Auditory streaming emerges from direct excitation and slow delayed inhibition

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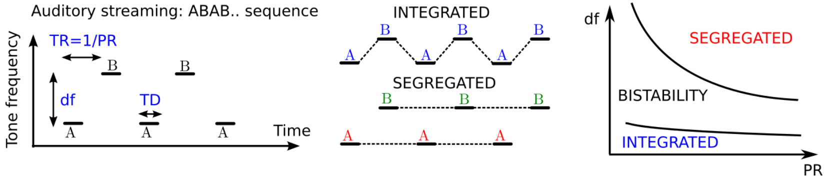
Dynamics Internal Seminar

26 March 2020

Motivation - The Auditory Streaming Paradigm (AS)

The psychoacoustic perceptual experiment - pure tones A and B



- Perception of integrated (INT) or segregated (S) rhythms
- Dependence on stimulus parameters

We explore a "minimal" firing rate model with periodic inputs driving oscillations

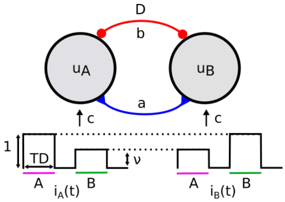
- Coupled neural CPG oscillators (Rubin & Terman, 2000)
- Perceptual rivalry via adaptation or synaptic depresstion (Shpiro et al, 2009)

Outline of the talk

- Model description
- Analyse symmetry and parameter constraints
- Analitical approach to classify dynamics (slow-fast decomposition)
- Exhaustive study of states in the space of parameters
- Interpretation of states with AS perceptions
- Show similarity with AS experiments
- Show a rich repertoire of dynamical states
- Computational analysis under smooth conditions

Model description

Two periodically forced neural populations (A and B) coupled by fast direct excitation and slow delayed inhibition



Periodically forced system of 4 delay differential equations:

$$
\begin{aligned}
\tau \dot{u}_A(t) &= -u_A(t) + H(au_B(t) - bs_B(t-D) + ci_A(t-t_0)) \\
\tau \dot{u}_B(t) &= -u_B(t) + H(au_A(t) - bs_A(t-D) + ci_B(t-t_0)) \\
\dot{s}_A(t) &= H(u_A(t))(1-s_A(t))/\tau - s_A(t)/\tau_i \\
\dot{s}_B(t) &= H(u_B(t))(1-s_B(t))/\tau - s_B(t)/\tau_i
\end{aligned}
$$

Heaviside activation $H(x) = \begin{cases} 1, & \text{if } x \geq \theta \\ 0, & \text{otherwise} \end{cases}$

Slow-fast regime $\tau/\tau_i << 1$

Parameters: activity threshold $\theta \in (0,1)$, synaptic strengths a and b, input strength c and onset t_0, inhibition time scale τ_i, activity time scale τ

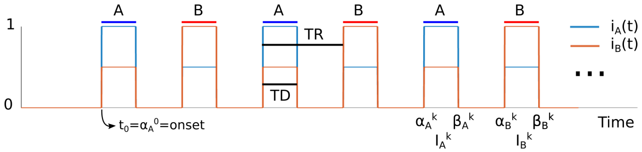
Periodic square-wave inputs

$$\begin{aligned} i_A(t) &= \sum_{k=0}^{\infty} \chi_{I_A^k}(t) + \nu \sum_{k=0}^{\infty} \chi_{I_B^k}(t) \\ i_B(t) &= \nu \sum_{k=0}^{\infty} \chi_{I_A^k}(t) + \sum_{k=0}^{\infty} \chi_{I_B^k}(t) \end{aligned}$$

$\chi_I(t) = 1$ for $t \in I$ and 0 otherwise, and scaling parameter ν ($d = \nu c$)

$I_A^k = [\alpha_k^A, \beta_k^A] = [2kTR, 2kTR+TD]$, $\quad I_B^k = [\alpha_k^B, \beta_k^B] = [(2k+1)TR, (2k+1)TR+TD]$

Where TD=tones' duration and PR=$1/TR$=tone repetition rate.



- Mimic stereotyped neural responses from Macaque auditory cortex (AC)
- Populations A and B are assumed to be located downstream AC

We introduce a bit of notation:

$$\Phi = \{R \subset \mathbb{R} : R = I_k^A \text{ or } R = I_k^B,\ \exists k \in \mathbb{N}\}, \quad I = \bigcup_{R \in \Phi} R$$

$\mathbb{Z}_2$ equivariance - asymmetrical cycles exist in pairs

Rewrite the model as a non-autonomous dynamical system

$$\dot{v}(t) = f(v(t), i_A(t), i_B(t)), \quad v = (u_A, u_B, s_A, s_B)$$

Map κ swaps A and B indexes in the system's variables

$$\kappa : v = (u_A, u_B, s_A, s_B, i_A, i_B) \mapsto (u_B, u_A, s_B, s_A, i_B, i_A)$$

Apply TR time shift on the inputs

$$i_A(t+TR) = i_B(t) \quad i_B(t+TR) = i_A(t), \quad \forall t \in \mathbb{R}$$

We have $\mathbb{Z}_2$-equivariance under transformation κ and TR time shift

$$\kappa(f(v(t), i_A(t), i_B(t))) = f(\kappa(v(t+TR), i_B(t+TR), i_A(t+TR)))$$

Main conclusion: any $2nTR$-periodic asymmetrical cycle (not in-phase nor anti-phase) has a co-existing κ-conjugate cycle $\kappa(v(t+TR))$

Constraining model into a realistic range of parameters

With no inputs ($i_A = i_B = 0$) there are two possible equilibria: a low-activity state $P = (0, 0, 0, 0)$ and a high-activity state $Q = (1, 1, 1, 1)$.

