HEGEL'S DIALECTICS AS AN INVITATION TO CLASS STRUGGLE IN MATHEMATICS

Roberto Ribeiro Baldino and **Tânia Cristina Baptista Cabral**

Universidade Estadual do Rio Grande do Sul, Brazil

rrbaldino@terra.com.br cabral.taniacb@gmail.com

Abstract

This paper is written for those who hate fascism and mathematics. Based on Hegel's narration of Kant's philosophy, we have created two characters, Speculation (Hegel) and Understanding (Kant) and trust in Marx's anatomy metaphor to show that these characters have been staging class-struggle in science since the time when Newton and Leibniz invented calculus. This mathematics lives in symbiosis with fascism through the pedagogy of credit system.

We point to the mathematics of the twentieth century (M20) a product of Understanding, as the reader's true object of hate. The mathematics necessary to follow our argument does not go beyond elementary school. We show that, from the simultaneous birth of calculus and capitalism in the seventeenth century, Speculation has been challenging Understanding with the concept of infinitesimal while Understanding takes refuge in arbitrary language conventions to ensure mathematical truth. No wonder so many hate M20. In the final section we lead Understanding to speak up the source of its silence about labor-power and surplus-value. This silence stems from the silence around qualified-labor-power, a special commodity produced in school. We answer Althusser's question: Why is the educational apparatus in fact the dominant Ideological State Apparatus in capitalist social formations, and how does it function? It is because the school credit system leads students to participate in an economic practice of production and seizure of surplus-value, the basic operation of capitalism, elicited by Marxism.

Key words: labor-power, infinitesimals, class-struggle, surplus-value, economic production in school.

Introduction[1]

When we are introduced to new acquaintances as "math teachers", it is common to hear the disclaimer: *I never did well in math*. If these people were to put it more bluntly, they would say: *I hate it*. This does not happen with other subject areas, such as physics or biology. If you too *hate it*, then this paper is written to you and for you. We do not intend any trickery to make you like mathematics, but perhaps we will help you understand why you came to hate it. From a historical perspective, we will focus on calculus, which was the crib of M20, the object of hate. We will show how two theories, one *hegemonic*, the other *subordinate*, still vie for the mathematics spotlight. Interesting enough, the capitalism that is assuming its fascist identity today, was born at the same time as calculus. Coincidence?

Calculus was born around the end of the sixteenth century, draped in physics and philosophy, amidst animated discussions about its validity. Bishop Berkeley used to say that it was easier to believe in God than in calculus. Roughly speaking, calculus deals with areas and tangent lines to curves. These problems, known since Ancient Greece, had to wait two thousand years for people to realize they were dealing with a single problem: the solution of one also solved the other. Newton, in England and Leibniz, in Germany, vied for the laurel for having found it first. Leibniz died accused of plagiarism.

We will refer to the development of calculus, from its origin to M20, as the struggle between two characters with different class positions: Understanding and Speculation. By the beginning of nineteenth century, Understanding had found support in Kant's philosophy, and Speculation in Hegel's. These two terms are actually due to Hegel who, by "Understanding" refers to Kant's philosophy, and by "Speculation" to his own. However, the class position that we attribute to these characters have been present even before the origins of calculus.

> The ecclesiastical authorities granted professional 'licenses to practice' because they were anxious to ensure that people were not led astray by individuals who set themselves up as teachers when they had no qualifications, and might turn out not only

> to be ill-informed, but dangerous demagogues at that. The twelfth century had seen a good few of those, and some of them had been 'academics' (Evans, 2010, p. 150).

What had been the struggle of sophists against philosophers in antiquity and would become that of Speculation against Understanding in the nineteenth century, was already there in the twelfth century, and must be thought of in terms of class-struggle, according to Marx's anatomy metaphor: "the human anatomy contains a key for to the anatomy of the ape" (Marx, 1973, p. 105). Understanding, rooted in Newton, became hegemonic and generated M20. Speculation, stemming from Leibniz, became subordinate but bequeathed the present notation of calculus to us.

