\,\psi-\frac{V\lambda}{L}}$$

$$\dot{\chi}=\frac{1}{T}\,(y-\chi)$$

Critical lag

Ordinary differential differential equation (increased order):

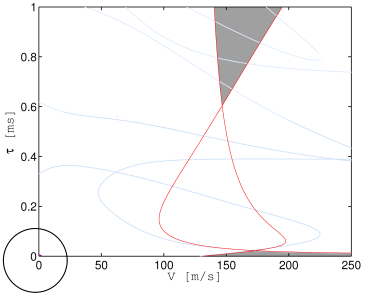
$$\dot{x}(t) = A_0 x(t)$$



Critical lag

Ordinary differential differential equation (increased order):

$$\dot{x}(t) = A_0 x(t)$$



Critical lag

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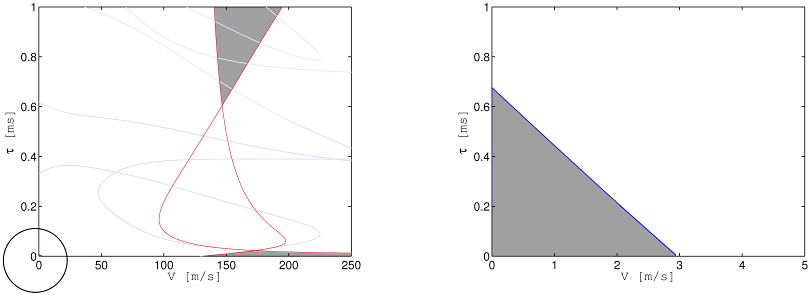

Good agreement for small delays

Conclusions

1. A model has been developed to study interaction between the landing gear and lateral fuselage dynamics
2. Given the right parameters, fuselage modes having lateral components can be excited during take-off and landing
3. Significant proportion of the excitation energy feeds modes of lower modal masses.
4. There is sufficient feedback between the two subsystems, so substructuring of the system is relevant
5. Even a slight delay may cause qualitative change in the behaviour

↓

why?

Thank you

Footnote: N.T. is looking for a postdoc...