If $a - b \geq \theta$ then P and Q co-exist, and dynamics is trivial

- If $c < \theta$: trajectories starting in the basin of P (Q) end in P (Q)
- $c \geq \theta$: any trajectory converges to Q

To avoid this unrealistic scenario

1. $a - b < \theta \rightarrow P$ is the only equilibium without inputs (no saturation)
2. $c \geq \theta \rightarrow$ units activate in the absence of inhibition ($b = 0$)

In addition, we consider input parameters contraints

- $PR \in [1, 40] Hz$: physiological conditions tested in experiments
- $TR > TD$: No active tone overlap $I_A^i \cap I_B^j = \emptyset$, $\forall i \neq j \in \mathbb{N}$
- $df \in [0, 1]$ defined as $\nu = \nu(df) = (1 - df^{1/m})$ to model the tone pitch difference

The fast and slow subsystem at fixed time $t \in I_A^k$

Change variables to the fast time scale $r = t/\tau$, $\tau \to \infty$

$$
\begin{aligned}
u_A' &= -u_A + H(au_B - bs_B(t-D) + c) \\
u_B' &= -u_B + H(au_A - bs_A(t-D) + d) \\
s_A' &= H(u_A)(1 - s_A) \\
s_B' &= H(u_B)(1 - s_B)
\end{aligned}
$$

Where $' = d/dr$. Units can be OFF (~ 0), ON (~ 1), or turning OFF or ON

- Synaptic variables may jump up or be constant (if A and B unit OFF)

Slow-dynamics

$$\dot{s} = -s/\tau_i$$

$\to s$ is decreasing (on either time scales) except when the units turn ON
Dynamics in $\mathbb{R} - I$ $(c = d = 0)$ is simple, WLOG $\tilde{s}_B \geq 0$, $\tilde{s}_A \geq 0$ (at steady state)

$$
\begin{aligned}
u_A' &= -u_A + H(au_B - b\tilde{s}_B) \\
u_B' &= -u_B + H(au_A - b\tilde{s}_A)
\end{aligned}
$$

$(0,0)$ is always a FP and $Q = (1,1)$ FP if and only if $a - b\tilde{s}_A \geq \theta$ and $a - b\tilde{s}_B \geq \theta$

Dynamics in the intervals with no inputs ($\mathbb{R} - I$)

Theorem

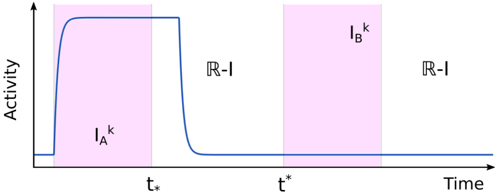
For any $t \geq t_0 \in \mathbb{R} - I$:

1. *If A or B is OFF at time t, both units are OFF in $(t, t^*]$, where t^* is:*

$$t^* = \min_{s \in I}\{s > t\}$$

2. *If A or B is ON at time t, both units are ON in $[t_*, t)$, where t_* is:*

$$t_* = \max_{s \in I}\{s < t\}$$



Consequence: no unit can turn ON in $\mathbb{R} - I$.

Definition (LONG and SHORT states)

- LONG - if at least one unit is ON at time t, $\exists t \geq t_0 \in \mathbb{R} - I$
- SHORT - if both units are OFF $\forall t \geq t_0 \in \mathbb{R} - I$

Synaptic decay and no saturated states if $TD+D<TR$

Lemma

Assume $TD+D<TR$. Defined Γ as:

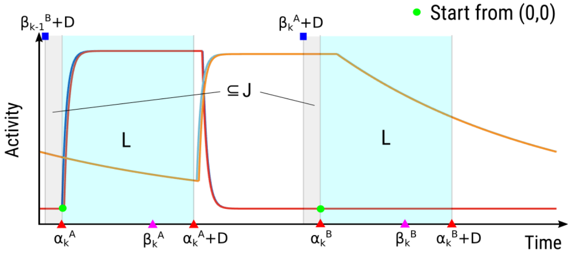
$$\Gamma = \{L \subset \mathbb{R} : L = [\alpha_k^A, \alpha_k^A + D] \text{ or } L = [\alpha_k^B, \alpha_k^B + D],\ \exists k \in \mathbb{N}\}$$

$s_A(t-D)$ and $s_B(t-D)$ are monotonically decreasing in L, $\forall L \in \Gamma$.

