

CRITICAL PHILOSOPHY OF MATHEMATICS BY OLE SKOVSMOSE: A REVIEW¹

Paul Ernest

University of Exeter, UK
p.ernest@exeter.ac.uk

The 2024 film *Lee* starring Kate Winslett is a biopic about the American photographer Lee Miller. A crucial scene depicts Lee Miller arriving at Dachau concentration camp shortly after its liberation in 1945. Miller's photographs of survivors, piles of emaciated corpses, dead bodies on transport trains and grim-faced guards horrified Americans and the British when they were published in *Vogue* magazine. The monstrosity of the Nazi final solution deeply disturbed the world then and continues to shock. The barbaric inhumanity of the elaborate apparatus for transporting Jews, Gypsies and others across Europe to death camps is the ultimate low point of humanity. Like many observers who entered the death camps during their liberation it was an experience never to be forgotten, and it was to haunt some, like Lee Miller, for the rest of their lives. Yet this atrocity was within our lifetimes, for Ole Skovsmose and I were both born in 1944, the year before this liberation.

When Eichmann was brought to trial in Jerusalem in 1961, he claimed he was just following orders. Hanna Arendt (1963) coined the term 'banality of evil' to describe the actions of this mastermind of the railway transportation to the death camps. Eichmann was not a murderous blood-crazed psychopath, wielding an axe in the grip of an awful passion. He was a quiet master of transport logistics who arranged complex transportation schedules with the quiet satisfaction of an administrator assigning teachers to classes in a school timetable. All classes covered and all teachers given the requisite amount of work.

Both might see themselves as merely fulfilling orders in a complex modern social system, although managing the timetable is, of course, defensible work. But where in schooling do we best learn to just follow the directives, the rules, the orders, without worrying about what situations to which they can be applied? Why in mathematics, of course (Ernest 2024).

¹ Skovsmose (2024) *Critical Philosophy of Mathematics* is published in the Springer series *Advances in Mathematics Education* edited by Gabriele Kaiser and Bharath Sriraman.

Ole Skovsmose begins his journey into the critical philosophy of mathematics with the advent of modernity. Descartes (1637) dismissed the medieval reliance on authority so favoured by theology and asked for a rational basis for all knowledge. In love with of Euclid he strove to remodel epistemology on an axiomatic basis, with all knowledge derived from indubitable foundations. Not just mathematics but science, philosophy and all knowledge was to be restructured and given a secure foundation. That which fitted this model was designated as real knowledge, that which did not was fake.

In a similar line of thought, David Hume (1748, sect. 12, pt. 3) asked of any learned volume, especially metaphysics:

Does it contain any abstract reasoning concerning quantity or number? No.
Does it contain any experimental reasoning, concerning matter of fact and existence? No. Commit it then to the flames: for it can contain nothing but sophistry and illusion.

Modernism followed this path through to the 20th century Logical Positivists. They asserted that only statements verifiable through direct observation or logical proof are meaningful in terms of conveying truth (Ayer 1946).

Notice what is omitted. There is no concern with ethics, society, caring or the human outcomes of knowledge. Modernism elevates mathematics and reason to god-like status and dismisses ethics and other human concerns. It argues that they are illegitimate considerations that besmirch the purity and validity of human knowing.

Ole Skovsmose shows how this very same reasoning enables the holocaust, the transatlantic slave trade, and the cruel exploitation in the name of empire. Once you elevate the reasoning, numbers and rules of mathematics above all else, and disallow ethical considerations, you risk allowing the abuses that ultimately led to the gas chambers.

Ole Skovsmose's critical philosophy of mathematics will not allow this ethical cleansing, nor what he terms the banality of mathematics, that does this work. Traditional philosophies of mathematics follow the path of modernism and look only inwards to foundational questions and technical issues. Thus, traditional philosophy of mathematics is not critical except in a restricted, inward philosophical sense. Purism reigns supreme in philosophical discussions of mathematics, and questions of where and what is mathematics are answered by pointing to the heaven of Platonism or the purgatory of formalism, which is in fact hell for many learners.

