A SHORT TEST ON THE HISTORY OF MATHEMATICS

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Before proceeding, the reader is invited to answer this question:

Which of the following was a mathematician?

Pythagoras

Omar Khayyam

Florence Nightingale

Lewis Carroll

What does "being a mathematician" mean?

To say that a mathematician is a person who does mathematics is obviously not helpful, because that requires answering the follow-up question "and what is mathematics?". In my judgment, that's a loaded and unproductive question (see Greer, Kollosche, and Skovsmose, in press). More constructive – and somewhat more answerable – questions are "What human activities get to be called mathematics?" and "Who gets to be called a mathematician?", which acknowledge the social agreement necessary for meaning, as argued by Wittgenstein, for one.

A salutary caution that the two questions are historically and culturally relative was provided by Cullen (2009, p. 592), writing in the context of Chinese culture, but with general relevance, when he warned against:

> [...] the idea that there is *a priori* a universal ahistorical cross-cultural "natural kind" called "mathematics" that can simply be located and

> studied once one can penetrate the linguistic barrier to see what it is called in Chinese, and on which one can simply impose all the structures and expectations that a modern person finds in the subject called "mathematics" in twenty-first-century English.

Høyrup (2013) provides typically detailed elaboration of this point in the context of Babylon. He suggests that being a mathematician goes beyond skill in calculation to require, in some sense, "fairly deep mathematical insight into the structures that are dealt with" (p. 116). If I understand it correctly, a central point that Høyrup is making is that the inference that such insight exists does not require documentation of formal proofs, but may be inferred from the sophistication of the methods described. It reminds me of Hardy receiving the famous pages from Ramanujan, consisting of simply a list of results – some known, some new, some surprising to Hardy, some incorrect. Hardy had no trouble in characterizing these as the work of an extraordinary mathematician.

Following that cursory treatment of a fascinating, complex, and consequential debate, I return to the test with which I began, put forward what I consider to be the most appropriate answers, and draw some important points from each case.

Before I turn to discussion of the four individuals, let me make it clear that in terms of knowledge of the history of mathematics I am at best a fascinated dilettante.

Pythagoras (c. 580-500 BCE)

> Pythagoras the mathematician finally perished A.D. 1962 (Netz, 2003, p. 272)

Some years ago, I was taken aback when I read the line above. I thought Pythagoras was *some kind* of a mathematician – precisely what kind I did not know. I was fairly sure that he had nothing to do with the original discovery of the theorem attached to his name, as opposed to what Høyrup (2024) refers to as the Pythagorean rule.

In the quotation above, Netz was referring to a book by a German historian of mathematics (Burkert, 1962/1972) who very carefully analyzed the available historical record. He summarized that Pythagoras represented "not the origin of the new, but the survival or the revival of ancient, pre-scientific lore or wisdom, based on superhuman authority and expressed in ritual obligation" (from the preface to the 1962 edition, as translated in the 1972 edition).

A comprehensive entry on Pythagoras, which draws heavily on Burkert, can be found within the online Stanford Encyclopedia of Philosophy (SEP), which makes abundantly clear how unreliable the writings about Pythagoras are. None were contemporary or even nearly so, and particularly striking is how many times forgeries are mentioned. The article ends by stating that the consensus among scholars is that *Pythagoras was neither a mathematician nor a scientist.*

Gainsford (2016) conveys the flavour of Pythagorean thought by quoting Plutarch:

> One might conjecture that the Egyptians hold in high honour the most beautiful of the triangles, since they liken the nature of the Universe most closely to it … This triangle has its upright of three units, its base of four, and its hypotenuse of five, whose square is equal to that of the other two sides. The upright, therefore, may be likened to the male, the base to the female, and the hypotenuse to the child of both, and so

> Osiris may be regarded as the origin, Isis as the recipient, and Horus as perfected result. Three is the first triangular odd number, four is a square whose side is the even number two; but five is I some ways like to its father, and in some ways like to its mother, being made up of three and two.

