A MATHEMATICAL SATAN: AN INTERDISCIPLINARY INQUIRY INTO HUMAN NUMBER SENSE COGNITION AND WHAT IT MAY MEAN FOR MATHEMATICS EDUCATION

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Abstract:
This paper ponders the philosophical ramifications of the latest research of number cognition in the fields of neuroscience and anthropology, as it relates to mathematics education. To begin, the author discusses the latest evidence of how humans develop number sense in the neuroscience research. Then, she explores the latest anthropological evidence to understand how humans developed a cardinal understanding of number. Woven in this narrative is a philosophical presupposition that mythology, a metaphorical knowledge, is a useful tool for research within philosophy of mathematics education discipline.

Keywords: mathematics education, philosophy, number sense, anthropology, neuroscience

Introduction

It is fall 2024 in upstate New York, U.S.A as I write this paper, just before our presidential election. It is hard to escape the news reports about the polling data gathered from "reputable sources." It is difficulty to not be swayed by the mania encompassing American daily lives complete with messages of joy and cries of deceit. Everyone seems to have an opinion backed up by the corporate driven news sources they have on in the background as they wash their evening dishes or feed their cats. Everyone seems concerned with food prices and inflation, even though most of them do not fully understand what the latter actual is and does. Harris is up nationally by 2 points one morning we are told. She raises 3.7 million dollars in a few hours, we are supposed to be impressed. A reporter exclaims Trump needs a swing state with its large number of electoral votes to win. Another reporter reminds us that he was found guilty of 37 felony counts and we are supposed to feel disdain. Meanwhile, a historian utilizes a robust mathematical model to confidently predict who will win the 2024 American election regardless of what the polling numbers show. My point for beginning this paper is not to insinuate my political affiliations or lack thereof, but to bring to the reader's attention just how powerful, pervasive, and utter illusive, the mathematical concept of numbers is in modern western society.

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What exactly are numbers and why does their awareness hold so much power? Philosophically, we can argue if they are “invented” or “discovered,” thus proclaiming our ontological and epistemological stance, but that will only get us into perpetuating more complex terminology, paradoxes, and theories, ending most likely with Gödel. We may ponder the universality and beauty of number concepts through Plato and Pythagoras, but that has been done by much more brilliant minds than myself. Or we may discuss the differences between cardinal, ordinal, and nominal numbers and their relationship to the infinite sets of numbers Cantor first made us aware of. However, this all seems superfluous these days since scholarship of mathematics education and philosophy of education research rightly focuses on societal oppressive systems, which are governed by numbers (e.g. Atweh, 2007; Bishop, 1988; D’Ambrosio, 2001, 2024; Ernest, 1988, 2004). This work is certainty paramount to counter social inequities since interdisciplinary inquiry into how oppressive systems emerged and are perpetrated may lend further knowledge into how to combat them. Regardless of how we feel about mathematics, it does stand “at the heart of science and technology impacting on the economic performance of societies since ancient times.” (OECD, 2010, 1) Therefore, any efforts for change to a more just peaceful world, must consider not only how to utilize mathematics and how it utilizes us, but also what the very nature of the tools we use and are being used by actually are and how we possess any ability to perceive them. Given the latest research in neuroscience about number cognition and the recent evidence in anthropological research in understanding how humans have evolved a number sense, the time may be ripe for another look into the out of fashion foundational philosophical debates around numbers.

First, let’s be clear about what we are talking about, which is a lot more difficult than one might imagine. Skipping over a review of the foundational debates concerning the nature of numbers in the early 19th century, I am concerned here with the set of all natural numbers (1,2,3,4,…) and how humans have come to a conceptual understanding of them through our evolutionary history. The budding research in mathematics ability in humans from such different fields of neuroscience and anthropology could significantly influence how we think about mathematics education. The presupposition for this work is that some of the take for granted epistemological claims about how humans come to understand mathematical concepts needs to be altered given the evidence from these fields.

The capacity to understand and then alter external phenomenon by creating robust mathematical models that can predict and explain our reality is unarguably central to the modern humans, particularly western post-industrial societies. This postulation begs several questions:

1. How did humans develop the mathematical knowledge utilized today in the modern world?
2. What correlation is there between mathematical systems of knowledge or lack thereof to a culture’s evolutionary path?

3. What might be lost and gained through this knowledge and what can we do from here?

I will begin this paper, by attempting to answer the first question by summarizing the latest research of number cognition in neuroscience. Later, I examine question 2 by reviewing the recent anthropological evidence of number sense in ancient humans, indigenous people, and primates and other animals. Before exploring what implications, this inquiry yields for mathematics education, I take a short tangent to explore how an ancient myth can shed light onto our recent discoveries.