Never do traditional philosophers of mathematics point to the everyday world where mathematics originates and where it is applied in every device and in most of our thinking.² But it is in this mundane realm where mathematics must be held in check, and held accountable, by our social consciences.

Western philosophy from Descartes through to the Logical Positivists and beyond is a great vanishing act. Epistemology, mathematics, science are all pure and ethically cleansed, which is only one consonant away from ethnically cleansed, the curse of society, the trumped-up goal of modern populist leaders and demagogues.

However, Ole Skovsmose does not turn his back on philosophical tradition. He argues that it is important that a critical philosophy of mathematics recognises the position of established philosophies of mathematics, while at the same time pointing out their limitations. In fulfilling this, a critical philosophy of mathematics must avoid any isolation and glorification of mathematics as well keeping well clear of any version of absolutism. For absolutism is a dogma that makes any critique of mathematics superfluous. If things cannot be different, any critique becomes pointless. Absolutism vaccinates mathematics against critique.

Ole Skovsmose argues that mathematics is constantly under construction and will never reach a definitive or final form. There is no Platonic reality about which mathematics might tell absolute truths. Mathematical reality is a constructed reality, comparable to architectural constructions. Mathematics is indefinite with respect to concepts, proofs, topics, applications, power, and culture. Indefinite in the sense of unfinished, changing, flowing, becoming, like the thesis in dialectics, which becomes a new and changed thesis after confrontation with the antithesis. Mathematical concepts can gain new meanings, they may be swallowed up by more general ideas, they may even become out-dated and abandoned. What is considered to be a proper mathematical proof changes over time and from one mathematical paradigm to another. The very idea of mathematical rigour is contested.

Mathematics is indefinite with respect to topics and applications. The concepts, ideas, and techniques that are crucial mathematical issues for one period may be consigned to the museum of obsolete mathematics. New topics and methods enter the scene and come to play prominent roles. Mathematics always finds new and often unexpected applications. What was considered to be a pure, innocent, and harmless mathematical research area, such as number theory, can come to have great value for

² I refer later to philosophers of mathematical practice who do look to the world of working mathematicians' everyday practices. But note Cellucci's (2022, p 67) challenging claim that "the philosophy of mathematical practice is continuous with mainstream philosophy of mathematics ... [and] ... shares the shortcomings of the latter."

technological, economic, or military purposes. All the while, mathematics is deeply entangled with power, for mathematics contributes to the formation of new schemes of production and profit making. It makes new forms of social control possible; it is used for algorithmic decision making, and it contributes greatly to military research.

However, the critical philosophy of mathematics recognises that mathematics can also work non-instrumentally for truth, justice, and the improvement of society. Mathematics can establish new and powerful ways of reading and writing the world (Gutstein 2006), and form part of the struggle for social and environmental justice.

Mathematics is indefinite with respect to culture, for it can be encultured in various ways. Hitherto, the history of mathematics has been shaped according to a Eurocentric worldview that narrates mathematics as a European achievement. Thus, mathematics has been colonised, which makes a decolonisation of mathematics an important critical task.

This indefiniteness is reminiscent of some aspects of the work of the artist Francis Bacon. His vision of isolated suffering, existential angst, the horror, the horror ..., may be one lesson of modernity, consonant with Ole Skovsmose's critique. But more to the point, Bacon has the insight that we humans, and the same applies to the whole world and its creatures, are never wholly here, never fully in focus, always pulsating with life, always becoming but never arrived, fixed or definite. Just like every aspect of mathematics, full of life, growth, uncertainty and indeterminacy.

However, it is not just a case of applying a critical philosophy of mathematics to defang mathematics by pointing to its abuses. Because mathematics is indefinite in all these ways, a critique of mathematics is not only possible, but necessary. A critical philosophy of mathematics is essential for the vital task of revealing the real nature of mathematics and forcing the philosophy of mathematics to wake up to its uncertainties.

