INTEGRATING ROCK CLIMBING INTO THE MATHEMATICS CURRICULUM — AN ENACTIVE AFFORDANCE-BASED APPROACH

Gabriela Hernandez

University of California, San Diego
gmhernandez@ucsd.edu

Christian Kronsted

Merrimack College, Massachusetts
kronstedc@merrimack.edu

Abstract

This paper proposes a novel approach to mathematics education by integrating rock climbing into the math curriculum. Based on embodied cognition and affordances, we argue that both mathematics and rock climbing involve embodied problem-solving processes. Current mathematics education contributes to an exclusionary culture hostile to marginalized students. Rock climbing offers a unique opportunity to address these issues by providing a concrete and embodied context for mathematical problem-solving. Through the lens of enactivism and ecological psychology, we demonstrate the overlap between the cognitive processes involved in rock climbing and mathematical problem-solving. Both activities involve the continuous cycle of planning, execution, and assessment through affordance-facilitated agent-environment couplings. This overlap allows rock climbing to serve as a powerful analogy for the problem-solving processes inherent in mathematics. Affordances are crucial for understanding why rock climbing and mathematics offer rich problem-solving experiences. Climbing routes and mathematical equations become meaningful through training and enculturation, revealing possibilities for action within an action-perception-affordance loop. The integration of rock climbing in mathematics education can foster a more engaging learning environment, enhance problem-solving

skills, and bridge the gap between seemingly abstract (in-the-head) mathematical concepts and real-world applications.

Introduction — Rethinking Mathematics in the Name of Equity

Despite decades of evidence that learning is better facilitated through activities and movement, much of high school and college education is still overwhelmingly sedentary (Becker, 2023). Simultaneously, education in the United States (strongly exacerbated by the COVID-19 pandemic) is undergoing a crisis as students fall further and further behind the rest of the industrialized world across the humanities, critical thinking, collaboration, and STEM (Graesser et al., 2020; *NAEP*, 2023; Samuel et al., 2017; Schleicher, 2019). Due to inequity STEM as a profession is still predominately white straight and male (National Center for Science and Engineering Statistics, 2023). Furthermore, even the newest published volume on mathematical education is strikingly disembodied (Bicudo et al., 2023). As many education specialists have argued, it is urgently time to integrate art and movement into institutional education — kindergarten through college (Becker, 2023; Kisida & LaPorte, 2021; Mobley & Fisher, 2014).

We are addressing the need for a more equitable and just culture in mathematics by transitioning from the deeply entrenched and rigid binaries of Cartesian dualism still prevalent in math education today to a more embodied learning approach. We here use autopoietic enactive cognitive science to demonstrate why rock climbing can be effectively integrated into the math curriculum. Through this embodied approach to mathematics, we aspire to change the culture at large as the perception of what it means to do math and who can do math is expanded and diversified.

Using enactivism, we demonstrate that there are overlaps between mathematical cognition and rock climbing. This approach is to show how a future (fun, engaging, and effective) math curriculum can be built around the incorporation of climbing into math. This type of embodied integration with math is not unique to rock climbing; it can be done with dance and many other forms of physical activity (Kronsted & Gallagher, 2021; Moyer-Packenham et al., 2016; Stern & Bachman, 2021). However, we here choose to focus on rock climbing because rock climbing requires an explicit form of problem-solving that easily transfers to the domain of mathematical problem-solving.

Specifically, we argue that both rock climbing and solving math problems involve the process of explicitly creating a cognitive "plan of action" that consists of expectations and planned movement sequences (sensorimotor and attention schemes), in accordance with social-cultural rules and material conditions. Such planning involves the body recombining its own sensorimotor dispositions, attentional patterns, and expectations and reordering the body's field and salience of affordances - All to meet the demands of the problem within a specific social, cultural, material, and embodied circumscribed frame. Notably, both rock climbing and math problem-solving involve the periodic "backing-off" to asses and update the cognitive plan. In this way, there is a skillful recursive loop between execution and planning that undergirds both math and rock-climbing problem-solving. Training the skillset of one is also training the skillset of the other. Thus, rock climbing can fruitfully be implemented into mathematics courses.

Furthermore, enactive cognition as a theoretical framework is still often accused of being unable to deal with so-called "representation-hungry," higher-order, "offline," abstract cognition such as simulation or planning (Aizawa, 2010; Clark & Toribio, 1994; Edelman, 2003; Williams, 2018). In this line of argument, complex mathematics "in the head" is the prime example of a cognitive function that is offline, higher-order, and seemingly impossible without symbolic mental representations. While plenty of thinkers have already outlined accounts of higher-order non-representational cognition, demonstrating the overlap between rock climbing and mathematics is yet another blow against the "scaling-up" objection to embodied enactive cognitive science(Brancazio & Segundo-Ortin, 2020; Gallagher, 2017; Hutto & Myin, 2013; Kronsted et al., 2023; Oliveira et al., 2021; Zahnoun, 2021).

First, we provide a quick tour through the basics of mathematics and then the basics of rock climbing. We then proceed to explain these two domains through affordance-based autopoietic enactivism. We then look more explicitly at the overlap in cognitive processes in rock climbing and math. We end with notes about the next steps for concrete implementation into actual math classes.

Mathematics What is it? — Problems with Current Approaches

Math education often involves the instruction of “abstract” concepts, logical reasoning, one-way problem-solving, and precise discourse. Traditional practices in the math classroom often prioritize rote memorization, algorithmic procedures, and binaries of right or wrong. These features of the traditional mathematics classroom are prevalent across the traditional American education system. Though researchers are aware that these outdated methods (Becker, 2023)--informed by behaviorism and traditional cognitive science--are ineffective, the field of math education is quite stagnant in its evolution to break beyond rudimentary instructional practices (Bicudo et al., 2023).

Mathematics, in particular, has an exclusionary culture that is shaped by assumptions, norms, and values that are informed by white supremacy and the patriarchy (Hottinger, 2016). Gendered and racial hierarchies are pervasive within mathematics classrooms, leaving women, Black, and Latinx students at the bottom (National Center for Science and Engineering Statistics, 2023). Additionally, many students perceive the culture surrounding mathematics to be extremely rigorous and elite, which naturally excludes students from marginalized groups (Leyva et al., 2021). Traditionally, mathematics has been used as a tool for imperialism and colonialism as it acts as a means to exclude and stratify by justifying and perpetuating binaries and hierarchies (Gutierrez, 2017).

Despite decades of scientific inquiry into the naturalistic origins of math, many students are still taught the "romance of math" (Lakoff & Núñez, 2000; Núñez, 2008). For instance, math as objective, abstract, pure, and rote procedures and formulas is still pervasive in the classroom (Battey & Marshall, 2023). Platonism concerning mathematics posits that abstract entities, which are objective and timeless, exist independently of the physical world and the symbols employed to denote them. Students around the world are still taught romantic platonic definitions of mathematics as nature's language of "truth." For example, one classic definition of math that still lingers in the literature and education practice of math is the National Research Council's 1989 definition:

> As a practical matter, mathematics is a science of pattern and order. Its domain is not molecules or cells, but numbers, chance, form, algorithms, and change. As a science of abstract objects, mathematics relies on logic rather than observation as its standard of truth, yet employs observation, simulation, and even experimentation as a means of discovering truth. (National Research Council, 1989, p. 31)

The surviving myth amongst scholars and laypeople alike is that math connects to objective, timeless platonic entities. As Nunez aptly summarizes the myth:

> Mathematics has a truly objective existence, providing structure to this universe and any possible universe, independent of and transcending the existence of human beings or any being sat all. Mathematics is abstract and disembodied–yet it is real. Human mathematics is just a part of abstract, transcendent mathematics (the concrete and mundane side of it) (Núñez, 2008, p. 340).

In extension of the romance of math is the myth that some people simply do not have the capacity to tap into the true nature of reality. In contrast, other people are innately gifted in understanding and manipulating the building blocks of existence.

Despite problem-solving being hugely important for math education, "problem-solving" is a nebulous concept in the literature still often connected to the platonic notion of mathematics as capital-T truth (Papert, 1980). Halmos (1980) even argues that problem-solving is at "the heart of mathematics" (p. 524), and Schoenfeld (1985; 1992) conceptualizes problem-solving as the very core of mathematics. As critical social scientist and math educator Gutiérrez argues, the culture of professional mathematics and math research still believes that; "mathematics carries with it something separate from humans that can be conveyed to individuals, thereby affording them a more powerful view of the world" (Gutiérrez, 2013). The upshot of the platonic romance of math is that mathematical problem-solving is seen as independent of earthly contexts such as emotion, hunger, fatigue, socioeconomic status, being a bored teenager, or any of the other embodied and social factors reserved for earthly creatures. Consequently, the myth of math as disembodied and context-free has led to severe equity problems in math education and the STEM workforce.