Lemma

If $TD+D<TR$ both units are OFF $\forall t \in J$, where:

$$J = \bigcup_{k\in\mathbb{N}} (\beta_k^A + D, \alpha_k^B] \cup (\beta_k^B + D, \alpha_{k+1}^A]$$



Dynamics in the the active tone intervals I assuming $D \geq TD$

If $D \geq TD$ the delayed synaptic variables are constant on the fast time scale (equal to 1 or exp decaying). For $t \in I_A^k$ (A tone active phase):

$$\begin{aligned} u_A' &= -u_A + H(au_B - b\tilde{s}_B + c) \\ u_B' &= -u_B + H(au_A - b\tilde{s}_A + d) \end{aligned}$$

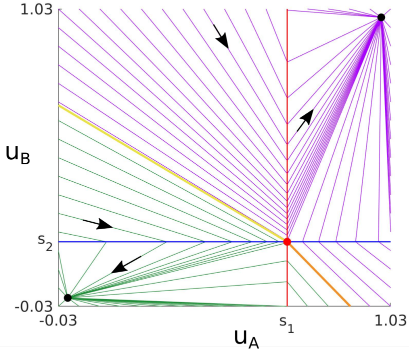
Where $\tilde{s}_A = s_A(t-D)$ and $\tilde{s}_B = s_B(t-D)$. Four possible FPs

1. $(0,0) \leftrightarrow c < b\tilde{s}_B + \theta$ and $d < b\tilde{s}_A + \theta$
2. $(1,0) \leftrightarrow c \geq b\tilde{s}_B + \theta$ and $a + d < b\tilde{s}_A + \theta$
3. $(0,1) \leftrightarrow a + c < b\tilde{s}_B + \theta$ and $d \geq b\tilde{s}_A + \theta$
4. $(1,1) \leftrightarrow a + c \geq b\tilde{s}_B + \theta$ and $a + d \geq b\tilde{s}_A + \theta$

Basin of attraction of $(0,0)$ and $(1,1)$

We note that only $(0,0)$ and $(1,1)$ can coexist if $a > 0$. We can rewrite system

$$\begin{aligned} u_A' &= -u_A + S(u_B - s_2) \\ u_B' &= -u_B + S(u_A - s_1) \end{aligned}$$



$s_1 = (b\tilde{s}_A - c + \theta)/a, \ s_2 = (b\tilde{s}_B - c + \theta)/a$

$0 < s_k \leq 1$ for $k = 1, 2$

- (s_1, s_2) is a degenerate saddle point where separatrices originate
- $(1,0)$ and $(0,1)$ converge to $(1,1)$

Differential convergence to $(1,1)$

Consider conditions when $(1,1)$ is the only FP and an orbit starting from $(0,0)$

1. $c - b\tilde{s}_B \geq \theta$ and $d - b\tilde{s}_A \geq \theta$ both units turn ON simultaneously

$$\begin{array}{rcl} u'_A & = & 1 - u_A \\ u'_B & = & 1 - u_B \end{array}$$

2. $c - b\tilde{s}_B \geq \theta$, $d - b\tilde{s}_B < \theta$ and $a + d - b\tilde{s}_A \geq \theta$ A turns ON before B
   Indeed $\exists u^* \in (0,1]$ for which:

$$a u_* + d - b\tilde{s}_A = \theta$$

   From the first condition the fast subsystem reduces to:

$$\begin{array}{rcl} u'_A & = & 1 - u_A \\ u'_B & = & -u_B + H(a u_A - b\tilde{s}_A + d) = -u_B + \eta(u_A) \end{array}$$

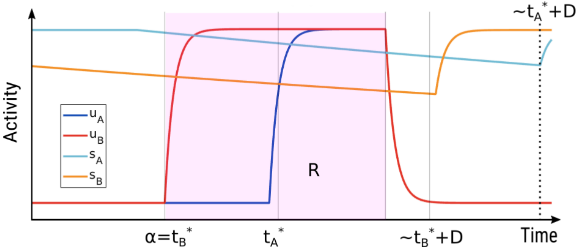
   - $u_A(r) \to 1$ exponentially reaching point u^* at time $r^* = \log[(1-u^*)^{-1}]$
   - $\eta(u_A(r)) = 0$, $\forall r < r^*$
   - $\eta(u_A(r)) = 1$, $\forall r \geq r^*$, and $u_A(r) \to 1$ following A dynamics at $r = 0$

   Back to $t = \tau r$ the B unit turns ON an infinitesimal delay $\delta = \tau r^*$ after A.
3. $d - b\tilde{s}_A \geq \theta$, $c - b\tilde{s}_A < \theta$ and $a + c - b\tilde{s}_B \geq \theta$ equal to 2 by swapping A and B.

Single OFF to ON transition during each active tone interval $R \in \Phi$

Lemma

Assume $TD + D < TR$ and $D > TD$. Let $R = [\alpha, \beta] \in \Phi$, and A (B) be ON at time $t \in R$, then
(1) A (B) is ON $\forall s \geq t$, $s \in R$
(2) $\exists! t^* \in R$ when A (B) turns ON
(3) $s_A(t - D)$ ($s_B(t - D)$) is decreasing in $[\alpha, t^* + D]$