As the preceding remarks show, and the topics covered, overwhelmingly this book is a deep inquiry into the ethics of mathematics. No single-authored dedicated volume on the ethics of mathematics has yet been published. The 'ethical turn' in philosophy, humanities and the social sciences (Davis & Womack 2001, Goodman & Severson 2016, Voloshin 1998) is only just now reaching mathematics and mathematics education. Ernest (2024) with colleagues opens up the ethics of mathematics education and announces it as a new area of research. But no counterpart exists in the philosophy of mathematics. So, this book ventures into a wide-open and burgeoning field. It offers a unique theoretical analysis and will be an

enduring and instantly classic contribution on ethics in the philosophy of mathematics.

It is no accidental oversight that the ethical perspective has long been neglected in mathematics. As we have seen mathematics, according to traditional philosophies, holds itself aloof from ethics claiming that because it is objective and necessary it is beyond good and evil. It is undeniable that mathematics is almost always presented as objective, with all the marks of the struggles to discover, create and prove its theorems eliminated (Lakatos 1976). Such tidying up of its self-presentation, well known in such fields as theatre, cookery and psychology (Goffman 1971), is also a feature of mathematics (Hersh 1997).

But despite its similarly objective appearance, science has been deeply concerned about its social responsibility, most noticeably since the creation of the atom bomb. Even so, mathematics widely maintains its refusal to accept any ethical or social responsibility, as expressed in the words of mathematicians and fellow supporters of the absolutist ideology. This is despite the fact that it provides the essential language and theory that underpins the development of the atom bomb, the electronic battlefield, computer programmes formatting social relations and Artificial Intelligence, with all their potential dangers. Self-evidently, all of these developments have great ethical significance with great risks of harm, as well as strong possibilities of benefit. Yet there remains a great resistance to acknowledging ethics in mathematics.

The final part of the book brings together all the earlier work as a foundation for explicitly addressing the ethical challenges for mathematics. To do this Ole Skovsmose advocates a novel performative interpretation of mathematics, seeing mathematics and action as integrated phenomena. This draws on the performative interpretation of language, originating in the work of Wittgenstein and Austin and expressed in discourse theory. A discourse constitutes both our ways of seeing the world and of acting in the world. Like a discourse, mathematics forms our experiences, knowledge, assumptions, preconceptions, and ideologies. It forms categories for understanding as well as for misunderstanding. Mathematics forms our worldviews, and our life-worlds as well.

Traditional philosophies interpret mathematics as an objectively pure subject, made up of knowledge that is unrelated to the world. But this approach begs the question. How could such abstractions impact on our lives, let alone have real world consequences? Indeed, in philosophy of mathematics this is known as the Benacerraf

(1973) problem: How can abstract and objective mathematics, within its own disconnected realm, have any effects whatsoever on the physical world?

What Ole Skovsmose does is to reject the perception of mathematics as inert, and to offer instead a performative interpretation of mathematics. Mathematics is not a passive language but is a site of activity and is agentic in itself. It disguises the fact that power gets enacted through mathematics by making conclusions, observations, and statements appear objective and neutral. By being brought into action, mathematics is shaping, fabricating, specifying, doing, forming, disguising, articulating and manufacturing not just truths but social reality itself.

Mathematics contributes not only to the identification of social and environmental risks but is also implicated in the very formation these such risks. No risky technology is made and applied without the calculation of risk factors, in nuclear energy, carbon-based energy sources, space travel, military weapons, pharmaceuticals and so on. But the very calculation of risks licences and legitimates the taking of risks, and periodically leads to disasters at the cost of human lives and the environment. Information technology applications, which are themselves mathematical applications, are distributed widely without any calculation of risks, and assumed to be benign. Clearly, they are powerful, shaping communications, financial transactions, trade, marketing and governance at all levels. But their functioning involves ethically problematic outcomes, for example, in social media, programmes make extensive computations of addictivity and profitability and shape the Apps to maximise such measured outcomes. These measures are what drive further corporate developments. Mathematical actions are implicated not only in shaping social realities but also in reshaping human identities and the human soul (Rose 1993). Such actions are aimed at enslaving children and adult humans to modes of working, playing, thinking, voting and being that best suit corporations, governments and the most powerful interests.

The traditional excuse that mathematical activities take place in the separated realm of mind no longer washes. The mind-body dualism that separated the realms of pure reason from bodily functions, activities and ethics is rejected in the realist ontology adopted by Skovsmose and others.