Again, we see that at the center of the platonic myth of mathematics is the commitment to the idea that problem-solving in math is disembodied. Problem-solving is often framed as a novice to expert progression, characterized by an information-processing approach (Garofalo and Lester, 1985; Schoenfeld, 2013) and the concept of metacognition. Many math education researchers rely on Flavell's definition of metacognition in their conceptions of problem-solving (Garofalo and Lester, 1985; Stillman & Galbraith, 1998):

> Metacognition refers, among other things, to the active monitoring and consequent regulation and, orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective (Resnick, 1976).

Some researchers even argue metacognition is the driving force in problem-solving and is embedded within every phase of the problem-solving process (Lester, 1994).

Classically, while metacognition involves cognitive processes, it does not inherently involve embodiment. Instead, it focuses on cognitive processes such as monitoring, regulation, and orchestration, which are often considered disembodied mental processes. In metacognition, the focus is on mental activities rather than embodiment (Clark & Toribio, 1994).

If mathematics is platonic, disembodied, and context-free, then there is no reason to integrate advanced mathematics teaching into contextual, embodied, goal-oriented, and pragmatic teaching. Such an unwillingness to connect math education with real conditions on the ground, in turn, hinders entry into math professions except for a select few types of students. The tired old report, “I am just not a math kinda person,” is directly connected to structures that uphold the romance of math and, with it, the myth that there are “math” and “non-math” people.

Looking at the numbers, again we see that the platonic conception of math is not socially innocent. Historically, platonic math education has led to racial, gender, and class disparities in math learning and professionalization. In fact, these issues are still pervasive in STEM today (National Center for Science and Engineering Statistics, 2023). The lack of diversity is evident as women comprise only 28% of the workforce in STEM. The lack of women in the STEM workforce is connected to the lack of women majoring in STEM in college. In fact, men significantly outnumber women, where 21% of engineering majors and 19% of computer science majors are women (National Science Board, 2018). In addition to the lack of women in the field of STEM, Black, Indigenous, and people of color (BIPOC) students are also vastly underrepresented. In 2018, only 11.4% of engineering bachelor's degrees went to Hispanic students, and only 4.2% to Black students (Roy, 2019). Several key factors perpetuate this gendered and racialized gap, such as gender and racial stereotypes, male-dominated cultures, fewer role models, and anxiety surrounding mathematics (Spencer et al., 2016; Leyva et al., 2021). The underrepresentation of women and people of color in the field of STEM has significant drawbacks, especially considering the economic benefits of being in the STEM workforce (National Center for Science and Engineering Statistics, 2023). Holding on to the idea that math is context-free facilitates educators overlooking or ignoring equity issues in math education.

One way to change the equity problem in mathematics as a professional field is by changing *how we teach math*. While humans are embodied and kinetic beings, much math is still taught in a dry, uncontextualized, sedentary, disembodied fashion. From other fields, we know that improving the quality of instruction when students are first introduced to the subject matter improves the chances that students other than white, male, cis-gendered, wealthy, and rich students will enter the field. As an example, we can point to the success stories of the non-profit initiative Black Girls Code in getting young black women to enter computer science education and careers (https://www.wearebgc.org/). In short, better instruction methods mean more people entering a given field - hating math class does not produce more mathematicians. Loving math class (even despite not having an immediate affinity for math) does produce more mathematicians. Integrating various movement-based approaches to math education and learning (in this case, rock climbing) can help alleviate some of the inequities in the college-to-STEM pipeline.

Along with a growing field of researchers, we argue that the metacognition approach to mathematics and math learning is misguided. Math is not a matter of disembodied processes but is rather deeply embodied — for example, learning math has been shown to be tied to learning new sensorimotor behavioral schemes (Abdu et al., 2023; Lakoff & Núñez, 2000; Menary, 2015; Shvarts & Abrahamson, 2023; Stern & Bachman, 2021). We, therefore, add to the literature on embodied math by demonstrating the overlap between math and rock climbing. We can understand why these two activities cognitively overlap by looking at both through the lens of enactive embodied cognition and affordances. While the two activities do have some obvious dissimilarities, one does not get hurt when getting a math problem wrong, we here focus on highlighting the relevant similarities between math and rock climbing.

What is Rock Climbing?

Rock climbing is a physical and intellectual activity where individuals ascend either natural rock formations or artificial climbing walls made of plastic to make it to the top. For this paper, we will consider rock climbing, specifically at an indoor artificial climbing wall known as a *climbing gym.* The climbing walls consist of various routes— a pre-determined arrangement of climbing holds, delineated by different colors, intended to be scaled by individuals. The routes are often changed every two to three months, giving climbers plentiful time to work, or *project*, various routes. Climbers must use specialized equipment such as harnesses, ropes, and climbing shoes to safely and optimally navigate various types of climbing features. For the convenience of the reader, we have provided a glossary at the bottom of this section with some of the most commonly used specialized rock climbing terms.

Rock climbing comes in different disciplines, but we will only discuss the disciplines that are observed in indoor rock climbing:

a) **Bouldering.** In bouldering, climbers tackle short, challenging routes, known as *problems.* Bouldering is done without a harness or rope. In a gym, these routes are often 10-15 feet high, and safety is ensured by thick pads below that cushion falls. Bouldering is often conceptualized as a sprint, in which climbers must quickly, intentionally, and precisely move through powerful moves in a short time.

b) **Sport climbing**. Sport climbing is a discipline conceptualized closer to a marathon, as it is a challenge to endurance and technique over a tall route, which often ranges from 40 to 60 feet in the climbing gym. In lead climbing specifically, climbers clip into quickdraws for safety. Thus, if an individual falls on a sport climbing route, they will fall to their last point of safety. In top-roping, a rope runs through an anchor at the top of the route, and the climber is attached to the rope via a harness.

c) **Speed climbing.** Speed climbing is a discipline in which there is a set route that is intended to be climbed as quickly and efficiently as possible. The route requires immense power and coordination, and at Olympic levels, the best climbers are as fast as five seconds. The margin for error is minimal. Indeed, the perfection of pre-planning is essential to being a successful speed climber.

Rock climbing offers a distinctive problem-solving arena, exploring embodied cognition, collaboration, individual work, and unique action possibilities absent in traditional academic contexts because problem-solving is instantiated in the physical material. When rock climbing in the gym, climbers partner up on different routes. One person is climbing up the route while the other is belaying. When someone is climbing at their limit, they are challenged right beyond their skill level on a specific route. The person on the wall embarks on their journey up the route, facing unique challenges that must be overcome numerous times as they progress up the wall. When the climber is facing the crux— there may be several— they must stop and calibrate. This could mean that the climber finds a decent hold to rest on while calibrating, or they let go altogether and fully rest as they prepare for the challenging movement. Sometimes the climber incorrectly "anticipates the beta," and the climber must back away and reevaluate. Additionally, the belayer may assist the climber in overcoming the crux by offering "beta," but ultimately, it is up the climber to problem-solve their unique body positioning and physical and mental capabilities. Ultimately, what distinguishes an expert climber from a novice climber is one that climbs efficiently and conserves energy, especially through the most challenging parts. Elegance is the manifestation of efficiency and energy conservation. In essence, an expert rock climber is an expert elegant problem-solver.

In general, each difficult position that a climber finds themselves in is experienced as a unique sub-problem that must be solved. To advance up the wall the climber must both solve hold-to-hold problems, and the overarching problem of the whole route. It is often said by people in the climbing community that "rock climbing is just problem-solving with the body."

One of the ways we can see that rock climbing is problem-solving like math is the possibility of mental fatigue. These sub-problems that must be solved, particularly when a climber is at their limit, are both physically and cognitively demanding. This is especially the case for lead-climbing, since this form of climbing has added dimension of fear. In lead climbing in the gym, the climbers bring up the rope with them, clipping into quickdraws every 5-7 feet, meaning that if they fall they can fall much further and potentially more painfully than in a regular top-rope climb. This added layer of fear induces more intentional and mindful movement, heightening the cognitive load as climbers must strategize not only for physical performance but also for safety. Therefore, the mental fatigue experienced in rock climbing, especially in lead climbing, is very similar to the cognitive fatigue in mathematical problem-solving, that also stems from increased cognitive load. While math students typically will not hurt themselves if they fail at a math problem, the experience of being overwhelmed, at one's limit, or in front of an insurmountable obstacle remains analogous.

The connection between mathematical problem-solving and problem-solving in rock climbing becomes even more obvious when we look at bouldering, in which each route is literally called a "problem." Bouldering routes are lower to the ground, and are designed to be tricky with a specific solution. Each route requires concentrated physical and mental exertion, happening in very short periods of time. When entering a bouldering area, you will often see several climbers working on the same problem, sharing strategies and different techniques to make it to the top of the problem, just like math students in a classroom working together to solve a problem. However, once again, when on the wall, it is ultimately up to the climber to problem-solve, given their unique body positioning and capabilities.

Each individual has different constraints that influence their personalized approach. Climbers' diverse strategies, shaped by physical and mental constraints, not only enhance their problem-solving skills but also serve as valuable analogies, offering insights that can be applied to various academic problem-solving domains.