A Mathematical Satan

I have endeavored this project to answer a deeply personal question I have had for some time – why, when, and how did humanity "fall" from grace? The Judeo-Christian reference of this questions refers to "humanity" not as Homo sapiens as a species, but rather the cultural lineage that emerged from societies that had a monotheistic conceptualization of an omnipotent creator/creation. In such societies, there is often a mythology of the descent of humankind from a once beautiful innocent and perfect reality to a painful, dangerous, and unknown existence. The cause of this tragedy takes on many forms depending on the tradition, but the Judeo-Christian one is quite fascinating to consider.

In the Old Testament's Genesis story, we are told that the first humans, Adam and Eve lived in utopia. All their needs were met, and they only had one rule to follow:

"You are free to eat from any tree in the garden; but you must not eat from the tree of the knowledge of good and evil, for when you eat from it you will certainly die." (Bible, 15)

Of course, then came the "evil" snake (Satan) who enticed them to eat from this tree. We know what happens next, but perhaps a closer inspection of the translated words from the common King James edition of the Bible can give us some insight: God said to Adam

"Cursed is the ground because of you; through painful toil you will eat food from it all the days of your life. It will produce thorns and thistles for you, and you will eat the plants of the field. By the sweat of your brow, you will eat your food." (Bible 19)

Therefore, we can infer that whatever knowledge the first humans obtained from eating the apple from the tree of knowledge of good in evil resulted in not only them being banished from the Garden of Eden, but also to have to invent agriculture to survive. The Bible further tells us, at least according to the serpent, that the knowledge from the tree of good and evil will

"You will not certainly die," the serpent said to the woman. "For God knows that when you eat from it your eyes will be opened, and you will be like God, knowing good and evil." (Bible 7)

Joseph Campbell (1991) once said:

> “Mythology is not a lie, mythology is poetry, it is metaphorical. It has been well said that mythology is the penultimate truth–penultimate because the ultimate cannot be put into words. It is beyond words. Beyond images, beyond that bounding rim of the Buddhist Wheel of Becoming. Mythology pitches the mind beyond that rim, to what can be known but not told.”

I suggest that the “Fall from Grace” myth can be interpreted as metaphor for humankind’s acquisition of specific type of mathematical knowledge - the concept of cardinality past the number 3. Doesn’t mathematics “open our eyes” and give us “Godlike powers to do horrific acts of destruction as well as learn how to travel outside our own solar system? Could the “tree of knowledge of good and evil” be a metaphor for binary systems of mathematics, which evolved simultaneously with dichotomous syntax in new language patterns? As I explain in this paper, the concept of number, as discrete quantities of sets was cognitively very difficult to acquire as a species and took tens if not hundreds of thousands of years. After all, the development of agriculture occurred if not because of, at least concurrently with, the invention of number systems and the written languages that enabled them. Later in this paper, I will show that capacity to develop hierarchical societies that tend towards oppressive, and violence are tied to complex number systems that conceptualize quantities as discrete magnitudes. Perhaps our gain of number also made us lose something valuable? The famous quote by the mathematician Kronecker says “God made the integers, all else is the work of man.” Given the latest neuroscience research and anthropological evidence, it seems the reverse it true – Evolution (or God to utilize the mythology) gave us the real numbers (ratios, irrationals, transcendental numbers) and it was the work of humans that gave us the natural numbers (Gallistel & Gelman, 2000). Towards the end of this paper, I will contemplate what these ideas might mean for education of mathematics.