To convey these two theories to you, we will need a minimum of mathematics, not beyond elementary school. You certainly know the meaning of a devastated area in the Amazon Forest, the meaning of the area affected by an A-bomb, and the meaning of the area of the house you live in. You know what "area" means. You also know that, when turning a corner while driving at night, the headlight beams of your car do not illuminate the path you are following, but rather keep pointing straight, tangent to the car's trajectory. Some classic models, like the 1948 Tucker, had a central beam connected to the steering wheel, and modern cars have adaptive headlights that follow the curve ahead. Perhaps you have also seen films with WWII fighter aircraft firing shots while making a curved trajectory. The shot follows along the tangent line to the airplane trajectory. Finally, we mention the popular expression: "going off on a tangent". So you know what a tangent line is. We will trust what you know, and that will suffice.

For those who tolerate, or even love M20, essential texts include Stroyan and Luxemburg's "Non-Standard Analysis" (1976), often called 'the Bible' on the topic of infinitesimals, followed by the seminal work of Abraham Robinson (1966), who is credited with creating the subject, and Howard Keisler's (1986) calculus book intended for college students. The literature on this branch of M20 is enormous. There are pioneering texts about teaching infinitesimals, like Sullivan (1976), intended for gifted students, as well as elementary approaches from the point of view of mathematics education, like Monaghan et al (2024). There are plenty of historical texts focusing on the origins of infinitesimals, like Bair et al (2022), as well as applied texts for electrical engineering, like José & Kemel (2020), and texts about the foundations of infinitesimals on set theory, like Ponstein (2001). However, *none of these texts are intended for people who hate mathematics*.

Speculation challenges Understanding

Today, M20 is a dead body. You are told that the area of a circle of radius R is πR^2 and that the length of the circumference is $2\pi R$. You are required to memorize it for the exam on Monday morning. During the eighteenth century, when mathematics was alive and M20 had not yet been born, people would ask: *how do you know that the area of a circle is* πR^2? A debate between Understanding (U) and Speculation (S) might take the form of the following dialog.

> U: I know it because I evaluate the area of a regular polygon circumscribing the circle (Fig. 1). There is a formula for this area. Then I increase the number of sides of the polygon. It is just a matter of computation.
>
> S: If you are always calculating the area of polygons, you never get the area of the circle.
>
> U: As you can see, if the radius is equal to one, as I increase the number of sides I get numbers closer and closer to *Pi*, which is the true area (Table 1). It works.
>
> S: What do you mean by "it works"?
>
> U: I am comparing an approximation to the true value…
>
> S: If you already know the true value, you do not need approximations. If you don't know it, you cannot make the comparison and say that it works. If your method intends to find out the area of the circle, you must assume that you do not even know Pi.
>
> U: Look, my method produces numbers whose initial decimals stabilize at 3.1416 (Formula 1). I call Pi this stabilized number.
>
> S: All right, but what does this number have to do with the area of the circle? You have only calculated areas of polygons.
>
> U: (With irritation). Look, everybody has an idea of the area of a circle, be it in square centimeters, square meters or square miles. My method only evaluates that area precisely, by closer and closer approximation. Look at the figure.
>
> S: By "everybody", do you include the tribal populations of Brazil and natives of colonized countries? Or do you leave them out? Is the perceived view of area in our society a parameter to evaluate the inferiority of people who do not share it with you?

According to Hegel, when Understanding is cornered by the dialog, he reacts aggressively. In the present case, Understanding would have to admit that the presupposition of knowledge in the school credit system leads to the social segregation that Speculation calls class-struggle in school.

$$A(n) = n\,R \tan\left(\frac{360}{2n}\right)$$

Formula 1

Pi	3.1415926540

Table 1

# sides	area of the polygon
10	3.2491969630
100	3.1426266050
1000	3.141602 9890
10 000	3.1415927570

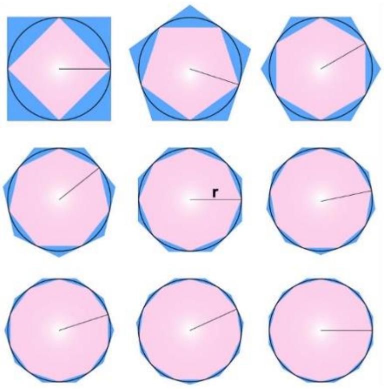

Figure 1

At the time when this dialog might have occurred, European colonization of other peoples was going on full-steam. The dialog elicits the two philosophies that support the struggle between the two philosophical positions, Understanding and Speculation, during the nineteenth century.