As a climber makes their way up a climb that is appropriately challenging for their skillset, they must constantly renegotiate their body positioning to preserve energy and attentional resources. Similarly, in the realm of math problem-solving, students are using both energy and attentional resources — solving math problems is inherently connected to schemes of attention (Mason, 2023). The explicit physical nature of rock climbing demands a strong sense of attention and presence in the moment, as physical limitations and a fear of falling force the climber to be attentionally anchored in the present moment. The climber must intensely focus on their body, movements, and environment. Overall, learning the embodied and attentional techniques of rock climbing can be directly transferred back into the domain of math problem-solving because the activities require similar cognitive and attentional skills.

Climbing Glossary

Term	Explanation
Backing-off	Taking a step back from a climbing sequence to reassess the movement. This is often done in a somewhat restful position.
Belaying	Managing the safety rope system to protect a climber from falling while they ascend or descend the wall.
Beta	The specific/subjective sequence for a section of a route (one possible solution for a problem).
Bouldering	A type of climbing where climbers tackle short, challenging routes, known as problems. Bouldering is done without a harness or rope. In a gym, these routes are often 10-15 feet high, and safety is ensured by thick pads below that cushion falls.
Crux	The hardest parts of the route.
Cruxing Out	When the climber is actively at the hardest part of the climb.
jug	A type of climbing hold that is easy to grip, hang off of, and conserve energy on. A jug provides possibilities for further movement upwards.
jug-able	To grab a hold optimally to conserve energy while grabbing it.

Lead-climbing	A type of climbing where climbers clip into quickdraws for safety. If an individual falls while lead climbing, they will fall to their last point of safety.
Projects	A route at a climber's limit that takes an extended amount of time to send.
Route	A pre-established climb/path in the gym, designated by different colored holds and varying in difficulty.
Send	Completing a route without falling.
Take	The act of the belayer taking slack in the rope which is often followed by resting.
Top-roping	A type of climbing where a rope runs through an anchor at the top of the route, and the climber is attached to the rope via a harness.

Affordances

Relations of Possible Action

To connect rock climbing and mathematics, we need to understand the conceptual apparatus of affordances. Originally coined by James Gibson, affordances are possibilities for action that exist between an animal and features in its environment (Gibson, 1977, 1979). While left fairly vague and barebones by Gibson, research on affordances has blossomed into ecological psychology, enactivism, the skilled intentionality framework, and more. While there are scholarly debates about the ontological status of affordances (are they representations, are they objective or subjective), we here follow the enactive and embodied route of Chemero — affordances are *relations* of possibility that exist between an agent and its environment (Chemero, 2009). In the literature, the most classic (but boring) examples include a cup affording graspability, a chair sit-ability, a bike ride-ability, and so on. In short, cognitive agents (humans and animals alike) experience their environments in terms of what they can do with that environment — in terms of possible actions. This is so even if that environment is virtual, abstract, or laden with symbols (Kronsted & Gallagher, 2021).

Since affordances are relations, a cup does not afford drink-ability without any agents present. A cup also does not afford drinkability if the agent is a spider or some other organism that cannot use the cup whether for embodied, social, or other reasons. Affordances only exist as relations of possibility between specific agents, with specific bodies, and their specific environments (Baggs & Chemero, 2019). For example, for the untrained person, a configuration of American Football players is simply experienced as an intimidating wall of meat and armor. However, for the trained agent, the configuration affords a wealth of possibilities, passability, tackle ability, loopholes, openings, trick shots, runs etc. Yet the untrained player or untrained audience simply experiences bodies violently piling on top of one another. The actionable affordance only exists as relations between specific agents and specific conditions. Similarly, the trained mathematician looks at a complicated board full of equations and sees a wealth of affordances, while the untrained student experiences opaque hieroglyphics with a hidden meaning. Affordances are relations for actions that are brought into being as links between specific agents and specific environments.

As relations, affordances exist for each agent depending on their physical structure, capabilities, current embodied states (fatigue, hunger, anxiety, joy, excitement, ability disability, etc.), and more. For example, because of the general structure of the human hand, a crevice on a rock wall affords gripability. However, despite being far more agile the same crevice does not afford grip-ability for a cat since they do not have fingers and thumbs. In reverse, the top of the bookshelf affords rest-ability for a cat, given its size and ability to jump, but does not afford rest-ability to a human. Furthermore, affordances are highly dependent on skill. The rock wall crevice only affords grip-ability for a human who has mastered the right technique, has the right strength, and is not injured. A novice climber looking at a “V12, 8A+” wall will experience very few affordances; the expert, on the other hand, will experience a wealth of possibilities. Only with training does the whole wall become experienced holistically as a series of “jug-able” movement series (Rucińska, 2021).

Training and Skill

To the untrained person, differential equations mostly look like mysterious hieroglyphics, perhaps with an aesthetic of depth and complexity (think here of any movie in which the film is attempting to demonstrate the intellect of the character by placing them in front of a whiteboard full of equations). However, for the person with mathematical training, the “mysterious” quality of the math-hieroglyphics is replaced by meaningful symbols that afford direct actions for solving. This is similar to the experience of hearing a language one does not understand versus learning that language. Once the language is learned, it will never sound the same again because it is now meaningful and affords responding (Cuffari et al., 2015). Whether doing something “manual” like working a Two-

Person Crosscut Saw or manipulating symbols on a screen, affordances become meaningful with skill and enculturation and continually reveal themselves through action in an action-perception-affordance loop.

However, having a wealth of affordance available does not necessitate action. Luckily, agents do not always have to choose between their entire landscape of affordances. Our individual *field of affordances* consists of the affordances that are available and *relevant* to us (Baggs & Chemero, 2019; Rietveld & Kiverstein, 2014). Not all affordances stand out to the agent with equal salience. Rather, depending on embodied factors (thirst, hunger, fatigue, mood, sickness, and health, etc.), social factors (norms, laws, relationships, personal finances, identity, etc.), material factors (rain versus sunshine, cramped space versus wide open, obstacles, etc.), and many other factors, affordance stands out at different strength (Brancazio, 2020; Dings, 2018, 2020; van Dijk & Rietveld, 2017). The affordance of bike-ability does not stand out strongly to a person with a broken leg. Eat-ability does not stand out strongly to the person with an upset stomach. In reverse, the purchase-ability of Romeo Santos concert tickets stands out strongly to the person who loves bachata. The drinkability of the water bottle is highly salient to the thirsty. The scalability of the wall is salient to the rested and excited climber. Across a range of dynamically changing factors, an agent’s field of affordances is constantly changing, including the distribution of affordance salience (Gallagher, 2020).

Nested Social Cultural Affordances

Affordances are not just about immediate action. Affordances are often dense and "nested" — they tell the agent what can be done multiple steps ahead (Araújo et al., 2019; Gaver, 1991; Hacques et al., 2021; Rucińska, 2021). As agents gain more expertise, they start seeing not only the immediate actions available in the environment but, with each affordance, the chain of actions across various timescales that acting on the affordance leads to. That is affordances set up the agent for future action. Experiencing the grasp-ability of the cup is also experiencing the drink-ability of water from the tap. This nested information is present in short-term high-dynamic interactions like sports or dance (Araújo et al., 2019; Kimmel & Rogler, 2018; Kronsted, 2021) and less volatile activities such as solving math problems. With the right training, looking at a quadratic equation means the equation affords not just the first step but the next several steps of solving the equation.

Nested affordances also set up agents for future action across longer time scales; for example, checking the subway schedule also affords going to work (Brancazio & Segundo-Ortin, 2020). Through skill development and socialization, our short and long-term intentions bring about information-rich nested affordances in the environment, that sets us up for future engagement with other affordances. For example, in climbing, expert-level affordances include seeing the whole wall as affording various climbing styles (Rucińska, 2021). Similarly, with training, the mathematician will see equations as affording styles of mathematical engagement (for example, factorable, reducable, integrateable, etc.). Long-term intentions such as planning to see Beyonce in concert next year or finishing a Ph.D. make cascading changes to the agent's dispositions so that they will interact with different affordance with different saliences (Bratman, 1987, 1999). Skill, socialization, and planning bring into being nested affordances, which change available future nested affordances (Gallagher, 2020).

No matter how seemingly "internal" and sedentary the activity (for example, traditional math), acting on affordance is still a matter of embodied action (Kyselo & Di Paolo, 2015). Using pen and paper, gesturing to oneself in the air, modulating once facial features, changes in brain activity, hormone production, breath patterns, and more are all part and parcel of a cognitive process in which the body attune itself to the environment (Beer & Di Paolo, 2023; Rucińska & Gallagher, 2021). "Thinking very intensely in the head" is a process that is facilitated by the environment and a state that the agent is actively perpetuating (much like choosing to hold one's breath). Everything the agent was doing up until the moment of the intense internalized thinking is sensorimotor activity, and even when “thinking in the head” the agent is still actively moving their eyes across the visual field, changing or holding a posture, controlling their breath etc. This view of “internal thought” is supported by the empirical research on brain and body sensorimotor activation patterns which demonstrate that thinking about actions (for example, throwing a ball, or moving a variable from one side of the equation to the other) activates the same sensorimotor circuits in the brain and body short of performing the action (Gallese, 2016, 2020; Gallese & Sinigaglia, 2011; Rizzolatti et al., 2001).