Lemma

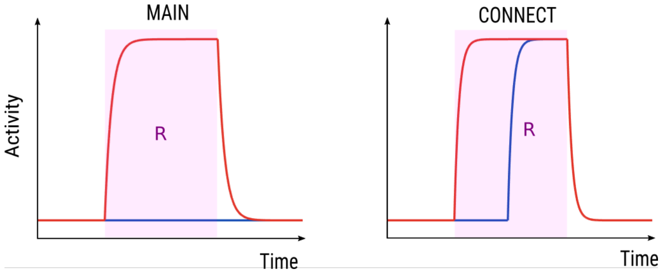
1. *A (B) turns ON at time $\alpha \Leftrightarrow A$ (B) is ON $\forall t \in (\alpha, \beta]$*
2. *A (B) if OFF at time $\beta \Leftrightarrow A$ (B) is OFF $\forall t \in R$*

MAIN and CONNECT states

Definition

Assume $TD + D < TR$ and $D > TD$. A solution (state) is:

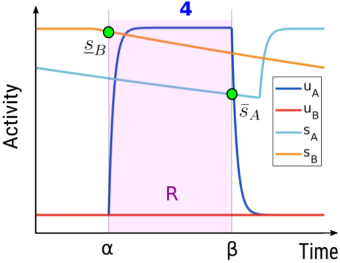
- MAIN if $\forall R \in \Phi$, if $\exists t^* \in R$ turning ON time for A or B, then $t^* = \min(R)$
- CONNECT if $\exists R \in \Phi$ and $\exists t^* \in R$, $t^* > \min(R)$ turning ON time for the A or the B unit



Classification of MAIN states

From the FP conditions of the fast system - dynamics in $R = [\alpha, \beta] \in \Phi$

- **Both units turn ON at time** α - $(1,1)$ is the only FP at time α.
  1. $f(\underline{s}_B) \geq \theta, g(\underline{s}_A) \geq \theta \rightarrow$ both units instantaneously turn ON
  2. $g(\underline{s}_A) < \theta$, $f(\underline{s}_B) \geq \theta$ and $a + g(\underline{s}_A) \geq \theta \rightarrow$ B turns ON after A
  3. $f(\underline{s}_B) < \theta$, $g(\underline{s}_A) \geq \theta$ and $a + f(\underline{s}_B) \geq \theta \rightarrow$ A turns ON after B



- **A(B) turns ON at** α **and B(A) is OFF at** β
  4. $f(\underline{s}_B) \geq \theta$, $a + g(\bar{s}_A) < \theta \rightarrow (1,0)$ FP in R
  5. $g(\underline{s}_A) \geq \theta$, $a + f(\bar{s}_B) < \theta \rightarrow (0,1)$ FP in R

- **Both units are OFF at** β $(0,0)$ FP at β:
  6. $g(\bar{s}_A) < \theta$ and $f(\bar{s}_B) < \theta$

$$f(s) = \begin{cases} c - bs, & \text{if } R = I_A^k \\ d - bs, & \text{if } R = I_B^k \end{cases}, \quad g(s) = \begin{cases} d - bs, & \text{if } R = I_A^k \\ c - bs, & \text{if } R = I_B^k \end{cases}$$

$$\underline{s}_A = s_A(\alpha - D), \quad \bar{s}_A = s_A(\beta - D), \quad \underline{s}_B = s_B(\alpha - D), \quad \bar{s}_B = s_B(\beta - D)$$

$$M = \{ \text{ MAIN states } \} = \{ s = s(t) \text{ states satisfying one of } 1 - 6, \forall R \in \Phi \}$$

Matricial form of MAIN states

Theorem

Let $R \in \Phi$. There is an injective map ρ

$$\begin{aligned} \rho\colon M &\to B(2,2) \\ s &\mapsto V = \begin{bmatrix} x_A & y_A \\ x_B & y_B \end{bmatrix} \end{aligned} \qquad \begin{aligned} x_A &= H(f(\underline{s}_B)), \quad y_A = \begin{cases} 1 & \text{if } a x_B + f(\underline{s}_B) \geq \theta \\ 0 & \text{if } a x_B + f(\bar{s}_B) < \theta \end{cases}, \\ x_B &= H(g(\underline{s}_A)), \quad y_B = \begin{cases} 1 & \text{if } a x_A + g(\underline{s}_A) \geq \theta \\ 0 & \text{if } a x_A + g(\bar{s}_A) < \theta \end{cases} \end{aligned}$$

Moreover:

$$Im(\rho) = \{V = \rho(s) : x_A \leq y_A,\ x_B \leq y_B,\ x_A = x_B = 0 \Rightarrow y_A = y_B = 0\}$$

Element $s \in M$ satisfying condition $1-6$ have one of the following images $\rho(s)$:

$$(1)\ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad (2)\ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad (3)\ \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad (4)\ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad (5)\ \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \quad (6)\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Classification of CONNECT states and matricial form

For the classification we consider the following cases:

- (1-2) **A(B) and B(A) turn ON at** α **and** $t^* \in (\alpha, \beta]$ **respectively**
- (3-4) **A(B) is OFF at** β **and B(A) turns ON at** t^***,** $\exists t^* \in (\alpha, \beta]$
- (5) **A and B turn ON at** $t^*, s^* \in (\alpha, \beta]$

$C = \{$ CONNECT states $\} = \{s = s(t)$ states satisfying one of $1 - 5, \exists R \in \Phi\}$