A Neuroscience perspective

Several decades ago, researchers tended to think mathematical knowledge of numbers was an innate evolutionary characteristic, perhaps even a uniquely human one. We now know that was mistaken. Not only do humans possess no such inherent ability, but even the concept of number and how we come to understand is being revised. One of the first researchers in the field was Stanislas Dehaene (1997), who argued that there is an evolutionary structural component to the human brain that can distinguish quantities of perceptional stimulus, but it doesn’t work the way we might have previously imagined. Based on empirical studies of primates, preverbal human infants, and patients with neurological disorders, Dehaene’s hypothesis is that “we are born with multiple intuitions concerning numbers, sets, continuous quantities, iteration, logic, or the geometry of space.” (p, 31) Since then, neuroscientists’ studies on infants and primates have offered a robust body of knowledge to understand in greater detail human’s ability to understand numbers (e.g. Feigenson, Dehaene, & Spelke, 2004). The emerging data reveals that humans are endowed with two types of number cognition systems, yet it is still unclear how they work together to formulate number sense in the way we understand it today. The first system is called “approximate number system” (ANS), which many animal species also possess (e.g., Merritt, DeWind, & Brannon, 2012); cognitive experiments have proven that this system is inherent at birth in humans (e.g., Izard, Sann, Spelke, & Streri, 2009). Basically, this system allows for an ability to perceive magnitude of objects/tones/light in relate to themselves. This perceptional ability follows Weber’s law of ratio comparisons (Cantlon et al. 2009b; Dehaene 1997; Feigenson et al. 2004). It isn’t yet clear if this approximation skill relates to cardinality or magnitude and the role visual phenomena plays in its perception. (Ross J, Burr DC. 2010)

The other endowed number system that humans are born with is called the “precise number system” (PNS) or “exact number system (ENS),” which allows humans (and infants, primates, and many animals) to perceive quantities up to 3. (e.g. Anobile G, Castaldi E, Turi M, Tinelli F, Burr DC. 2016). In the educational world this innate ability is referred to as “subitizing.” Basically, this means that animals, like humans, can perceive immediately a group of three or less objects. Additionally, studies have shown that human infants, as well as our closest evolutionary relatives, the great apes, can do simple arithmetic with these small number quantities (Rugani et. al. 2010). However, it is not clear if the participants in these studies, be them infants, or other animals, have a conceptual grasp of cardinality of numbers one through three, or more like a “sense of magnitude” that enables discrimination between difference continuous magnitudes, rather than as a characteristic of the number of objects in a group/set. (e.g. Leibovich et al. 2017). Recent studies also suggest that numerosity processing might not be as automatic as previously assumed; the field of neuroscience is still relatively new, with only many research agendas. Nevertheless, the current research is providing a clearer picture of how humans have developed an understanding of numbers as discrete entities/concepts.

Researchers believe brain imaging can help bridge that gap by understanding what parts of the brain are utilized when conceptualizing numbers. Brain imaging has detected specialized neural networks when humans use these innate abilities (Zamarian et al., 2009). These networks are in the parietal and prefrontal cortices, but also involve brain regions in occipital cortex, subcortical regions, and the cingulate cortex. (R. Cohen Kadosh et al., 2008; Dehaene, 2009; Zamarian, Ischebeck and Delazer, 2009). Recent studies also found that numerosity is not processed independently of continuous magnitudes, as proposed by now outdated number sense theory. Latest neuroscience studies have shown that there is a gray area in both left and right hemispheres of the brain that fires when humans are thinking mathematical (Vogel, S. E., & De Smedt, B. (2021).

To gain an awareness of quantities past three, theorists in the neuroscience field belief humans needed to develop a larger working memory (e.g. Adams, Barmby, & Mesoudi, 2017). Studying how young children acquire number sense, researchers offer a lengthy trajectory starting at birth to kindergarten. This includes, object tracking system, one to one correspondence (equinumerosity), Analog magnitude system, successor principle knower, all necessary preconditions before children can be referred to as “cardinal principle knowers” (e.g., Carey, 2001 Sarnecka & Carey, 2008). It is no wonder children struggle with learning mathematics at an early age, and many struggle throughout their education. One thing is very clear - learning what we believe is the foundation of mathematics – numbers – is a lot harder than what was once believed. Additionally, to learn mathematics, children need language, and many experiences with adults, teachers, and the entire cultural community, to develop a grasp of modern-day western number concepts. This brings us to the work of linguists and anthropologists, who content that mathematics cognition is culturally mediated and developed over thousands of years of human evolution. (e.g. Núñez, 2017; Ascher, 1991, 2002). To understand how language and culture have mediated our understanding of numbers, we will need to delve into the current anthropological evidence.

An Anthropological perspective

To begin this survey into the anthropological evidence of number cognition, I first turn to some classical theorists. It used to be believed that mathematics developed around five thousand years ago in Mesopotamia. Later, researchers saw similar developments in China and Mesoamerica, although time frames were often disputed. Mathematics seemed to evolve in human civilizations that possessed some type of written language and were complex in organization. Lakoff and Johnson (1980) theorized that mathematics evolved through systematic metaphorical expressions that were linguistic in origin and developed simultaneously with the development of number systems. These underlying metaphors of phenomenon is one of the key reasons why early humans were able to develop a concept of numerical quantities past three.