Finally, It is crucial to understand that affordances are also cultural (Rietveld & Kiverstein, 2014), social (De Jaegher et al., 2010), and tied to identity (Brancazio, 2020). We live in dense cultural environments that we navigate with ease because we have been socialized into certain socially and culturally relevant skills and habits that exist uniquely in our social material niches. Lying, for example, is a skill that depends on mastering available social and cultural affordances (Kronsted et al., 2023). While climbing is a very physical skill the activity is still dependent on several social affordances that only become live between the agent and environment once the agent is properly enculturated into a climbing community. As a simple example, climbers are only allowed to use holds of the same color on each wall. Similarly, there are various forms of etiquette around when to climb, how to support others, ask for feedback, and so on. Math is also highly reliant on social and cultural affordances. The symbols used in math only become meaningful affordances within human cultures within specialized math-making contexts. Whether the physicality of an escalator, a rock wall, a traffic sign or a math problem, all require enculturation into a rich social environment of practices (Raja & Heras-Escribano, 2023; Rietveld & Kiverstein, 2014; van Dijk & Rietveld, 2017). The affordances provided by symbols and signs only become affordances to agents socialized into the practice environments.

Autopoietic Enactivism

As a research field enactivism comes in multiple varieties, including sensorimotor enactivism (O'Regan & Noë, 2001), radical enactivism (Hutto & Myin, 2013, 2017), autopoietic enactivism (Di Paolo et al., 2017; Thompson, 2007; Varela et al., 1991), scientific enactivism (Beer, 2023) and others (Meyer & Brancazio, 2023). As our theoretical foundation, we here use autopoietic enactivism (From here, "enactivism" will be shorthand for autopoietic enactivism). On the enactive account, there is no separation between perception, action, and cognition. Only as scientific heuristics do we separate these processes.

On traditional cognitivist models (which are often inspired by or integrated with the romance of math), cognition is understood as an input-computation-output process (Cummins, 1996; Ramsey, 2007; Stokes, 2021; Wilson, 2002). On such a view, “the mind” turns sensory input into mental representations with content that is intentionally directed at the world. Such representations are then manipulated before “the mind” produces an output signal (This is overarchingly still true even for newer models such as predictive processing despite their claims of being anti-cognitivist (Parr et al., 2022). Most varieties of enactivism are decidedly anti-computational and anti-representational. Enactivism strongly rejects, mental representations, computation, and the input-output model of the mind. In extension, many enactivists and embodied cognition researchers also reject the romance of math (Abrahamson & Sánchez-García, 2016).

Enactivism grounds all cognition in the ongoing activity of the physical body. In simplified sales pitch form, to “think” is to move. Activity in the world does not come with some antecedent mental processing because the movements just *is* the processing. For example, on the enactive account, gesturing wildly to explain a story or a math technique is not the expression of some antecedent cognitive state. Rather, when doing math (or anything else for that matter) the gesturing, words, direction of attention, volume of speech, posture, hormonal flow, neuronal firing, and sensorimotor processes all together constitute cognition (Di Paolo et al., 2017, 2018; Gallagher, 2017, 2020).

Perception and cognition co-occur in ongoing action-perception-affordance feedback loops. That is, we act on affordances in the world, which brings about experience and, in turn, more action (Varela et al. 1991). Rather than building a model of the world, on the enactive approach, cognizers and their environments in joint interactive coupling *bring forth their world* (Di Paolo, 2023; Varela et al., 1991). While we here stay somewhat agnostic to the strong ontological commitments behind this claim, the more important point is that cognition is always coupled and interactive between agents and environments (Di Paolo et al., 2017). Paraphrasing Malafouris, rather than asking “what” is an agent it is more fruitful to ask “when” is an agent (Malafouris, 2013). The upshot here is that cognition is a relational and distributed process. Cognition is coupled activity between brain-body-world as a joint system.

Notice the opposite nature of enactivism from the “Romance of Math.” While the platonic romance of math thinks of mathematical objects as existing eternally and context-free, enactivism posits math as a series of actions that unfold in interaction with the environment. Math is a series of actions that necessarily and constitutively require physical, cultural, and social, environments. So, on this view, math is not something that exists independently in nature that is then discovered. Math, just like rock climbing, is an activity co-constituted by agents and environments. Doing multiplication on the abacus is a collection of sensorimotor skills, enculturation, and material conditions (namely, the presence of an abacus on a table in a context that demands math-ing). An enactive approach to math does not need to posit ontologically eternal unchanging mathematical forms, since math is equivalent to the practices of math. In other words, math is a set of social-cultural practices used to *describe nature* without necessarily "being" nature.

Math is a matter of embodied action facilitated by a rich nexus of social-cultural and material affordances. Math and math learning, is not something that an agent produces "within" and then asserts onto inert matter (for example, paper or a whiteboard). Math is an activity that dynamically takes place in a system between human agents, materials, and facilitating social-cultural background institutions (Gallagher, 2020; Malafouris, 2013, 2019; Menary, 2015). No materials, no institutions, and no enculturated agents to facilitate the learning - then no math. Math symbols, math environments, and math learning situations all contain nested cultural affordances. Many people can perform rapid addition or multiplication on the abacus without necessarily being "good at math in the head." Using the abacus is primarily an affordance-based sensorimotor skill in which the agent and the abacus become a joint system for mathematical deduction (we will return to "math in the head" shortly).

Sensorimotor Habit

From the enactive perspective, cognitive activity is carved out in terms of habit and skill. In pace with Ryle (Ryle, 1976), enactivism thinks of cognition, even traditionally “off-line” and “higher-order” cognition such as math, as a matter of skillful affordance-based habit (Di Paolo et al., 2018; Gallagher, 2020; Maiese, 2022). However, unlike Ryle, who originally conceptualized habits as root and automatic, the enactive project (mostly) follows John Dewey in conceptualizing habits as flexible, contextual, and plastically open to revision (Dewey, 1922). Habits then are cascading sensorimotor schemes of affordance engagement with the environment aimed at staying in optimal attunement with that environment across material, social, cultural, metabolic, and other conditions (van Dijk & Rietveld, 2017). As agents physically act on affordances in their environments, they slowly fine-tune their sensorimotor schemes to the demands of these various contexts across timescales and conditions (Di Paolo et al., 2018):

> In the context of rock climbing, the optimality of the climb is not a static property of the environment to be picked up and processed (even if the environment itself is unchanging); the decision about what the most optimal move is, is made in action. The suggestion is that the optimal way of climbing is not a piece of information dependent on representing or calculating the parameters of the wall and bodily factors. The decision-making process, including planning, re-planning of the route, and visualizing

> oneself on the route, is a dynamic activity, relating to the ongoing interaction of the agent with his/her environment (Rucińska, 2021, p. 5249)

Both in the case of math and rock climbing there are optimal and less optimal courses of action dictated by the context. Being "good" at math or "good" at climbing means having developed habitual modes of engagement with affordances in those contexts in which the agent is dispositionally ready to act correctly within that context. The rock climber can switch their center of gravity, weight, and holds so as to minimize the strain on the body as they ascend. The mathematician can move hand and pen in patterns that produce valid patterns of inference in accordance with culturally transmitted mathematical rules. Our constant attunement of sensorimotor activity is always normatively determined by contextual affordances.

The complex contextual behaviors required to operate the social world smoothly come in hierarchically organized and dynamically constraining and enabling "bundles of habits" (Maiese, 2022). On a more fine-grained level of detail, each habit consists of mutually constraining and enabling "sensorimotor schemes" (Di Paolo et al., 2018). Rather than mental representations that produce a motor command, we can *heuristically* think of agential behavior as adaptive behavioral loops of sensing affordances and acting. Rudimentary sensorimotor schemes, such as reaching, grabbing, and walking that are learned in early infancy slowly stack and scale to develop more advanced sensorimotor schemes (Delafield-Butt & Trevarthen, 2015; Kelso, 2016; Thelen & Smith, 1994). Even folk-psychological concepts such as intentions and “mindreading” begin with the repetition and adaption of sensorimotor processes often facilitated by the narration of caretakers (Hutto, 2008). Our sensorimotor schemes change and update to stay adaptive to cultural, social, material, and embodied demands. Schemes, therefore, exist in dynamically integrated bundles of behavior that can perturb, activate, reduce, increase, and combine each other in self-organizing dynamically flexible patterns (Di Paolo et al., 2017, 2018).