Theorem

Let $R \in \Phi$. There is an injective map ρ

$$\rho \colon C \to B(2,3)$$

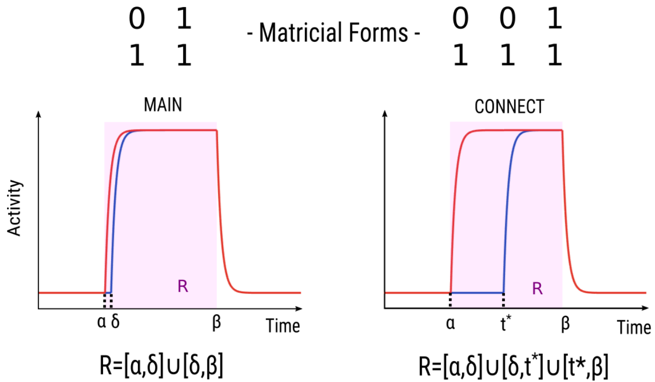
$$s \mapsto W = \begin{bmatrix} x_A & y_A & x_A \\ x_B & y_B & z_B \end{bmatrix}$$

$$\begin{aligned} x_A &= H(f(\underline{s}_B)), & x_B &= H(g(\underline{s}_A)), \\ y_A &= H(ax_B + f(\underline{s}_B))), & y_B &= H(ax_A + g(\underline{s}_A))), \\ z_A &= H(ay_B + f(\bar{s}_B))), & z_B &= H(ay_A + g(\bar{s}_A))) \end{aligned}$$

$$Im(\rho) = \{W : x_A \leq y_A \leq z_A, x_B \leq y_B \leq z_B, x_A = x_B = 0 \Rightarrow y_A = y_B = 0, y_A < z_A \text{ or } y_B < z_B\}$$

Dynamics visualisation via the Matricial form V

The units' dynamics in R is given by 1st and 2nd rows of V



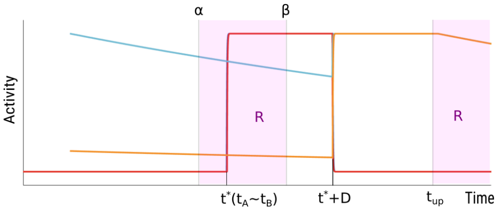
LONG states

Lemma (LONG equivalence)

If $D \geq TD$ and $TD + D < TR$. A state is LONG if and only if $\exists R = [\alpha, \beta] \in \Phi$

1. *A and B turn ON at times t_A^* and $t_B^* \in R$, respectively.*
2. *$a - bs_A(\beta) \geq \theta$ and $a - bs_B(\beta) \geq \theta$, with $\beta = \max(R)$*

Moreover, both units are ON for $t \in [\beta, t^* + D]$, turn OFF at time $t^* + D$, and are OFF $\forall t \in (t^* + D, t_{up}]$, where $t^* = \min\{t_A^*, t_B^*\}$ and $t_{up} = \min_{s \in I}\{s > t\}$.



$2kTR$-periodic MAIN and CONNECT states

Theorem

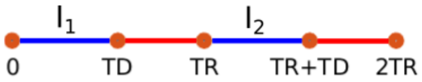
Any periodic state $\psi = \psi(t)$ must be $2kTR$-periodic, $\exists k \in \mathbb{N}$

$$\psi(t + 2kTR) = \psi(t), \quad \forall t \geq t_0, t \in \mathbb{R}$$

Definition

- M_k^S and M_k^L - sets of SHORT and LONG $2kTR$-periodic MAIN states
- C_k^S and C_k^L - sets of SHORT and LONG $2kTR$-periodic CONNECT states

Case $k = 1$



- Activity of A and B during I_i can be represented by matrices V_i, $i = 1, 2$
- V_i depend on the delayed synaptic variables at α_i and β_i

$$s_A^{i-} = s_A(\alpha_i - D), \quad s_B^{i-} = s_B(\alpha_i - D), \quad s_A^{i+} = s_A(\beta_i - D), \quad s_B^{i+} = s_B(\beta_i - D)$$

$$\alpha_i = (i-1)\,TR, \quad \beta_i = (i-1)\,TR + TD$$

Matricial form for $2TR$-periodic MAIN states

For $2TR$-periodic SHORT states these delayed synaptic values depend on

$$N^- = e^{-(TR-TD-D)/\tau_i}, N^+ = e^{-(TR-D)/\tau_i}, M^- = e^{-(2TR-TD-D)/\tau_i}, M^+ = e^{-(2TR-D)/\tau_i}$$

Theorem

There is an injective map:

$$\rho \colon M_1^S \to B(2,4)$$

$$\psi \mapsto V = \left[V_1 \mid V_2\right] = \left[\begin{array}{cc|cc} x_A^1 & y_A^1 & x_A^2 & y_A^2 \\ x_B^1 & y_B^1 & x_B^2 & y_B^2 \end{array}\right]$$

Where, for $i=1,2$, V_i is the matricial forms of ψ during the interval I_i:

$$s_B^{i\pm} = N^\pm y_B^j + M^\pm (1-y_B^j) y_B^i, \quad \text{and} \quad s_A^{i\pm} = N^\pm y_A^j + M^\pm (1-y_A^j) y_A^i, \quad \forall i,j=1,2, i\neq j$$

$$Im(\rho) = \{V = \left[V_1 \mid V_2\right] : V_1 \in Im(\rho^{I_1}), V_2 \in Im(\rho^{I_1}) \text{ satisfying 1-4 below}\}$$