What is so important in this move is that mathematics is no longer isolated from the world of actions. Ole Skovsmose points out that much philosophy of action has focused on individual and or collective actions. He also adds in structural action. These need not be deliberately performed by any individual, team, group, or institution. Structural actions can be extremely powerful and have a profound and

also devastating impact. He introduces the performative interpretation of mathematics by relating mathematics to individual, collective, as well as structural actions. Mathematics is performative when embodied in digital systems that enact decisions across the whole of society. These extend from pricing, allocation, sales, taxation, benefit calculations, criminal sentencing recommendations to the selection of content to display to individuals on social media, via search engines and throughout information technology applications. Further structural actions include face recognition and military target selection technology.

No action, including any mathematics-based action, takes place in an ethical vacuum. The performative interpretation of mathematics brings any mathematical practice face to face with a critical response and an ethical challenge. This challenge can be addressed to any form of mathematical practice, application or action. What is the purpose of this activity? What are the ethical implications of performing it? Who gains and who loses?

Preparations for making such critical challenges can also be built into the mathematics classroom by engaging students in reflective inquiries. Such inquiries can concern the nature of mathematical ideas and reveal that alternatives are possible with respect to mathematical concept- and theory-building. Reflective inquiries can concern economic inequalities and racist manifestations. But they will never end in definite conclusions. In the world we live in there are no absolutely right answers. Answers are at best optimal solutions for answering particular questions in the contexts of sets of assumptions and chosen parameters. Only in the artificial world of traditional school mathematics or published research mathematics can one meet claims of necessity and certainty, and then only because one has filtered out any real-world considerations.

This fits with Giambattista Vico's (1710) famous criticism of Descartes' cogito argument. Vico argues that a sceptic may agree with Descartes that basically we only know beyond doubt that we think, and hence that we exist. But Vico goes on to posit the *verum-factum* principle which holds that one can only know the truth in what one makes. From this we can conclude that we only know mathematics with certainty, and mathematical necessity only exists, insofar as we made it ourselves. We made mathematics, we decided its rules, and so we made its certainties. If modernism had taken note of Vico's critique of Descartes, epistemology might have evolved in a more human and indeed humane way. Once you apply mathematics to the real world the necessity vanishes. We only have certainty in our own constructions and games, where we decide and impose the rules. Appealing as this might be in social reality, it is only absolute dictators who get near to controlling the people and world in this

way. Furthermore, their control is never absolute and such regimes are the most detestable to democratic thought.

The very act of considering educational as well as real world applications sets Ole Skovsmose's critical philosophy of mathematics apart from almost all preceding philosophies. For traditionally, the philosophy of mathematics has limited itself to the philosophy of pure mathematics. Very strict demarcations are enforced between philosophy of mathematics and applications and responsibilities of mathematics. Applications have been ruled out not only as irrelevant but also as ugly, dirty and demeaning to mathematics.

The great irony implicit in Sergio Leone's film title, "The good, the bad and the ugly" is that it implies a worsening sequence in which ugliness is even worse than evil. But because purists in the philosophy of mathematics have banished good and bad from their universe, ugliness is the greatest evil.

Hardy (1941) divided mathematics into the pure which can be beautiful and applied which is necessarily ugly. He says "beauty is the first test, there is no place in the world for ugly mathematics"

The same holds true in traditional Anglo philosophy but is not defended so floridly. Bringing the contexts of discovery, or the reality of power, or the formation of the subject through education into epistemology has always been taboo. The polite society of Anglo epistemologists and philosophers of mathematics will not countenance such unseemly matters at their dinner table. Like Plato, practical matters or applications of mathematics are relegated to the province of lesser beings, while the exalted ones sit ensconced in their elevated pure considerations with their purified discourses.