Mathematical Cognition is Still Sensorimotor Coordination

While it is perhaps easy to think of classically sensorimotor tasks such as “sports” being non-representational, online, and non-computational, it is harder to see in the case of traditionally “abstract” and off-line cognition such as math. However, enactivist “holds the line” by arguing that even “thinking in your head” and abstract symbol manipulation is still a matter of embodied habitual skill acting on affordances. For one, doing the vast majority of complex mathematics involves using the body to create symbols (on a board, on paper, on a screen, etc.). Such symbols are then manipulated in a law-like step-by-step fashion, which involves physically moving the body to re-arrange the symbols in accordance with what the symbols *afford*. Knowing how to solve math equations is the body doing specific actions when coupled with nested affordances. In other words, abstract knowledge, such as the quadratic formula, *is* a disposition to skillfully apply a set of sensorimotor operations when in contact with the right affordances. Knowing "quadratic equations" means doing the steps of symbol manipulation procedures. As an example, let's examine the introductory summary of the quadratic formula on the popular internet math-self-help website Khan Academy:

> The quadratic formula helps us solve any quadratic equation. First, we bring the equation to the form $ax^2+bx+c=0$, where a, b, and c are coefficients. Then, we plug these coefficients in the formula: $(-b\pm\sqrt{(b^2-4ac)})/(2a)$ (Khanacademy.org, retrieved 01/19/2024)

Notice the immediate move from a concept into a summarized explanation of the concept in terms of step-by-step *actions*. The enactivist point is that even when we are manipulating complex conceptual thoughts "in our heads" we are doing sensorimotor actions in engagement with worldly affordances, and the whole body is involved in the production and manipulation of those affordances (Gallagher, 2017). To think deeply about utilitarianism vs. deontology, "in the head" is for one's neurons, hormones, hands, and so on to plastically reorganize their action dispositions so that the agent will act differently in future interactions with relevant ethical situations with ethical affordances. In this case, for example, to act in accordance with the categorical imperative rather than out of utility concerns the next time one is in a morally complex situation. As Rucsinka, ‘a la’ Ryle, argues explicitly about rock climbing:

> First, there are no two parallel activities (cognitive and not-cognitive) occurring during an intelligent, embodied act. Second, there is no second mental process causing the intelligent act. In this way, the concept of heeding applied to rock climbing serves as an alternative to the ideas that rock climbing involves two distinct activities (cognitive planning of the climb and mindless execution of the climb, ala Dreyfus), or that mindful rock climbing must involve a special second process in a cognitive architecture where planning really occurs (ala Evans and Stanovich). Heeding can help us think of rock climbing as one mindful activity, where planning (including route reading and visualizing) are themselves qualities of that activity (Rucińska, 2021, p. 5296).

This way of reconceptualizing higher-order thinking as the rearranging of affordance-based dispositions for embodied action aligns with the empirical literature on neuroscience. For example, across much of neuroscience, rather than looking at brains as modular, the brain has now been demonstrated to be an organ that constantly plastically rewires its own self-organizing circuits (Anderson, 2014). In fact, the recursive, dynamic, self-organizing, and plastically updating cascading activation loops between body and brain demonstrate that the brain is better conceptualized as an organ that *resonates* between world and body (Damasio, 1994; Fuchs, 2018; Kelso, 1995; Raja & Anderson, 2019). As Hutto and Myin forcefully point out, we have no scientifically respectable account of how information in the environment is supposed to become meaningful, contentful mental representations (Hutto & Myin, 2013, 2017). However, what we do have is convincing growing evidence of recursive dynamic interactions of synchrony and resonance between brain-body-world as a coupled system (Raja & Anderson, 2021; Tognoli et al., 2020). Even predictive processing accounts of the brain and behavior now use a dynamic and world mediated account of cognition (Friston, 2010; Parr et al., 2022; Ramstead et al., 2020). The point of this detour through neuroscience is again to demonstrate that it is perfectly coherent to think of higher-order cognition as embodied, enactive, and skill-based —even in cases such as planning or doing math “in the head.” Furthermore, as we shall see in the next section, doing math “in the head” is often also a matter of doing an embodied simulation — a kind of embodied activity just short of overtly performing the action.

As we saw in previous sections, both in the case of rock climbing and math problem-solving, practitioners must plan and "back-off" to reassess. Both math and climbing significantly depend on higher-order planning for the engagement with future affordances. Next we look more explicitly at the role of planning and backing-off in math and in rock climbing to justify their potential integration into a future math curriculum.

Overlap Between Climbing Problems and Mathematics Problems

In our investigation of rock climbing problem-solving and math problem-solving, we have found that both activities involve planning and "backing off." Seen through the lens of embodied cognitive science, the underlying cognitive mechanism is in both cases the same; acting on nested affordances to produce a plan (the re-orienting of one's bodily action dispositions) and periodic pausing to assess the current situation against the plan. With each assessment pause, the plan is updated, and the loop continues — planning-execution-assessment to planning-execution-assessment. Learning how to reorganize attentional patterns and getting better at embodied simulation while under various physical and cultural constraints can (with or without explicit instruction from an educator) be used in the mathematical context. As has been shown with other activities such as dance and theater embodied activities that involve various degrees of attention training, embodied critical thinking, empathy, and simulation improves critical thinking skills in students even without explicit instruction from an educator (Becker, 2023; Giguere, 2006, 2011; Lee et al., 2020).

So far many of the cognitive processes we have covered take place in daily skillful action but in a mostly sub-personal or unreflective fashion. However, both in the case of mathematics and rock climbing attention shifting and embodied simulation for the sake of planning is voluntary and overt. Both math and rock climbing forces the agent to make reflective and voluntary cognitive processes that we in daily life typically rely on "under the radar." In the case of math and rock climbing, the planning and attentional shifts have to be overt and intentionally directed.

While neither endevour is particularly asocial, it is ultimately up to the individual to leverage their skills, abilities, and mental fortitude to solve the problem. Further, both endeavors do involve a relatively static environment. The static environment of the rock climbing gym enables the climber to take more initiative in their environment. Similarly, the affordances of a math equation will not begin to change until the agent acts on the problem. In contrast, we can think of something like surfing. In surfing, there are many environmental elements that are out of the control of the agent. One can paddle out to the surf break with an idea of how they want to perform, but at the end of the day a rogue wave will switch the agency to being outside the agent's control. Similarly in tennis, the agent has to constantly respond/react to their opponent or teammate, once again displacing much of the agency to outside the agent. In rock climbing most of the agency within the organism-environment system is skewed towards the initiative of the agent. In contrast in activities such as surfing or tennis the agency of agent-envrionment coupling lies more skewed towards the environment. Rock climbing and math both have rigid activity structures in which the locus of agency for the agent-environment system lies mostly with the agent.

Both math and rock climbing involve very rigid prescribed boundaries for doing a task. In indoor rock climbing, there are specific starting holds and finishing holds where the agent is only allowed to use a prescribed color up the entire route. However, there is space of adaptation and creativity, but this is dependent on the skill level of the agent. For instance, a skilled climber can use different sequences of moves, body position tactics, and rest strategies to optimize their ascent. Similarly in math, the agent often begins a task with a very specific set of parameters, like equations, formulas, or starting values, and with a very clear end goal. There is also space for creativity, yet this is contingent on the skill level of the agent solving the problem. A skilled agent might discover a more creative or efficient way to solve the problem, prove the same concept using a different approach, or even invent new techniques to solve the problem efficiently. Indeed, both math and rock climbing require a balance between following very specific rules and procedures while also encouraging exploration and creative problem-solving.

Rock climbing and math require learning to overtly and voluntarily reorient attention, and simulate movement for future execution. Bringing attention and simulation from the pre-reflective to the reflective are two of the reasons why rock climbing is a great candidate for math curriculum integration. The activity trains the agent to make explicit, voluntary, and intentional, attentional, and simulative strategies that are also used in the same way in mathematics. Put simply, rock climbing and mathematical problem-solving overarchingly use the same cognitive process. Training in one domain, therefore, improves capabilities in the other domain.

Finally, we wish to address one classic objection often faced by researchers who look at niche activities; “what about this other activity X?” Whether researching dance, rock climbing, martial arts, or whichever subdomain, researchers are often asked, “what is so special about this activity?” We are not claiming that rock climbing is the only embodied activity that jives well with problem-solving. Neither are we claiming that rock climbing is the best activity to pair with mathematical problem-solving — chess is probably a more obvious and straightforward candidate, but that is exactly beside the point. Other activities similarly utilize embodied simulation, and attention shifting for cognitive planning, and that's fine. Rock climbing does not have to be unique to be effective. We are claiming that given the particular nature of rock climbing, the activity does lend itself well to incorporation with mathematics curricula. While other activities, for example, playing chess, might also lend itself easily to overlap with math, so does rock climbing. However unlike chess, rock climbing is arguably more fun, engaging, and cooler.

Planning and Backing-Off as Shifts in Attention

As we have seen both in math and in climbing, expertise involves being open to new possibilities while staying within the constraints of the problem; skillful math and skillful climbing involve "seeing" paths for solutions by shifting one's attention to different aspects of the problem (Mason, 2023; Rucińska, 2021). When climbers are afraid on the wall, it is common to over grip and over-attend to one specific thing, whether it be a hold or a specific path. Similarly, in math, students often become overly attached to a certain solution path, sending them down a 'wild goose chase' (Schoenfeld, 2013) as they cling on to their solution. In both domains, mastery involves skillfully shifting attention in context-appropriate patterns to follow new lines of affordances.