1. $y_A^1 = y_B^2 = 1 \Rightarrow x_A^1 = x_B^2$ *and* $y_A^2 = y_B^1 = 1 \Rightarrow x_A^2 = x_B^1$
2. $y_B^1 = y_B^2 \Rightarrow x_A^1 \geq x_A^2$ *and* $y_A^1 = y_A^2 \Rightarrow x_B^1 \geq x_B^2$
3. $y_A^2 = 1 \Rightarrow x_B^1 \leq r$ *and* $y_B^1 = 1 \Rightarrow x_A^2 \leq r$*, for any entry* r *in* V
4. $y_A^2 = y_B^2$, $y_A^1 = y_B^1 \Rightarrow x_A^1 \geq x_B^1$ *and* $x_B^2 \geq x_A^2$

Matricial form and conditions for all $2TR$-periodic SHORT MAIN states

- Using the conditions over V we find 13 possible states (algorithmically)
- Conditions for the well-definiteness of all Vs and simplification
- Add conditions contained in the LONG equivalence theorem

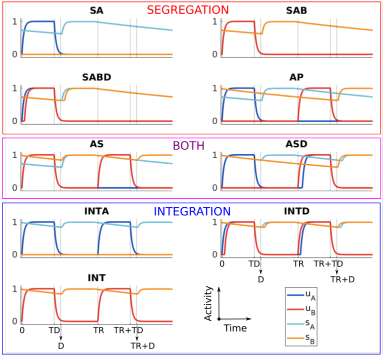
$$C_1 = d, \quad C_2^{\pm} = a - bM^{\pm} + d, \quad C_3^{\pm} = c - bN^{\pm}, \quad C_4^{\pm} = c - bM^{\pm}, \quad C_5^{\pm} = a - bN^{\pm} + d,$$
$$C_6^{\pm} = a - bN^{\pm} + c, \quad C_7^{\pm} = d - bN^{\pm}, \quad C_8^{\pm} = d - bM^{\pm}, \quad C_9 = a - bM^{+} \quad C_{10} = a - bN^{+}$$

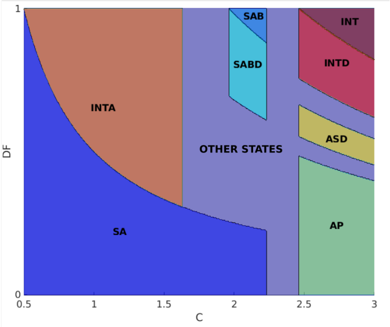
S_1	SAB_1	SD_1	AP	AS_1	ASD_2	INT_1	$INTD$	INT
1100 0000	1100 1100	1100 0100	1100 0011	1111 0011	1101 0011	1111 0000	1101 0111	1111 1111
$C_1 < \theta$ $C_2^+ < \theta$ $C_3^+ < \theta$	$C_3^+ < \theta$ $C_8^- \geq \theta$ $C_9 < \theta$	$C_4^- \geq \theta$ $C_2^- \geq \theta$ $C_3^+ < \theta$ $C_8^- < \theta$ $C_9 < \theta$	$C_2^+ < \theta$ $C_3^- \geq \theta$	$C_3^- \geq \theta$ $C_5^+ < \theta$ $C_8^- \geq \theta$ $C_{10} < \theta$	$C_2^- \geq \theta$ $C_3^- \geq \theta$ $C_5^+ < \theta$ $C_8^- < \theta$ $C_{10} < \theta$	$C_1 \geq \theta$ $C_6^+ < \theta$	$C_3^- \geq \theta$ $C_5^- \geq \theta$ $C_7^- < \theta$ $C_{10} < \theta$	$C_7^- \geq \theta$ $C_{10} < \theta$

The next slides...

- Time histories and 2D regions of existence of $2TR$-periodic SHORT MAIN STATES
- Examples of $2TR$-periodic SHORT CONNECT and LONG MAIN states
- Cycles with higher periods and cascades
  - Period doubling of SHORT MAIN states
  - Segrated switching SHORT MAIN and CONNECT states
- MAIN states under the case $D < TD$ and comparisons with experiments
- Numerical study with smooth inputs and gain function, and non slow-fast regime

2*TR*-periodic SHORT MAIN states time histories and regions of existence





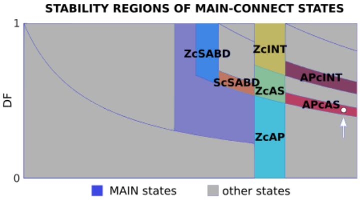
- Integration - at least one unit responds to both tones
- Segregation - no unit responds to both tones

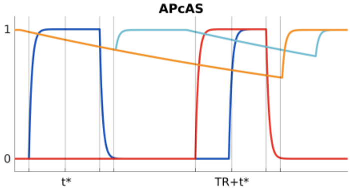
Theorem (Multistability)

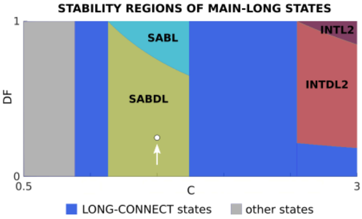
No 2TR-periodic SHORT MAIN states can be bistable except for INTA and SAB (not shown)