Of philosophy we have the concept that it is class- struggle in the realm of theoretical practice (Althusser). For Kant, only our cognitive procedures are accessible to us; trough these processes, we approach the object of knowledge but never actually reach it. The *thing in itself* cannot not be known. For Hegel, *understanding* consists of black-and-white separations, clearly a rightist philosophy in search of hegemony. He says that if you want to know an object, you should not impose exterior determinations on it; simply dwell in the object and let it speak for itself.

Note that Speculation does not confront understanding. As in the above dialogue, Speculation does not oppose Understanding with another concept about the area of the circle, and neither does she try to change Understanding's ideas. She simply makes Understanding confess his own contradictions. This method is what Hegel calls *dialectics*. Understanding fights in order to wipe out Speculation; Speculation fights to continue fighting. She knows that, if she could eliminate understanding, she would die too, for lack of an object upon which to exercise herself.[2] It is worth noting that the mature Marx declared himself a pupil of Hegel.[3]

Speculation dwells in the circle

It would be useless for Speculation to try to convince Understanding to change his position about the area of the circle, but Speculation can justify herself to us. Dwelling in the circle, Speculation realizes that, looking to a point *x* on the circumference with a very powerful microscope, she would see the point as the center of a small segment of straight-line. *Hum…* she would think, *what if I stretch this segment out of the microscope's focus*? She realizes that from a global perspective, the straight line segment would produce a full line tangent to the circle (Fig. 2).

At this point, Speculation realizes that the microscope only produces an optical illusion. *I have only produced the illusion that the curvature of the circle has disappeared*. If you, the reader, has access the zoom feature of a software like CorelDraw or GeoGebra, you can share in this initial illusion and in the realization. Speculation continues: *if I imagine that the microscope is infinitely powerful… I would enter a new universe, a Lewis Carol kind of universe of the infinitely small. From this universe, looking back, I would see the center of the circle as infinitely far away…* In its new universe, Speculation would see all the points of the circumference that are infinitely close to *x*; this set is called the *monad* of *x*. In the figure, we have chosen a certain length on the paper to represent the monad. This length is a kind of yardstick for Speculation's infinitesimal universe (first balloon in Fig.2).

Speculation continues her reflection. *In this universe, I have the infinitesimal segment on the tangent line. The point x is common to the circle and to the tangent, but… what if I look at a point a little to the right or left of x?* Speculation marks the point $x+\alpha$ where α is infinitesimal compared to the length of the monad. *The circle is bent… at* *this point it cannot coincide with the tangent… How come I do not see their separation?* Speculation is puzzled. Finally, she exclaims: *Aha! To see this separation I need another microscope, infinitely more powerful than the one I am thinking of. This separation is in another universe, infinitely smaller than mine*. Indeed, separation has stumbled on what is called a *second order infinitesimal* (second balloon in Fig. 2).