Challenging the powerful ones in society has always been regarded at best as a breach of manners, and at worst as a rule breaking, illegal rebellion, needing to be constrained and banished to beyond the pale. This is the risk Ole Skovsmose willingly takes with his philosophical apostasy. His key move is to reject the preconception that mathematics is just the inert architecture of Platonic heaven. For the critical philosophy of mathematics, it is necessary to recognise mathematics as being performative. Mathematics contributes to the formation of our worldviews as well as to our life-worlds. This raises ethical challenges for any mathematical practice, including research, application, and education. His critical philosophy of mathematics faces such challenges and recognises that uncertain situations and that questioning, scepticism and disbelief are all an ineliminable and necessary part of

human life. This includes questioning the narratives of knowledge, government and power. Mathematical applications can never provide certainty, and every aspect of mathematical discourse, and every mathematical-based conclusion includes uncertainty, incompleteness, and a whole range of open future possibilities. Mathematical certainty can never transfer to the empirical or everyday world, it is trapped forever in the constructed box we have made for it, and made it in.

But in addition to simply incorporating ethics and uncertainty, Ole Skovsmose's critical philosophy of mathematics fills a giant hole in contemporary philosophy. It offers a philosophy of *applied* mathematics, something that traditional philosophers have overlooked or turned away from. This neglect is in part this due to the ideology of purism that values pure over applied mathematics, and that of absolutism, which locates the objects and truths of mathematics in some superhuman other world. This leads to a philosophical attitude that turns away from practical philosophies and philosophies of practice.

Focussing on a philosophy of applied as opposed to pure mathematics is not a reduction but an expansion of ambition. A philosophy of applied mathematics not only treats the ontological and epistemological issues that a traditional philosophy of mathematics does. Nor does it expand its goals by merely including the themes of a philosophy of mathematical practice. Ole Skovsmose's philosophy is much more ambitious, for it includes the nature, possibilities and roles of the applications of mathematics throughout citizens' lifeworlds, throughout society and across the world we live in.

The open-endedness this discussion reflects that fact that none of the chapters in this book aim or claim to offer final or definitive answers to the questions of the ethics and critical philosophy of mathematics. They raise questions rather than offering definitive answers, although convincing provisional answers are provided. This particular inquiry is but one strand in the great continuing conversation of humankind. It is a strand to be strengthened and built upon by the scholars and researchers that already are following in Ole Skovsmose's path finding steps. They will extend his analyses, sharpen his conceptualisations, and further develop the remedies he proposes. But make no mistake, those that follow in his footsteps will carry an ineliminable debt to his explorations. And he is not done yet.

The book is named for, and proposes, a critical philosophy of mathematics. It is an adjunct to critical mathematics education (Skovsmose 2023) which is located within educational theory and proposes four main things. These are first, a critical look at pedagogy and education, second, a critical look at the role of teaching mathematics

and mathematics education in society and its political consequences, third, a critical look at mathematics itself, fourth, a critical look at role of mathematics within society and the distorted power relations and injustices that arise from its applications.

Critical mathematics education and more generally critical education draw their critiques of society from neo-Marxist theory. This theory can be mediated by the Frankfurt school of critical theory, by Paolo Freire's critical conscientization programme, by Gramsci's ideas, or by other theories such as post-structuralism. Ole Skovsmose (1985) began his published work in critical mathematics education drawing explicitly on Frankfurt school ideas. Since then, he has broadened his base and draws from many other philosophers and theorists. But his primary focus is critical mathematics education. From early days he has made it clear that this involves a philosophical reconceptualisation of mathematics. And he draws on the most significant published work in the philosophy of mathematics. But the problems he addresses are those that bedevil critical mathematics education. These include reshaping a philosophy of mathematics to accommodate education for critical citizenship and social justice, to include ethical issues, to incorporate political critique. So, one can ask the question: Which audience is this book addressing and will it meet their standards?

Ole Skovsmose addresses mathematics teachers, mathematicians, educational researchers but not primarily philosophers of mathematics. He does not neglect the traditional problems of the philosophy of mathematics such those of epistemology and ontology here. These include: how do we justify mathematical knowledge? What is the status and nature of mathematical objects? But he does so within the new outlook he presents here, his new system.