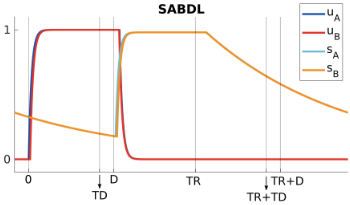
2TR-periodic SHORT CONNECT and LONG MAIN states

A similar analysis can be carried out for all combinations of SHORT/LONG and MAIN/CONNECT states









Period doubling MAIN and CONNECT states

Extension to conditions for period doubling cascade using quantities

$$L_k^- = e^{-(kTR - TD - D)/\tau_i}, \quad L_k^+ = e^{-(kTR - D)/\tau_i}, \quad L_0^{\pm} = +\infty$$

$$S_k \qquad \begin{array}{cc} A & B \\ x_A^1 y_A^1 & x_A^2 y_A^2 \\ x_B^1 y_B^1 & x_B^2 y_B^2 \end{array} \quad \rightarrow \quad \begin{array}{cccc} A & B & A & B \\ x_A^1 y_A^1 & 0 & 0 & x_A^2 y_A^2 \\ x_B^1 y_B^1 & 0 & 0 & x_B^2 y_B^2 \end{array} \quad \rightarrow \quad \begin{array}{cccccc} A & B & A & B & A & B \\ x_A^1 y_A^1 & 0 & 0 & 0 & 0 & x_A^2 y_A^2 \\ x_B^1 y_B^1 & 0 & 0 & 0 & 0 & x_B^2 y_B^2 \end{array}$$

$$I_k \qquad \begin{array}{cc} A & B \\ x_A^1 y_A^1 & x_A^2 y_A^2 \\ x_B^1 y_B^1 & x_B^2 y_B^2 \end{array} \quad \rightarrow \quad \begin{array}{cccccc} A & B & A & B & A & B \\ x_A^1 y_A^1 & 0 & 0 & x_A^2 y_A^2 & 0 & 0 \\ x_B^1 y_B^1 & 0 & 0 & x_B^2 y_B^2 & 0 & 0 \end{array} \quad \rightarrow \quad \begin{array}{cccccccccc} A & B & A & B & A & B & A & B & A & B \\ x_A^1 y_A^1 & 0 & 0 & 0 & 0 & x_A^2 y_A^2 & 0 & 0 & 0 & 0 \\ x_B^1 y_B^1 & 0 & 0 & 0 & 0 & x_B^2 y_B^2 & 0 & 0 & 0 & 0 \end{array}$$

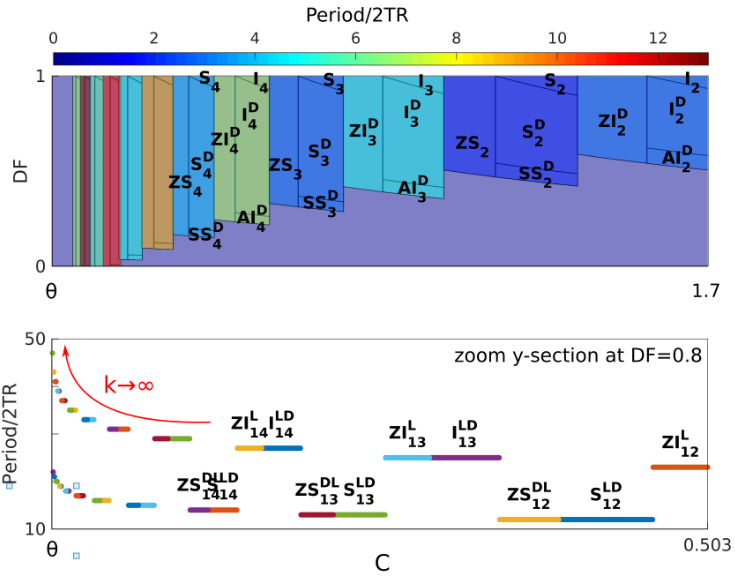
$S_k = SAB_k, SA_k$ and $I_k = INTD_k, INT_k$ PERIOD: $T_{S_k} = 2kTR$ and $T_{I_k} = 2(2k-1)TR$

Theorem

Period doubling solutions of the other MAIN states cannot occur: AP, AS, SA, INTA

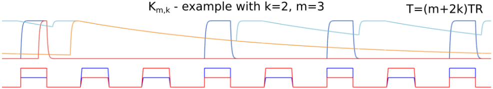
$\rightarrow$ similar condiderations can be made with CONNECT states