Caleb Everett’s (2017) book, *Numbers: and the making of us*, explains how number systems developed in some human societies over thousands, perhaps tens of thousands of years. Everett’s argument is that numbers are “conceptual tools, that are culturally dependent and acquired by some human societies due to linguistic innovations.” (Everett, 2017, 5). While this is not completely new, the time frame of mathematical innovation in human ancient societies has been challenged by archeological evidence recently found in sites across the earth, dating back tens of thousands of years. Paleolithic tools for tracking the lunar cycle were found in many archeological sites. There is now several undisputed evidence of ancient Paleolithic human tallying marks, on antlers, on bones, on wood were used to mark lunar cycles, probably for hunting purposes (Everett, 2017). Everett theorizes that humans invented number words, and through a variety of adaptations where able to utilize those conceptual tools to develop mathematics as we know it in the modern world. Such an ability changed the course of human history. This invention of number words enabled the precise and consistent discrimination of quantities greater than three. How did this happen?

Everett begins by citing that finger counting is a “ubiquitous practice across the world cultures.” (42) It is no accident that many of the world’s numerical systems are base 10, or base 12 (the number of digits in one hand minus the thumb). There are, of course, exceptions, but almost all of them have a basis of human anatomy structure. Furthermore, anthropologist are beginning to view cave paintings of human hands in a numerical sense, rather than purely metaphorical. This mathematical discrete counting took a long evolutionary time and was made possible by other evolutionary occurrences such as bipedalism, symmetry of our bodies, our innate number system (APS), our larger brains for memory, changing landscape causing humans to decipher natural phenomena more effectively. “Humans learn to unite their innate mathematical capacities only if they are exposed to numbers after being embedded in a numerate culture while speaking a numeric language.” (Everett, 120) Everett claims that there is some hardwiring of the human brain that predisposes humans to conceptual numerical quantities, but this does not give us numbers as such. We needed number words, and for that we needed language. Humans needed to scaffold if you will our awareness of natural phenomena, like we do colors, to make any sense of it. While we now have a clearer picture of how certain human cultures may have developed numbers and thereby mathematics as we know it today, we still have much to learn about the centrality of numbers in modern culture and the role they play. And let’s not forget the obvious question - how and why did some cultures “invent” numbers and others did not

Anthropological research confirms that number systems are not universal; in fact, there are several cultures that have no concepts for numbers past three. It is well established that many languages around the world do not have words for exact numerosity’s beyond ‘four’ or ‘five’ and that native speakers of these languages seem to reason only approximately about numericities above that range (Pica, Lemer, Izard, & Dehaene, 2004). By studying anumeric societies, anthropologists can glean much about how some human societies have come to have any understanding of numbers. In indigenous societies today we find similar tally systems as we do in Paleolithic artifacts, but not necessarily words for numbers past three. For example, the Piraha culture an anumeric culture, do not require precise quantity differentiation, not in their hunting practices, their kinship or governing organizations, nor even in their housing structures. This suggests that “Not all cultures value numbers in the same way, even if they are concerned with mathematical topics.” (Everett, 130)
Scholars have for some time tried to create a trajectory or a classification system of mathematics cognition to map the evolutionary history of our species with the cultural evolution of mathematics. (e.g., Beller & Bender, 2008; Feigenson, Dehaene, & Spelke, 2004; Nunez, Cooperrider, & Wassmann, 2012; Wiese, 2003, 2007). But, numeration systems do not always evolve from simply to more complex and from specific to abstract systems” (Beller & Bender, 2011) For example, Bender & Beller (2017) studies two oceanic societies and found evidence of a complex binary system as well as a ENS for smaller number quantities. Interestingly, the binary system was used primarily for quantification of surplus and establishing systems of economics.

It is well documented that number systems enabled trade and more complex agricultural techniques, thereby allowing for larger populations. These larger societies then yield bigger networks of minds sharing the same language, through which “new numerical tools can quickly diffuse” (Everett, 2017, 222-223) Interestingly, Everett (2017) claims numbers were quite likely foundational to the advent of writing around the world and not the other way around. While it is not clear which came first or the causal effect of each cultural tool, it is clear that mathematical systems evolved in humans at the same time as written language systems, with both systems becoming (generally) increasingly more complex and abstract. Surely mathematics contributed to economic, technological and scientific advances in some human societies, but it is not clear how mathematical awareness also contributed to other cultural aspects such as religion and social hierarchical structures.