Whether climbing on the wall or grinding through difficult equations, when following affordances, the revealing of new affordances often leads to shifts in the agent's attention (Mason 2023; Abrahamson and Bakker 2016; Abdu et al. 2023).

When being still on the wall and evaluating their current position and situation, climbers also reconfigure their attention, which lets them pursuit new lines of affordances. When backing off to reconsider a math problem, pondering the problem for new insight often leads to subtle changes in attention that reveal a new path through the problem — that is, new affordances (Mason 2023). Both mathematicians and climbers periodically need to reassess the situation to find new paths through their tasks by shifting their attention to perceive new affordances.

In fact, empirical evidence strongly suggests that students develop new sensorimotor schemes and attentional schemes when they learn new mathematical problem-solving techniques. That is, across activity domains, it is shown that mathematical learning involves movement and shifts in attention. For example, across several studies Abrahamson and colleagues use eye tracking to demonstrate that students can learn the concept of "ratio" through hand eye coordination in video games (Abdu et al., 2023; Abrahamson & Sánchez-García, 2016; Howison et al., 2011). Without any instruction these students use sensorimotor coordination and shifts in attention to solve problems that involve understanding, and holding ratios. Similarly, semester long studies, in which math students undertake dance training, or are allowed to move freely around the classroom demonstrate that students use sensorimotor skills, perspective, and attention change to learn math concepts and solve problems (Leandro et al., 2018; Stern & Bachman, 2021). In addition, being able to manipulate physical or digital objects in 3D to change perspective and attention improves mathematical learning (Landy & Goldstone, 2007; Landy & Linkenauger, 2010). In the same vein the use of augmented reality devices have been shown to be effective in mathematics education across multiple studies because AR helps students redirect their attention to relevant features of the problem environment (Ahmad & Junaini, 2020). In the arts music, dance and drama have all been shown to improve mathematical learning (Alam & Mohanty, 2023; Becker, 2023; Kisida & LaPorte, 2021; Lee et al., 2020; Winner et al., 2020).

Across the above cases we see math learning and concept formation take place through the development of sensorimotor schemes and attention schemes — being able to switch between movement patterns and attention patterns are core components of mathematical competence. With its emphasis on backing-off for attentional reassessment, rock-climbing deliberately trains the agent’s ability to shift attention and integrate the attention shift into sensorimotor activity.

With shifts in attention, new affordances move into salience, potentially activating various combinations of trained sensorimotor scheme clusters. Whether the sensorimotor schemes in question are “knowing” specific math techniques such as factoring, or how to leverage a foothold, shifts in attention let the agent ready themselves for executing that sensorimotor technique. As the technique is executed, new affordances emerge; new potential holds or new potential mathematical "moves," which leads to further execution of sensorimotor schemes, new shifts in attention, and possibly updating the plan. Note that this does not mean that the affordances exist independently of the agent and are somehow hidden and platonic. Rather the agent generates new affordances in their action, attending, planning loop. In this way, attention, planning, movement, and assessment become a virtuous cycle for problem-solving in climbing and math.

Planning and Backing-Off as Simulation

Both in the case of math problem-solving and rock climbing planning involves running an *enactive simulation* of solving the problem. Here the simulation is neither cognitively distinct from the act of actually solving the problem or relying on representations. Rather, when in front of the problem, whether math or rock, the nested affordances available allow the agent to actively use their whole body (including vision, hearing, gesture, posture, etc.), to explore the route forward by preparing the body through simulation — doing the actions just short of actually doing the actions so that the body is ready:

> Even though climbers spend 63% of the time in stationary positions during the climb, and 37% of the time ascending (Billat et al. 1995), being stationary does not mean being still. During stationary positions climbers

> engaged in exploratory actions that supported their climbing performance. Being stationary also involves action. In exploratory movements, the climber "(co-)constructs information through her/his actions", since the perceived patterns of stimulation are contingent on his/her motion (Seifert et al. 2018, p. 2). The key idea is that exploratory action, an equivalent of a "dry run", is executed by the agent in the world, not in some mental sphere. This provides a good basis to understand how further re-planning of the climb is achieved in the action. Engaging in exploratory action is a way of responding to and acting on the affordances of the wall. Such exploration, even if only visual, is an act that makes a practical difference as, for instance, it regulates the climber's posture (Rucińska, 2021, p. 5300)

Calibrating the body before engaging or recalibrating in the midst of problem-solving is not removing oneself from the process or "going back inside the head." Rather, in both math and climbing, the agent is actively exploring affordances in the environment that they can use to recalibrate the body's dispositions for the next burst of *overt* movement (Brancazio & Segundo-Ortin, 2020). This is consistent with research on mental simulation, which demonstrates that simulation is highly multimodal and activates bodily readiness short of doing the action (Gallese, 2020; Gallese & Guerra, 2019; Ilundáin-Agurruza, 2017). Planning and backing-off are embodied, enactive, and part and parcel of the problem-solving. Whether readying oneself for specific holds and configurations on the wall or using specific mathematical rules and techniques, active self-priming keeps the agent on task toward a solution.

The higher-order cognition involved in math and climbing requires an ongoing assessment and incorporation of the agent's bodily resources into the plan for further continuation. Good climbers always aim for efficiency — that is, good climbers always take the most energy-efficient route. A similar approach exists within mathematics; an "elegant proof" is the route through a proof with the fewest least complicated steps. To achieve the aim of efficiency is pragmatically also to deploy one's attentional resources optimally. Pragmatically both the mathematician and the rock climbers must preserve their attentional resources. Both activities require the use of the agent's limited attentional resources (simply think back to those long nights trying to finish math homework and reaching math exhaustion while parents keep fussing). Thus, implicit within mastery of these activities is an assessment of the agent's own attentional resources. Pursuing a specific type of route up the wall or route through the proof might be cognitively and physically demanding. As with the mastery of many other activities (Montero, 2016; Toner et al., 2021), the climber and the mathematician both must *continuously monitor the body in execution* while simultaneously executing sensorimotor engagement with the environment. For example, "do I have the energy to solve that one hold using a one-arm hang?" Or "if I factor this equation will the new equation be more confusing?" As climbers or mathematicians "backs-off" to assess their next move, they must both incorporate the current state of the body into their cognitive planning.

Here, we need a cautionary note about the direction of causality. In this paper we are mainly focused on the reasons why rock climbing can improve mathematical ability. We hope that educators will use this article as a starting point to integrate rock climbing into their math curricula. However, technically the argument is laid out so that the arrow of direction runs both ways — in theory math training could also make you a better rock climber. Again, the two domains use the same cognitive mechanisms. However, the physical demands of each activity does create important restraints that we must acknowledge. Rock climbing is extremely physically demanding. Typical mathematics is not physically demanding. Because of this asymmetry in task demands we cannot mathematize ourselves into great rock climbers.

Using Climbing and Other Physical Problem-Solving Activities to Teach Math — a Preliminary Guide to Educators

One of the overarching goals of this paper is to demonstrate the potential of rock climbing to be integrated into mathematics curricula. We have seen that both math and rock climbing utilize enactive planning, simulation, and reassessment. Both activities are ultimately based on affordance-based activity. The activity of math and rock climbing both move processes that are often sub-personal or pre-reflective into explicit, voluntary, reflective execution. Both activities train the agent into using attention and embodied simulation in a voluntary fashion to solve problems under physical and cultural the constraints. For educators, this means that several math techniques can be taught and improved through rock climbing. The role of the educator is to make the student realize the connection between the skills they are using in rock climbing and that they are using in mathematics. For example,

> Remember when you were on the wall and you were at the crux? What do you do when you are cruxing out on a problem? Remember how you took a deep breath, backed off, and looked around to figure out your next move? Think about that feeling of being on the wall and figuring out your beta. Try to use that same way of thinking when you are looking at the equation. Where can you go next?

Again, one of the many reasons we focus on rock climbing is that the activity requires explicitly using attention, simulation, and sensorimotor skills as a form of planning. It is the overt use of these cognitive processes that aligns rock climbing so well with math education. Depending on their approach of choice, the instructor can choose to point out this overt overlap. For example the instructor can integrate language from climbing in the classroom. Another subtly strategy is to use climbing language during instruction but make students realize the overlap in technique themselves as they work through problems.

A professor can develop classes that meet three times a week, in which one of those weekly meetings is at a climbing wall (many universities have indoor climbing walls for free.) Another approach is for educators to teach every session at a climbing facility in which conventional whiteboard mathematics is taught before and after climbs (use whiteboards on wheels to bring the board close to the actual wall). Another approach is to solve math problems between each climb.