Period doubling MAIN and CONNECT states

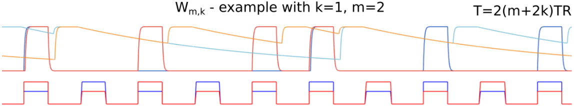


Segregated switching MAIN and CONNECT states K_k^m and W_k^m

m=# of skipped tone and k=# of segregated cycles



B	A	..	B	A	..	B	A
11	0	..	0	1	..	0	1
01	0	..	0	1	..	0	1
		k			m		

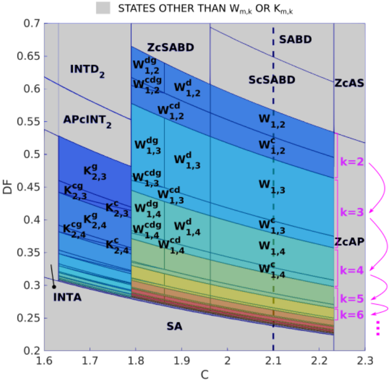


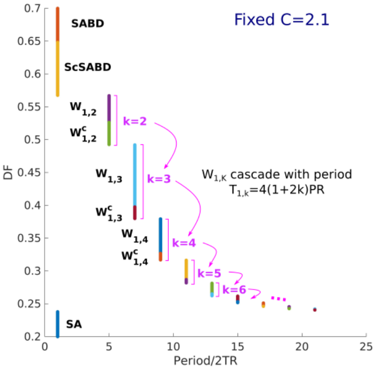
B	A	..	A	B	..	A	B	A	B	..	B	A	..	B	A
11	0	..	0	0	..	0	0	01	0	..	0	1	..	0	1
01	0	..	0	1	..	0	1	11	0	..	0	0	..	0	0
		k			m					k			m		

Theorem

W_m^k may exist for m <u>even</u>. K_m^k may exist for m <u>odd</u>

Cascades of W_k^m and K_k^m in 2D



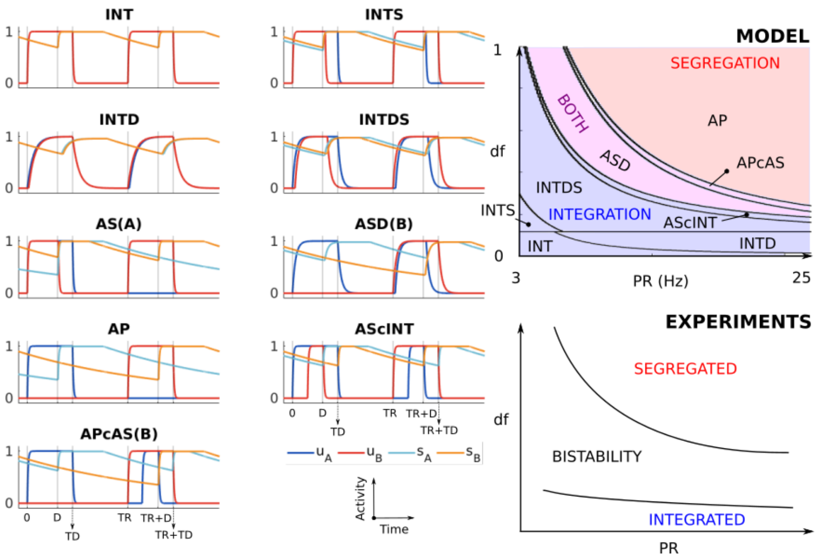


Extension to CONNECT states c, d, g, cd, cg, dg and cdg

The case $D < TD$ under $c - b \geq \theta$

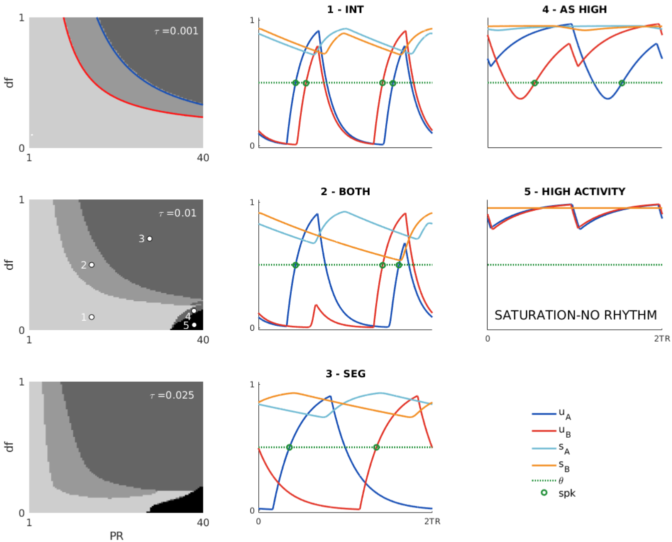
Theorem

If $c - b \geq \theta$ these are the all possible stable states (LONG states cannot exist)



Computational analysis under smooth gain and inputs, non slow-fast scales

Sidmoid $H(x) = [1+\exp(\lambda(x-\theta))]^{-1}$ with fixed $\lambda = 30$ and τ/τ_i up to $\sim 10^{-1}$



Provide insights on the case $TD + D > TR$

RECAP AND MORE...

- Detailed analytical study of a rich repertoire of model states
  - All possible $2TR$-periodic states
  - Cascades of period doubling and segregated switching
- Computational analysis under smooth conditions
- Link $2TR$-periodic states with AS perceptions
  - States occupying larger region of existence
  - Quantitative agreement when varying parameters influencing AS perception
- Neuro-inspired model of AS with biophysical parameters
  - Slow inhibition masks the perception of subsequent tones during segregation
  - Fast excitation enables integration for large tones' pitch differences
- More... (not shown in this presentation)
  - Bistability via inhibitory feedback from a third, intrinsically oscillating unit
  - Study of other exotic states (cycle skipping)

Thank you!!