As he comments, this isn't mathematics, it's numerology and wordplay. It is true that Høyrup maks a case for considering as mathematicians "those Pythagoreans […] who […] explored the properties of 'the odd and the even' and of triangular and square numbers" but I suggest that this kind of activity was incidental, not central. Moreover, his overall view (2024, Vol. 1, p. 52) is that:

> In spite of the prevailing opinion in popular histories of philosophy and science it seems most safe to disregard the incoherent accounts of Pythagoras the mathematician and Pythagoras the experimental physicist. In all probability he was much more of a guru, a spiritualized teacher, than a "scientist".

So why is there such a marked difference between the image of Pythagoras and the reality revealed, as far as that is possible, by painstaking scholarship? The only explanation I can propose is that it is to do with the mythologizing of history in the service of white (European) intellectual superiority.

Omar (or Umar) Khayyam (c. 1048-1126 BCE)

Some time ago, two Iranian students in my class, very familiar with Omar Khayyam's poetry, were amazed to be told that he was also an eminent mathematician. He was also a philosopher and an astronomer. For a summary of his mathematical contributions, in particular on solutions of cubic equations through graphical means (he foreshadowed Descartes in elucidating the relationship between algebra and geometry) and his critique of Euclid's parallel postulate and the Greek theory of ratios as numbers, see the entry on him in Stanford Encyclopedia of Philosophy).

Here I address two deeper questions. In the SEP essay, it is pointed out that "Whereas his mathematical works and poetry have been the subject of much discussion, his recently edited and published philosophical works have remained a largely neglected area of study". In his poetry, "he challenged religious doctrines, alluded to the hypocrisy of the clergy, cast doubts on almost every facet of religious belief, and appears to have advocated a type of humanism". Later we read that "Whereas Khayyam the philosopher-mathematician justifies theism based on the existing order in the universe, Khayyam the poet, for whom suffering in the world remains insoluble, does not talk about theism, or any type of eschatological doctrine, as a solution to the problem of the meaning of human existence". What I see in this is in stark contrast to those who – to this day – read into "the unreasonable effective of mathematics" (Wigner, 1960) evidence for the existence of "God".

The second point I want to make is that the general ignorance of Khayyam-as-mathematician is the flip side of the worship of the image of Pythagoras. In his book "99 Variations on a Proof", Philip Ordling (2019) reveals how his question "Why isn't there a Renaissance solution of the cubic by intersecting curves?" was answered by a reader "Because that approach was mastered [by Omar Khayyam] four centuries earlier". Ordling then recounts how he consulted Dieudonné's historical review of algebraic geometry only to find the discussion "jumping from Apollonius to Descartes without citing the contribution of a single Arab". Ordling's final comment:

"Just because some mathematicians like to think of themselves as being above the fray doesn't mean that our history isn't written by the victors". For more on Eurocentrism, "the Greek myth", and intellectual white supremacy, I recommend the work of Jens Høyrup (see Greer, 2022).

Florence Nightingale (1820-1910)

I deem her to have been a mathematician, though perhaps my opinion is more debatable in this case. I decline to discuss the opinion of some mathematicians that statistics is not a branch of mathematics. Even less can I take seriously Hardy's conceit of "real mathematics" which would rule out Nightingale as a mathematician since what she did was undeniably useful.

Her experience in a hospital during the Crimean War made clear the need for reform of treatment for wounded soldiers and she based her case on carefully gathered, organized, and presented data. To this end, she invented forms of data representation to facilitate communication and understanding (Andrews, 2022). And she successively argued her case for reform in the medical treatment of soldiers with the army establishment, a major achievement for a woman in Victorian England (she was a profound feminist in many respects). She interacted with major figures in the developing field of statistics applied to social phenomena, in particular Quetelet. Maindonald and Richardson (2004) argue that her work was foundational to the concept of evidence-based medical policy. She was central in conceptualizing and realizing the modern profession of nursing. And in the later part of her life, she acted as a statistical analyst and adviser for the government, promoted essential improvements in sanitation in both Britain and India, and wrote extensively on social justice, particularly in regard to British rule in India. On imperial indifference to famine, for example, she wrote "we say nothing of the famine in Orissa, when a third of its population was allowed to whiten the fields with its bones" (cited by Galeano, 2009, p. 225).

A quotation attributed to George Bernard Shaw (which seems impossible to authenticate) is that "the mark of a truly educated person is to be deeply moved by statistics". Florence Nightingale surely embodied this concept of "statistical empathy" (Mukhopadhyay & Greer, 2007).