For an even more involved curriculum educators can design math problems that are solved by performing certain activities on the wall. For example, students can pair up and go bouldering at a local climbing gym. They will have a checklist of different problem-solving strategies with them. As their partner attempts different routes, they can check off problem-solving strategies on the list as they see the different strategies on the wall from their partner (i.e, trying a different approach when one doesn't work; planning the problem out before getting on it). After both the climbers have had an opportunity to climb as well as watch their partners and use the check list, the students will engage in several math problems, using their check list to guide them through challenging problems. The goal here is to integrate explicitly embodied learning with mathematical problem-solving, leveraging the hands-on engagement of rock climbing to deepen students' understanding of problem-solving strategies and their application in mathematical contexts. While in this article we have focused on the theoretical concerns regarding math and rock climbing one of the authors of this article will later release a full curriculum for college mathematics with rock climbing.

References

Abdu, R., Tancredi, S., Abrahamson, D., & Balasubramaniam, R. (2023). Demonstrating mathematics learning as the emergence of eye–hand dynamic equilibrium. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-023-10279-0

Abrahamson, D., & Sánchez-García, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning*. https://www.tandfonline.com/doi/abs/10.1080/10508406.2016.1143370

Ahmad, N., & Junaini, S. N. (2020). Augmented Reality for learning mathematics: A systematic literature review. *International Journal of Emerging Technologies in Learning (iJET), 15,* 106–122.

Aizawa, K. (2010). The coupling-constitution fallacy revisited. *Cognitive Systems Research*, *11*(4), 332–342.

Alam, A., & Mohanty, A. (2023). Music and Its Effect on Mathematical and Reading Abilities of Students: Pedagogy for Twenty-First Century Schools. In *Interdisciplinary Perspectives on Sustainable Development* (pp. 342–346). CRC Press.

Anderson, M. L. (2014). *After phrenology*. MIT Press.

Araújo, D., Davids, K., & McGivern, P. (2019). The irreducible embeddedness of action choice in sport. *Handbook of Agricultural Economics*.

Baggs, E., & Chemero, A. (2019). The Third Sense of Environment. In *Perception as Information Detection*. Routledge.

Battey, D., & Marshall, B. L. (2023). Math Logics: Perpetrators of Whiteness in STEM Educational Spaces. In J. Ravulo, K. Olcoń, T. Dune, A. Workman, & P. Liamputtong (Eds.), *Handbook of Critical Whiteness: Deconstructing Dominant Discourses Across Disciplines* (pp. 1–11). Springer Nature Singapore.

Becker, K. M. (2023). *Learning Through Movement in the K-6 Classroom: Integrating Theater and Dance to Achieve Educational Equity*. Taylor & Francis.

Beer, R. D. (2023). On the Proper Treatment of Dynamics in Cognitive Science. *Topics in Cognitive Science*. https://doi.org/10.1111/tops.12686

Beer, R. D., & Di Paolo, E. A. (2023). The theoretical foundations of enaction: Precariousness. *Bio Systems, 223*, 104823.

Bicudo, M. A. V., Czarnocha, B., Rosa, M., & Marciniak, M. (2023). *Ongoing Advancements in Philosophy of Mathematics Education*. Springer Nature.

Brancazio, N. (2020). Being Perceived and Being “Seen”: Interpersonal Affordances, Agency, and Selfhood. *Frontiers in Psychology, 11*. https://www.frontiersin.org/article/10.3389/fpsyg.2020.01750

Brancazio, N., & Segundo-Ortin, M. (2020). Distal engagement: Intentions in perception. *Consciousness and Cognition, 79*, 102897.

Bratman, M. (1987). *Intention, plans, and practical reason*. Harvard Univ. Press.

Bratman, M. (1999). *Faces of intention* (1. publ.). Cambridge Univ. Press.

Clark, A., & Toribio, J. (1994). Doing without representing? *Synthese*, *101*, 401–431.

Cuffari, E. C., Di Paolo, E., & De Jaegher, H. (2015). From participatory sense-making to language: there and back again. *Phenomenology and the Cognitive Sciences, 14*(4), 1089–1125.

Cummins, R. (1996). *Representations, targets, and attitudes* (1st ed.). MIT Press.

Damasio, A. R. (1994). *Descartes’ error*. Avon Books.

De Jaegher, H., Paolo, E. D., & Gallagher, S. (2010). Can social interaction constitute social cognition? *Trends in Cognitive Sciences*, *14*(10), 441–447.

Delafield-Butt, J. T., & Trevarthen, C. (2015). The ontogenesis of narrative: from moving to meaning. *Frontiers in Psychology*, *6*, 1157.

Dewey, J. (1922). *Human nature and conduct*. Holt.

Dings, R. (2018). Understanding phenomenological differences in how affordances solicit action: An exploration. *Phenomenology and the Cognitive Sciences*, *17*(4), 681–699.

Dings, R. (2020). Meaningful affordances. *Synthese*. https://doi.org/10.1007/s11229-020-02864-0

Di Paolo, E. (2023). F/acts Ways of Enactive Worldmaking. *Journal of Consciousness Studies*, *30*(11-12), 159–189.

Di Paolo, E., Buhrmann, T., & Barandiaran, X. (2017). *Sensorimotor life* (First edition). Oxford University Press.

Di Paolo, E., Cuffari, E. C., & De Jaegher, H. D. (2018). *Linguistic bodies*. The MIT Press.

Edelman, S. (2003). But will it scale up? Not without representations. *Adaptive Behavior*, *11*(4), 273–275.

Friston, K. (2010). The free-energy principle: a unified brain theory? *Nature Reviews. Neuroscience*, *11*(2), 127–138.

Froese, T., Weber, N., Shpurov, I., & Ikegami, T. (2023). From autopoiesis to self-optimization: Toward an enactive model of biological regulation. *bioRxiv*, 2023.02. 05.527213.

Fuchs, T. (2018). *Ecology of the brain* (first edition). Oxford University Press.

Gallagher, S. (2017). *Enactivist Interventions: Rethinking the Mind* (Kindle Edition). Oxford University Press.

Gallagher, S. (2020). *Action and Interaction* (1st ed.). Oxford University Press.

Gallese, V. (2016). Finding the body in the brain. In *Goldman and His Critics* (pp. 297–317). John Wiley & Sons, Inc.

Gallese, V. (2020). *A Bodily Take on Aesthetics: Performativity and Embodied Simulation* (pp. 135–149). Springer.

Gallese, V., & Guerra, M. (2019). *The Empathic Screen: Cinema and Neuroscience*. Oxford University Press.

Gallese, V., & Sinigaglia, C. (2011). What is so special about embodied simulation? *Trends in Cognitive Sciences*. https://www.cell.com/trends/cognitive-sciences/fulltext/S1364-6613(11)00194-X

Garofalo and Lester. (1985). *Metacognition, Cognitive Monitoring, and Mathematical Performance*.

Gaver, W. W. (1991). Technology affordances. *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, 79–84.

Giguere, M. (2006). Thinking as They Create: Do Children have Similar Experiences in Dance and in Language Arts? *Journal of Dance Education*, *6*(2), 41–47.

Giguere, M. (2011). Dancing thoughts: an examination of children's cognition and creative process in dance. *Research in Dance Education*, *12*(1), 5–28.

Graesser, A. C., Greiff, S., Stadler, M., & Shubeck, K. T. (2020). Collaboration in the 21st century: The theory, assessment, and teaching of collaborative problem-solving. *Computers in Human Behavior*, *104*, 106134.

Gutiérrez, R. (01/2013). The Sociopolitical Turn in Mathematics Education. *Journal for Research in Mathematics Education*, *44*(1), 37–68.

Hacques, G., Komar, J., Dicks, M., & Seifert, L. (2021). Exploring to learn and learning to explore. *Psychological Research*, *85*(4), 1367–1379.

Howison, M., Trninic, D., & Reinholz, D. (2011). The Mathematical Imagery Trainer: from embodied interaction to conceptual learning. *Proceedings of the*. https://dl.acm.org/doi/abs/10.1145/1978942.1979230

Hutto, D. (2008). *Folk Psychological Narratives: The Sociocultural Basis of Understanding Reasons* (Kindle). MIT Press.

Hutto, D., & Myin, E. (2013). *Radicalizing enactivism*. MIT Press.

Hutto, D., & Myin, E. (2017). *Evolving Enactivism: Basic Minds Meet Content* (Kindle Edition). MIT Press.

Ilundáin-Agurruza, J. (2017). Muscular imaginings—A phenomenological and enactive model for imagination. *Sport, Ethics and Philosophy*, *11*(1), 92–108.

Kelso, S. (1995). *Dynamic patterns: The self-organization of brain and behavior*. MIT press.

Kelso, S. (2016). On the Self-Organizing Origins of Agency. *Trends in Cognitive Sciences*, *20*(11), 868.

Kimmel, M., & Rogler, C. R. (2018). Affordances in Interaction: The Case of Aikido. *Ecological Psychology: A Publication of the International Society for Ecological Psychology*, *30*(3), 195–223.