Traditionally the philosophy of mathematics has not welcomed new systems unless they are technical and foundational. Clever analyses and new arguments directed at outstanding philosophical problems are what is prized and are what are discussed in the top philosophical journals. A quick look through recent issues of *Philosophia Mathematica* reveals keywords in titles such as impredicative definabilism, sizes of countable sets, boffa set theory, formal proofs and conceptual complexity, non-well-founded set theories, analyticity and thin existence, the iterative conception of set, negationless intuitionistic mathematics. These are close-up detail issues. All terms are doubtless of philosophical significance, but nevertheless remain narrow, technical and internal in focus. These illustrate the claim of Cellucci (2022) that the professional philosophy of mathematics is irrelevant to professional mathematicians. What Ole Skovsmose proposes is not directed at an audience of academic philosophers of mathematics, nor does it fit well with their interests. Rather it is

primarily directed at mathematicians and mathematics educators, and others trying to understand mathematics as a whole, including its roles in society.

This audience should come as no surprise, since the book is published in the Springer series named *Advances in Mathematics Education*. One reason might be that philosophies of mathematics are widely regarded on having a larger impact on mathematics teaching and mathematics teacher education than on the practices of mathematicians (Ernest 1991). As Cohen (1971) pointed out, echoed by Hersh (1997), mathematicians are Platonists on weekdays and formalists on Sunday, ‘donning’ the latter philosophy only for public respectability. Platonism is not an explanation of epistemological or ontological issues. It is just a system of belief in the pre-existence of mathematical objects and entities. If anything, it raises more questions in these domains than it provides answers.

In contrast to Ole Skovsmose’s position, traditional philosophers of mathematics focus on their own inward concerns and problems rather than on the nature of mathematics and the role of mathematics in society. Among mathematicians and philosophers of mathematics the critiques of absolutism, definiteness, ethical neutrality and the philosophical significance and risks of applications fall on deaf ears. I am sensitive to this for as a companion philosopher of mathematics education and mathematics my own work suffers a similar fate.

An earlier co-authored work (Ravn and Skovsmose 2019) provides a full treatment of the history of the philosophy of mathematics schools and epistemological and ontological issues. In a review of that work François (2024, p. 139) judges the following.

The authors succeed in bringing together the canonical presentation of the ontological and epistemological dimensions of mathematics education together with the more challenging positions of the human dimensions of mathematics, these being the social and ethical aspects.

The present work elaborates on “the more challenging positions”, the social and ethical aspects of the philosophy of mathematics and is a companion to this earlier work (and Skovsmose 2023). Expanded and treated more fully here, these broader dimensions in the philosophy of mathematics represent the unique and lasting contribution of the present work.

However, the traditional questions and schools in philosophy of mathematics are not neglected. There is a substantial chapter on each on the themes of logicism,

formalism, Intuitionism and one on the philosophy of mathematical practice. The thoroughness of the review, and its extensive list of citations, challenges my claim that there is not yet a canon of work for the philosophy of mathematical practice movement. But welcome though this movement is, it still falls foul of the criticism that it fails to address the ethics of mathematics.

Interestingly, the François (2024) review of the earlier work is published in *Philosophia Mathematica*, the standard bearer for academic philosophy of mathematics. The review ends by recommending the publication to philosophers of mathematics (education) and philosophers of mathematical practices, but not, by omission, to traditional academic philosophers of mathematics.

I should make it clear, after criticising the narrowness of traditional academic philosophers of mathematics, that there are two newer and more liberal schools of thought in the philosophy of mathematics. These are the ‘maverick’ philosophers of mathematics, including Lakatos, Tymoczko, Kitcher and Hersh, and the even more recent emergence of the philosophy of mathematical practice movement (Sriraman (2021)).³ The latter are more concerned with the actual practices of mathematicians and their philosophical implications. Ole Skovsmose treats these more recent developments although he does not make this binary distinction. Hopefully, provided that news of this work is promulgated effectively among the mavericks and philosophers of mathematical practice, due recognition will be accorded to the philosophical import of the social and ethical dimensions within the philosophy of mathematics. But requiring more than credit, the new theoretical challenges posed by Ole Skovsmose’s groundbreaking work here and elsewhere should be taken up more widely by the philosophy of mathematics community, beyond that of the dedicated philosophy of mathematics education community. The ethical and social challenges facing mathematics in these times of global warming, spreading wars and global domination by information and communication technologies are the greatest we have ever faced. It is now urgent and vital that philosophers, mathematicians and indeed all citizens wake up to these challenges, expressed so forcefully and analysed so convincingly in this book.