Everett (2017) asserts:

> "We could go so far to say the development of number systems was pivotal to the creation of God(s). Or perhaps, to some, this development let to the accurate realization the God(s) existence…What is clear is that the growth of large hierarchical religions based around God(S) is a fairly recent trend. (248)

It is a trend that followed the numerically influenced agricultural revolution that enabled the growth of human populations in the regions where the religions in question then developed. (Everett, 250) Not surprising, mathematics, particularly numbers, are utilized in religious practices. Divination practices in almost every culture, I Ching being a prominent example. (Asther 2002). "Many contemporary scholars interested in cognitive processes believe that the creation of dichotomies, such as light/dark, thick/thin, odd/even, on/off, yes/no, up/down, and hence binary choices is fundamental to all human thinking." (Ascher, 2002, 34-35)

A philosophical perspective

Bookchin (2005), through a careful analysis of human history, elucidates that hierarchical thinking is strictly a human social activity. Certainly, we can witness hierarchical structures in nature and with our closest primate relatives, nevertheless Bookchin argues there is a categorical difference between human societal structures and all other living groups. While it is beyond the scope of this paper to paraphrase Bookchin's argument for the emergence of hierarchy, we can look specifically into his argument for the role mathematics played in created our modern society. In indigenous societies, or as Bookchin (2005) calls "ecological societies," abstract money systems did not exist, nor did bartering in the ways we understand it today. The origins of monetary and bartering systems, according to Bookchin, can be traced to Mesopotamia where cuneiform writings depict meticulous records made by temple clerks to record products given and received. Later, this system of quantification and standardization seized by certain groups of people to acquire prestige and power, was justified by abstract formulas given to the products in which they now claim to own. Unlike ecological societies, these new societies no longer operated under the principle of reciprocity, gift giving, and complementarity, but rather on competition, scarcity thinking, and later property rights.

The social evolution of hierarchy is woven in Western history, through Ancient Greece, Descartes's Cartesian linear system and to the standardization movement today. For example, hierarchical thinking became further entrenched into the Western psyche in Ancient Greece with the god Justitia, depicted with a scale and a blindfold to illustrate the need to measure with exactness, reducing all "qualitative difference to quantitative ones" (Bookchin, 2005, 223-224). And this quantifiable world became further entrenched in the 20th and now 21st century late capitalism and pseudo democratic societies such as the United States.

Porter (1995), a historian, who studies recent Western culture's pursuit of objectivity and its increasing trust in numbers; he associated the development of human manipulation of numbers with notions of modern citizenship and democracy. His work correlates well with STEM educational discourse centering around the "need" to "compete" at the world economic stage by developing citizens that have mathematical literacy (Wagner, D., & Davis, B. (2010).

Porter writes:

> "This is why a faith in objectivity tends to be associated with political democracy, or at least with systems in which bureaucratic actors are highly vulnerable to outsiders. The appeal of numbers is especially compelling to bureaucratic officials who lack the mandate of a popular election, or divine right. Arbitrariness and bias are the most usual grounds upon which such officials are criticized…Quantification is a way of making decisions without seeming to decide. As politicians and bureaucrats use numbers to claim objectivity, to mask their biases, and to legitimize their decisions, it could be said that the citizens, who have been enculturated in schools to put their trust in number, are being duped by number, not empowered to make informed decisions, and, of course, claims of objectivity are made by more people than politicians and bureaucrats." (8)

Modern ontological assumptions about mathematics allow it to be appropriated to serve global elite while perpetuating social and racial inequities (e.g. Atweh, 2007; Frankenstein, 1983; Skovsmose, 1994). Alan Bishop, (1988, 1990) who is known for his attention to the cultural nature of mathematics education, noted that exponential-based numbers, which comprise a relatively abstract system of representing quantity, made colonialism possible. Alain Badiou (2008) called for us to rethink our understanding of number and its relationship to human societies. Number, an ontological entity for Badiou, is not an objective measurement device, rather it is "a form of being." Further, our failed sense to understand what Number is has led to a collective amnesia that is the cause and effect of our human condition.