Kisida, B., & LaPorte, A. (2021). *Art for Life's Sake The Case for Arts Education* (p. 56). American Academy of Arts & Sciences. https://www.amacad.org/sites/default/files/publication/downloads/2021-Art-for-Lifes-Sake.pdf

Kronsted, C. (2021). Using Affordances to Teach Improvisational Dance. *Journal of Dance Education*, *21*(3), 168–175.

Kronsted, C., & Gallagher, S. (2021). Dances and Affordances: The Relationship between Dance Training and Conceptual Problem-Solving. *The Journal of Aesthetic Education*, *55*(1), 35–55.

Kronsted, C., Gallagher, S., Tollefsen, D., & Windsor, L. (2023). An enactivist account of the dynamics of lying. *Adaptive Behavior*, 10597123231166421.

Kyselo, M., & Di Paolo, E. (2015). Locked-in syndrome: a challenge for embodied cognitive science. *Phenomenology and the Cognitive Sciences*, *14*(3), 517–542.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from* (1. ed.). Basic Books.

Landy, D., & Goldstone, R. L. (2007). How Abstract Is Symbolic Thought? *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *33*(4), 720–733.

Landy, D., & Linkenauger, S. (2010). *Arithmetic Notation…now in 3D*.

Leandro, C. R., Monteiro, E., & Melo, F. (2018). Interdisciplinary working practices: can creative dance improve math? *Research in Dance Education*, *19*(1), 74–90.

Lee, B. K., Enciso, P., & Brown, M. R. (2020). The effect of drama-based pedagogies on K-12 literacy-related outcomes: A meta-analysis of 30 years of research. *International Journal of Education & the Arts*, *21*(30). http://www.ijea.org/v21n30/

Lester, F. K., & University-Bloomingt, I. (1994). *Musings about Mathematical Problem-Solving Research: 1970-1994*.

Maiese, M. (2016). *Embodied Selves and divided Minds* (1st ed.). Oxford University Press.

Maiese, M. (2022). *Autonomy, Enactivism, and Mental Disorder: A Philosophical Account*. Taylor & Francis.

Malafouris, L. (2013). *How things shape the mind*. MIT Press.

Malafouris, L. (2019). Mind and material engagement. *Phenomenology and the Cognitive Sciences*, *18*(1), 1–17.

Mason, J. (2023). Specifying Defining, Generalising and Abstracting Mathematically All Seen as Subtly Different Shifts of Attention. In M. A. V. Bicudo, B. Czarnocha, M. Rosa, & M. Marciniak (Eds.), *Ongoing Advancements in Philosophy of Mathematics Education* (pp. 103–124). Springer International Publishing.

Maturana, H. R., & Varela, F. J. (1980). *Autopoiesis and cognition* (Vol. 42). Reidel.

Menary, R. (2015). *Mathematical Cognition*. https://doi.org/10.15502/9783958570818

Meyer, R., & Brancazio, N. (2023). Enactivism: Utopian & Scientific. *Constructivist Foundations*, *19*(1), 1–11.

Mobley, K., & Fisher, S. (2014). Ditching the Desks: Kinesthetic Learning in College Classrooms. *The Social Studies*, *105*(6), 301–309.

Montero, B. G. (2016). *Thought in Action - Expertise and the Conscious Mind* (1st ed.). Oxford University Press.

Moyer-Packenham, P. S., Bullock, E. K., Shumway, J. F., Tucker, S. I., Watts, C. M., Westenskow, A., Anderson-Pence, K., Maahs-fladung, C., Boyer-thurgood, J., Gulkilik, H., & Jordan, K. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal*, *28*(1), 79.

NAEP Long-Term Trend Assessment Results: Reading and Mathematics. (2023). https://www.nationsreportcard.gov/highlights/ltt/2023/

National Center for Science and Engineering Statistics. (2023). *Diversity and STEM, Women, Minorities and Persons with Disabilities*. National Science Foudnation. https://ncses.nsf.gov/wmpd

Núñez, R. (2008). Conceptual metaphor, human cognition, and the nature of mathematics. *The Cambridge Handbook of Metaphor and Thought*, 339–362.

Oliveira, G. S. de, Raja, V., & Chemero, A. (2021). Radical embodied cognitive science and "Real Cognition." *Synthese*, *198*(S1), 115–136.

Papert, S. (1980). *Mindstorms: children, computers, and powerful ideas*. Basic Books.

Parr, T., Pezzulo, G., & Friston, K. J. (2022). *Active inference: the free energy principle in mind, brain, and behavior*. MIT Press.

Raja, V., & Anderson, M. L. (2019). Radical Embodied Cognitive Neuroscience. *Ecological Psychology: Gibsonian Neuroscience*, *31*(3), 166–181.

Raja, V., & Anderson, M. L. (2021). Behavior considered as an enabling constraint. *Neural Mechanisms: New Challenges in the Philosophy of Neuroscience*, 209–232.

Raja, V., & Heras-Escribano, M. (2023). Behavior settings, enabling constraints, and the naturalization of social norms. *Places, Sociality, and Ecological Psychology: Essays in Honor of Harry Heft*, 91.

Ramsey, W. (2007). *Representation reconsidered* (1. publ.). Cambridge Univ. Press.

Ramstead, M., Kirchhoff, M., & Friston, K. (2020). A tale of two densities: active inference is enactive inference. *Adaptive Behavior*, *28*(4), 225–239.

Resnick, L. B. (Ed.). (1976). Metacognitive aspects of problem-solving. In *The Nature of Intelligence* (p. Flavell, J. H.). Lawrence Erlbaum Associates.

Rietveld, E., & Kiverstein, J. (2014). A Rich Landscape of Affordances. *Ecological Psychology: A Publication of the International Society for Ecological Psychology*, *26*(4), 325–352.

Rizzolatti, G., Fogassi, L., & Gallese, V. (2001). Neurophysiological mechanisms underlying the understanding and imitation of action. *Nature Reviews. Neuroscience*, *2*(9), 661–670.

Rucińska, Z. (2021). Enactive planning in rock climbing: recalibration, visualization and nested affordances. *Synthese*, *199*(1), 5285–5310.

Rucińska, Z., & Gallagher, S. (2021). Making imagination even more embodied: imagination, constraint and epistemic relevance. *Synthese*, *199*(3), 8143–8170.

Ryle, G. (1976). *The Concept of Mind* (Kindle edition). Penguin Books.

Samuel, G., Katharina, H., & Maida, M. (2017). Mapping an Experiment-Based Assessment of Collaborative Behavior Onto Collaborative problem-solving in PISA 2015: A Cluster Analysis Approach for Collaborator Profiles. *Journal of Educational Measurement*, *54*(1), 103–122.

Schleicher, A. (2019). PISA 2018: Insights and Interpretations. *Oecd Publishing*.

Schoenfeld, A. H. (2013). Reflections on problem-solving Theory and Practice. *The Mathematics Enthusiast*, *10*(1-2), 9–34.

Shvarts, A., & Abrahamson, D. (2023). Coordination Dynamics of Semiotic Mediation: A Functional Dynamic Systems Perspective on Mathematics Teaching/Learning. *Constructivist Foundations*. https://constructivist.info/18/2/220.shvarts

Stern, E., & Bachman, R. (2021). Pattern Play: The Case for Dance in College Mathematics and Beyond. *Journal of Dance Education*, *21*(3), 158–167.

Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real world connections: metacognitive characteristics of secondary students. *Educational Studies in Mathematics*.

Stokes, D. (2021). *Thinking and Perceiving*. Routledge.

Thelen, E., & Smith, L. B. (1994). *A dynamic systems approach to the development of cognition and action*. The MIT Press.

Thompson, E. (2007). *Mind in life*. Harvard Univ. Press.

Tognoli, E., Zhang, M., Fuchs, A., Beetle, C., & Kelso, S. (2020). Coordination Dynamics: A Foundation for Understanding Social Behavior. *Frontiers in Human Neuroscience*, *14*, 317.

Toner, J., Montero, B., & Moran, A. (2021). *Continuous Improvement: Intertwining Mind and Body in Athletic Expertise*. Oxford University Press.

van Dijk, L., & Rietveld, E. (2017). Foregrounding Sociomaterial Practice in Our Understanding of Affordances: The Skilled Intentionality Framework. *Frontiers in Psychology*, *7*. https://www.frontiersin.org/article/10.3389/fpsyg.2016.01969

Varela, F., Rosch, E., Kabat-Zinn, J., & Thompson, E. (1991). *The Embodied Mind: Cognitive Science and Human Experience*. Mit Press.

Williams, D. (2018). Predictive Processing and the Representation Wars. *Minds and Machines*, *28*(1), 141–172.

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, *9*(4), 625–636.

Winner, E., Hetland, L., Veenema, S., Sheridan, K., & Palmer, P. (2020). Studio thinking: How visual arts teaching can promote disciplined habits of mind. *Creativity and the Arts*, 189–206.

Zahnoun, F. (2021). On representation hungry cognition (and why we should stop feeding it). *Synthese*, *198*(1), 267–284.