Lewis Carroll (1832-1898)

Charles Lutwidge Dodgson (the real name of Lewis Carroll) was a mathematics lecturer at Oxford so would qualify to be called a mathematician on that account alone. Beyond that, his mathematical accomplishments were by no means trivial, and beginning to be more recognized (e.g. Moretti, 2015). His important contributions (notably in logic, the relationship between natural language and mathematics, problem solving, and the practicalities of voting systems and cryptology) are well summarized in Wilson and Moktefi (2019) to which I refer the interested reader. Here I concentrate on something I consider of great importance he illuminated that it seems has largely gone unrecognized.

It is now widely acknowledged that what are called "word problems" or "story problems" exemplify a bizarre genre. In such texts, typically, what appears to be a problem relating to aspects of what I will call (without attempting to enter that philosophical black hole) "the real world" does not bear scrutiny. The issue is discussed at some length in Verschallel, Greer, & De Corte (2000), including analysis of this problem set by Carroll in a popular magazine:

> If 6 cats kill 6 rats in 6 minutes, how many will be needed to kill 100 rats in 50 minutes? (Verschaffel et al., 2000, pp.132-134)

As a normative practice, this problem can be "solved" arithmetically by a routine procedure, the method of double proportion. Carroll violated the norm by insisting on making specific the assumptions as to how the killing might take place and by taking into account the physical reality of cats and rats – in other words treating the exercise as one of mathematical modelling. That led him to suggest three alternative answers. It is notable that when the problem is presented in Elran (2021) no hint is given of any other possibility than executing the standard procedure. When teaching at Oxford, Carroll used similar examples in which extreme choices of the parameters should have set off alarms, but generally did not.

Some mathematicians (e.g. Toom, 1999) take the view that word problems, however unrealistic, are merely vehicles for learning the "real" decontextualized mathematics. I would argue that what such a position does is to inculcate in children a blind faith in simplistic modelling which has dire consequences when they are, as adults, faced with cases of "formatting" of complex situations, as Ole Skovsmose has written about for decades. I assert that there is no reason why children from an early age should not learn to discriminate among cases of exact models, approximate models (to some degree of accuracy), and altogether inappropriate models.

Conclusions

I have told the stories of how a cult leader, a polymath with deep views on the nature of existence and humanist skepticism about the divine in mathematics, an important figure in the development of social statistics, and an original and insightful mathematician morphed in the public imagination into one of the founders of mathematics, a poetic oenophile, "the lady with the lamp" and a wonderful writer of children's stories. The main point of the exercise was to illustrate that common beliefs about mathematicians and of historical figures in general may be false. The image of “the lady with the lamp” for example diminishes the real historical figure of Florence Nightingale. The first two examples illustrate, as Ordling put it, the writing of the history of mathematics by the winners, both reflecting and promoting intellectual white supremacy. Lewis Carroll satirically made a point that is vital about the difference between the unreasonable effectiveness (Wigner, 1960) of models of physical phenomena and the reasonable ineffectiveness of models of social phenomena.

Any reader who teaches mathematics might care to give the test to their class and then invite them to read this paper. Other examples could be added. One that comes to mind is M. C. Escher. He did not do well in mathematics at school, by the usual criteria, and did not like the subject. His early work was largely intuitive but later in his career he interacted with a number of mathematicians, notably Polya and Penrose (for a brief discussion of Høyrup’s views on relationships between aesthetic productions and the mathematical structures that may be seen in them, see Greer (2022).

What this essay should make clear is that doing history of mathematics well is extremely hard work, especially in relation to times long past. And with regard to the case of Pythagoras, in particular, might we not ask if it is all right to lie to children in school (see Raju, 2017) and to maintain myths in prestigious publications. For example, the “fact-checked” Encyclopedia Britannica describes him as a philosopher and mathematician and has an illustration entitled “Pythagoras demonstrating his Pythagorean theorem in the sand” full of symbolism (and with a steam train in the background). It is time to interrogate the chasm between what mathematicians do, taking into account the historical, cultural, societal, and political contexts, and the images projected in school and beyond.

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