He writes "

> If the reign of number – in opinion polls or votes, in national accounts or in private enterprise, in the monetary economy, in the subjectivising evaluation of subjects – cannot be authorized by Number or by the thinking of Number, it is because it follows from the simple law of the situation, which is the law of Capital. This law assures, as does every law, the count-for-one of that cannot make any claim to truth; neither to a truth of Number, nor to a truth which would underlie that which Number designates as form of being." (p. 149-150)

Badiou strongly upholds the notion that the present modern world adheres to a classical schema, which through the centuries has given humankind the tools and methods for learning about the reality in which we live. Indeed, without mathematics very little of humankind's accomplished could have been achieved much less imagined. Scientific inquiry would be vacuous without the use of mathematics. Mathematics, the language of ontology for Badiou, is precisely the discourse needed to rid us of the "slavery of numerosity" of our own device. Badiou furthers this ontological paralysis, stating "But we don't know what a number is, so we don't know what we are" (ibid., p. 3). Philosophers of education have attempted to decode Badiou's cryptic statement for decades. Some have even tried to apply it to mathematics education (e.g. Brown, 2010; Lewis & Cho, 2005). Given the latest evidence in neuroscience and anthropology, we can revisit his statement with a broader understanding. Applied to elementary mathematics education, Badiou's axiom becomes an educational opportunity to envision the teaching and learning of numbers radically different.

Rethinking mathematics education

Now that we know about our evolutionary numeric past and our cognitive embodied present, what implications does this knowledge have for mathematics education? Given that mathematics education directly influences societal norms, traditions, and deep-seated values, considerations in its pedagogy and curricula have the potential to change the cause of human cultural evolution. It has in the past, after all, why couldn't it now? To begin this inquiry, let's first consider the epistemic conditions in which mathematical knowledge is transmitted through educational and cultural processes.

Neurological studies show that developing exact number quantities takes a long time and with explicit experiences and training. First children are exposed to number words and counting practices. The realization that the last number word said in a set happens gradually (the cardinality principle). Evolutionary speaking, this realization and later utilization took tens, if not hundreds of thousands of years. How then can we expect children to acquire this knowledge in a few short years of their formative life? The fact, children that do not understand the cardinality principle by a certain age are in danger of having a limited understanding of arithmetic is an important part of critical education's mission for equity. Many children fall behind at this point, and even more once fractions are introduced in elementary school. This dominant curriculum exemplifies the critical argument that mathematics education is used as a quantitative sorting mechanism for social reproduction (Frankenstein, et al. 2011; Gutierrez, 2008; Gutstein, 2008; Martin, 2003; Stinson & Bullock, 2012). Scholars have envisioned ways mathematics education could not serve the global elite (e.g. Shulman, 2002; Warnick & Stemhagen, 2007), but given what we now know about number cognition in humans, I feel more work can be done.

Traditionally, teachers of mathematics believe that students can only grasp complex principles by first learning simpler ones. This linear spiral model is now outdated and needs a complete revision. Educational practices take a long time to change, but if we truly care about developing a mathematically literate populace, then we must revision the ways in which we teach mathematics. Of course, this might not be the goal for the global elite/captains of industry that are more interested in creating a emotionally driven consumers and sedated workers. Maybe it is too late, but as a professor of mathematics education, whose primarily job is to train future teachers, I cannot throw in the towel just yet. I propose that one way to fulfill this duty is to envision a new elementary mathematics education that works with children's inherent number sense and fuses that with their innate curiosity and wonder about the world around them.

Why isn't the beauty and wonder of mathematics and how it elegantly explains our natural world not be prevalent in mathematics education, especially at an elementary level where children are generally curious about the world? Why are we concentrating on developing children's "number sense" through discrete cardinality principle by first insisting on boring, soul crushing exercises in rote memory, and monotonous tasks of number groupings? What if we starting where children are innated gifted with as part of our generic heritage – approximate number system? I am proposing that we don't begin mathematics education with "natural numbers" since they aren't "natural" at all. Rather, we should begin with "nature" numbers, which technically would be the real number system, primarily focusing on the irrationals and transcendental numbers. Of course, we may not name them as such in kindergarten, nevertheless we can develop experiences through which children play with such concepts in a meaningful mathematical way.

Marks-Tarlow (2008), a child psychologist, believes that children are inherently drawn to a self-similarity form; this idea is related to fractal geometry where the pattern of the whole reflects itself in various sizes or time scales. Marks-Tarlow illustrates this by a story about how a 5th grade patient of hers and his idea of infinity. The story is about a shoebox in the closet of a giant whose universe in turn was in a larger box in a closet of a yet larger giant, and this pattern goes on to infinity. I have witnessed similar educational moments in my own classroom and with my own children. Humans naturally see patterns in their perceptual awareness. Visually or mentally seeing patterns of self-similarity in mathematics provides that necessary link for the human imagination to flourish. It is precisely the imagination, provoked by self-similarity, that can aid a struggling mathematics learner to understand complex concepts and algorithms. For instance, a student that has trouble with equivalence of percentages, fractions, and decimals can be helped by a visual aid depicting these different versions of expressing the same quantity. Explaining how ½ is equal to .5 and to 50% would be easier with a picture, story or song. Moreover, if these equivalent forms of expressing the same number concept could be initially explain before the complex algorithms taught, students would have a clearer image of what operations they are performing, why they work, and what answers they yield.