November 2024
London and Exeter

³ A canonical set of references for the philosophy of mathematical practice is not yet established although one might include Sriraman (2021), Cellucci (2022), Corfield (2003) and Mancuso (2008).

References

- Arendt, H. (1963). *Eichmann in Jerusalem, A Report on the Banality of Evil*. New York, United States: New Yorker Magazine.
- Ayer, A. J. (1946) *Language, Truth and Logic*, London: Gollancz.
- Benacerraf, P. (1973). Mathematical truth. *Journal of Philosophy*, Vol. 70: pp. 661–679.
- Sriraman, B., Ed., (2021). *Handbook of the history and philosophy of mathematical practice*. Cham, Switzerland: Springer.
- Cellucci, C. (2022) *The Making of Mathematics - Heuristic Philosophy of Mathematics*. Cham, Switzerland: Springer.
- Cohen, P. J. (1971). Comments on the foundations of set theory. In D. Scott, Ed. *Axiomatic Set Theory, Proc. Symp. Pure Math.* Vol. XIII, Providence: American Mathematical Society, 1971: pp. 9-15.
- Corfield, D. (2003) *Towards a Philosophy of Real Mathematics*, Cambridge University Press: Cambridge.
- Davis, T. F. & Womack, K., Eds. (2001). *Mapping the Ethical Turn: A Reader in Ethics, Culture, and Literary Theory*. Virginia, USA: University of Virginia Press.
- Descartes (1637) *A Discourse on Method*. Translation reprinted in Descartes, R. *Philosophical Works*, Volume 1, Dover Press, New York, 1955.
- Ernest, P. (1991). *The Philosophy of Mathematics Education*, London: Routledge.
- François, K. (2024) The Interest of Philosophy of Mathematics (Education), *Philosophia Mathematica*, Vol. 32, No.1, (Feb. 2024): pp. 137-142, <https://doi.org/10.1093/philmat/nkad026>
- Goffman, E. (1971). *The Presentation of Self in Everyday Life*. London: Penguin Books.
- Goodman, D. & Severson, Eds. (2016). *The Ethical Turn. Otherness and Subjectivity in Contemporary Psychoanalysis*. London: Routledge.
- Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward a pedagogy for social justice*. New York: Taylor & Francis.
- Hardy, G. H. (1941). *A Mathematician's Apology*. Cambridge, U. K.: Cambridge University Press.
- Hersh, R. (1997). *What Is Mathematics, Really?* London, Jonathon Cape.
- Hume, D. (1748). *An Enquiry Concerning Human Understanding*. Reprinted as D. Hume, *An Enquiry Concerning Human Understanding: A Critical Edition*. United Kingdom: Oxford: Clarendon Press, 2000.
- Lakatos, I. (1976). *Proofs and Refutations*. Cambridge, UK: Cambridge University Press.

- Mancosu, P. Ed. (2008). *The Philosophy of Mathematical Practice*. Oxford: Oxford University Press.
- Ravn, O. and Skovsmose, O. (2019). *Connecting Humans to Equations: A Reinterpretation of the Philosophy of Mathematics*. Cham, Switzerland: Springer.
- Ravn, O. and Skovsmose, O. (2019). *Connecting Humans to Equations: A Reinterpretation of the Philosophy of Mathematics*. Cham, Switzerland: Springer.
- Rose, N. (1989). *Governing the Soul: The shaping of the private self*, London: Routledge.
- Skovsmose, O. (1985). 'Mathematical Education versus Critical Education', *Educational Studies in Mathematics*, Vol. 16, No. 4 (November 1985): pp. 337-354.
- Skovsmose, O. (2023). *Critical Mathematics Education*. Cham, Switzerland: Springer.
- Skovsmose, O. (2024). *Critical Philosophy of Mathematics*. Cham, Switzerland: Springer.
- Voloshin, B. R. (1998). The Ethical Turn in French Postmodern Philosophy. *Pacific Coast Philology*, Vol. 33, No. 1, pp. 69-86.