Theorists coming from Vygotskian scholarship, such as Davydov (1995), contend that human cognitive development descends from abstract thinking to concrete. This theory drastically changes how we ought to think mathematics should be taught.

Davydov explains that mathematics is a relational system that cannot be meaningfully understood through explicit concrete problem solving. Rather children should be able to explore the theoretical and structural components latent in the mathematics they are studying, which according to Davydov, would ground their concrete experience with mathematical problem solving. This way of teaching is implicitly recursive since it follows from the larger concept to the smaller ones, each playing a role in the overall teaching and learning of mathematics. Although, recursive learning seems counterintuitive, empirical evidence has proven its possible large-scale effectiveness.

With the publishing of *The Fractal Geometry of Nature*, Benoit Mandelbrot (1983) set forth a new episteme, enunciated by the radical idea that nature encompasses fractal patterns. He identified the four basic characteristics of fractal geometry, which later played a key role in developing a new field of mathematics – complexity theory. The recursive characteristics in Fractals that Mandelbrot uncovered is currently being used to understand how seemingly random events create complex mathematical patterns that may be used to explain all kinds of natural phenomenon. Children can explore natural phenomenon all around them by simply taking a walk outside, counting the petals on flowers, tracing the spirals on shells, or carefully witnessing ripples in water or a drip of a faucet. This would not only make mathematics much more enjoyable to teach and learn, but also deeply connects higher order mathematics to children's experiential knowledge. "Numbers" would be experienced as relationships, or artful relations among things.

Due to the 20th century push for formalizing mathematics into a common abstract logical language, much of the ancient ways of knowing mathematically were pushed to the periphery. Symbols instead of shapes became more common and thinking geometrically or visually was replaced with thinking abstractly or conceptually. Mathematics is an art form and has much in common with other art forms. This truth claim "can lead us beyond ordinary existence and can show us something of the structure in which all creation hangs together, is no new idea. (Brown, p. v cited by Kobayashi, 2009, 59). Mathematics has been demarcated from other modes of understanding our world. Since the 19th century, mathematics became increasingly divorced from its mystical and aesthetic roots. In ancient times, mathematics was not "only a discipline in which to inquiry about empirical phenomena, but a way of knowing about the world and of expressing it in its purest form" (Pfenninger, p. 167).

Scholars have proclaimed the aesthetic dimension of mathematics as the key characteristic of the mathematical learning experience (e.g. Sinclair, 2001;Tymoczko, 1993; Wang, 2001). Indeed, great mathematicians from Poincare to Gödel have asserted that their practice of mathematics is latent with aesthetical experiences (Devlin, 2000). Even the U.S. National Council of Mathematics Teachers asserts that a connection to art and music ought to be achieved in the mathematics classroom. A particular example of a philosophy of mathematics based on aesthetics is Resnik's (1981) notion of mathematics as a study of patterns and Shapiro (1997) mathematics as a study of structures. Within these views, mathematicians and philosophers of mathematics are not concerned with the ontological properties or truth-values of numbers themselves, but only the structures and relationships that bind them together. Shapiro explains that on all versions of structuralism, the nature of objects in the places of structure does not matter – only the relations among the objects are significant. A simple way of understanding of teaching this to a child is to depict relations in a spatial-temporal way that children can experience through an embodied sense. For example, you are only short compared to someone taller; a thousand dollars is either a lot of money or not that much depending on where you live and the lifestyle you are accustomed to. Conceiving numbers as relations, rather than static entities or culturally meaningless terms, may elicit a more aesthetic experience in the practice of doing mathematics. This has great implications for education, especially at the elementary level when numbers are first introduced. Thinking about mathematics as a discipline that attempts to understand patterns and relationships provides an alternative to the historical aim of education as well as the pedagogies that have come forth to meet them. Here, mathematics education would aim to provide an aesthetic experience of doing mathematics, which would in turn inspire the imagination and bring forth the necessary cognitive apparatus needed to learn mathematics well. In addition, and this is my most far-reaching claim, if we learn mathematics as a system of relations and patterns, our ways of conceptualizing our world and ourselves might change as well so that we would view connections to be explored rather than quantities to be measured.

Final Thoughts

Ian Stewart (1995) in *Nature's Numbers* calls for inventing a new branch of mathematics to help us understand the “why” and not just the “how” of natural phenomenon: “… a new kind of mathematics, one that deals with patterns as patterns and not just accidental consequences of fine-scale interactions. (p 144). He terms this mathematics “morphomatics,” which is “a way of thinking creatively innovatively. This new branch can help mathematics solve, and more importantly understand all kinds of unknowns today, the largest one being the unified physics theory. Some parts of this new mathematics exist, proclaims Stewart, in dynamical systems, complexity theory, cellular automata, etc. “It's time we started putting the bits together. Because only then will we truly begin to understand nature's numbers – along with nature's shapes, structures, behaviors, evolutions, revolutions.” (151) Certainly, mathematicians and physicists are collaboratively working frantically and passionately today all over the world as they always have done. After all, it is always them that usher in paradigmatic shifts in human society. Capitalism may have dampened their quest over the last century, nevertheless, hope is never lost. For empires rise and fall, but humanity's search for meaning and understanding always prevails. Sometimes even that desire leads to unthinkable human evil yet given the entirety of human history that we know of thus far, there seems to be a balance of good vs evil. Yes, we invented the atomic bomb and horrifically used it, but maybe, just maybe that same mathematical theories that enabled us to do such evil can also figure out how to harness energy that is free and clean, drastically alter human systems of oppressive worldwide.

Bateson (1972) writes: “The major problems in the world are the result of the difference between how nature works and the way people think” (p. 470). What if our

understanding of how nature works has a mathematical component that we have misunderstood or categorized incorrectly? Due to the 20th century push for formalizing mathematics into a common abstract logical language, much of the ancient ways of knowing mathematics were pushed to the periphery. Symbols instead of shapes became more common and thinking geometrically or visually was replaced with thinking abstractly or conceptually. Mathematics, thus, demarcated from other human modes of understanding our world, became a vehicle for the rational scientific logical paradigm, still dominant today. This rationalistic paradigm is intrinsically connected to the divisive economic and social practices of the modern era. To challenge this dominant paradigm and to offer viable alternatives for a more just socially and ecologically equitable society, the way we conceptualize mathematics itself needs to be questioned as well as how we
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Gregory Bateson and Mary Catherine Bateson ponder an interesting question from cybernetic research that relates to our mission here: "What is a man that he may know a number, and what is a number that a man may know it?" (2005, 177). This reversal question is significant since it changes what can be asked and therefore known and experienced. To push this inquiry further, we can ask what kind of relationship can we envision between mathematics and humans that is non-binary? In other words, how can we reimagine humans as part of, and not separate from, a natural world that is always in the process of knowing itself? Gregory Bateson (1972) writes: "We are not outside the ecology for which we plan–we are always and inevitably a part of it" (512).

The yearning for a more peaceful, just, and healthy world, I feel, is the underlying desire of all humanity. Perhaps, in a small way, teaching and learning mathematics in such a different way can be that additional push humankind needs in our evolution, which is always social, political, and spiritual. If we only experience the world within a prescribed linear progression and believe we are separate numerable parts of such a reality, not only will our sense of freedom and agency crumble, but so will our ability to think in creative and novel ways, characteristics essential to the survival of the human species. If, instead, we imagine a world of fluctuation, beauty and ever-changing interrelated parts, and ourselves as an aggregate of endless potential, then our sense of hope and joy can flourish and a vision of a peaceful, just reality may emerge.

There is no going back to the proverbial "Garden of Eden," but perhaps if we understand why and how we left, we can glean some possibilities for the future of modern humanity**.** Back to the Judeo-Christian creation mythology, Numbers originated in the "naming process" through which humans sought power over the world" (Restivo et al, 1993). Perhaps if cultural linguistical metaphors can facilitate new expressions of our reality, thus enabling us to think mathematically in different ways, we can conceptualize, through a different understanding of number, the world and our relation to it and ourselves differently. Equipped with this new mathematical perception, we can set in motion a way of asking different kinds of political questions, never imagined before. We may look for elegance and beauty in our societal organization, especially its political infrastructure. We may dream of a new pattern of social networking that enables plurality as well as commonality. We may understand that our cultural habits are part of an infinite chain of causations that are intricately woven together. We may even conceive ourselves not in Cartesian dualism, but as a beautiful aggregate of ever-changing parts. And just maybe, at least in in few short thousand or tens of thousands of years of our evolution, we can return to the garden…

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