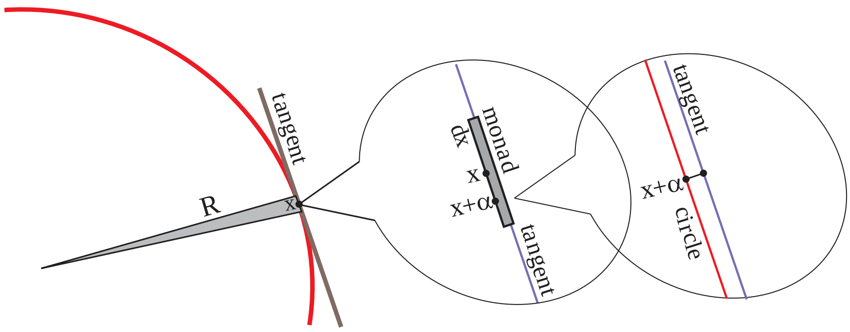

Figure 2: second order infinitesimal

Speculation has made the circle talk. The circle has told us that there is an infinitely small of the infinitely small of the infinitely small… and so on. Returning from its imagination to our universe, Speculation realizes that, by drawing the straight lines from the center of the circle to the endpoints of the monad she obtains a triangle with height equal to *R* and base of length *dx*. (In the figure, the base *dx* has been exaggerated.) She realizes that the area of the circle will be the sum of the areas of all these triangles, as the monads vary on the circumference. However, this voyage by Speculation has to be made with imagination, as it is impossible, in our universe, to draw a monad that is infinitely small. (You, the reader may rely on meditation if you prefer.) The difference to Understanding is striking. *Speculation knows what the area of the circle is*. The circle, itself, has spelled out its concept. Only now does it make-s sense to use Understanding's table of areas, compare it with a known object and evaluate its numerical determination.

Drawing a tangent line

Instead of trying to convince Understanding of her view, Speculation shows him this picture (Fig. 3).

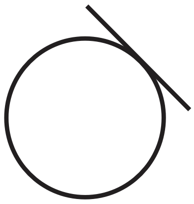

Figure 3: a circle

S: What is this?

U: It is a circle with a tangent line.

S: Yes, it is a circle and a tangent. I just stretched out an infinitesimal segment of the circumference.

U: There is no meaning in what you say; infinitesimals do not exit, they are self contradictory.

S: Precisely. We now have a circle, its tangent, the story of my voyage and your denial. All this will be there whenever you look at this picture from now on. Our dialog has changed something in the world.

Hegel uses the verb *aufheben*, generally translated by *to supersede*, to [4]refer to this change. Speculation has elicited the four unavoidable moments of dialectics; they were contained in the first answer by Understanding (Hegel, 1966, V. 2, p. 478). More common views of dialectics do not count the first moment and start counting one at Speculation's exposition. Fichte called these moments *thesis, antithesis, synthesis*.

Speculation continues:

S: Why did you say it is a circle and its tangent?

U: This is evident.

S: Not to everyone. Most people would have answered differently. Why did you say *circle* and *tangent*?

U: (Showing irritation.) For a certain level of culture, it is evident.

S: Precisely. You could not devalue yourself with a vague answer. You must zeal for the value of your own qualified-labor-power. The preservation of *your capital* directs your answer.

In Baldino & Cabral (2015) we developed the idea that the labor-power which has its value increased by a diploma is treated as a kind of capital by its owner. Understanding risks devaluing it in a debate with Speculation.

M20 as a convention of language

Understanding and Speculation lively discussed their positions throughout the eighteenth century. We doubt that you would have hated that. However, in 1824, a mathematician, Augustin-Louis Cauchy, willing to make the dialogs more precise, stroke a death blow on live mathematics. From the hegemonic stand he stated: “one says that…” (*on dit que*). In the first dialog we presented, about the area of the circle, he would have laid things to rest by stating: *one says that the area of the circle is this stabilized number, produced by increasing the sizes of the circumscribed regular polygons.* This would have ended the discussion. Cauchy’s blow eliminated all possible divergent ideas and reduced mathematics to a *convention of language*, thereby blocking out all other philosophical arguments, except those from Understanding, now on its way to become hegemonic.

Later in the century, another mathematician, Karl Weierstrass, stroke the *cup de grace* on live mathematics. He organized language in terms of logic, so that each *on dit que* would have one single meaning. Understanding was definitively enthroned as hegemonic. The corpse of mathematics exploded into uncountable new results, for the delight of mathematicians, who could now publish new results, explore new avenues and increase the value of their qualified-labor-power to vie for academic positions (Baldino & Cabral, 2013). M20, *the hegemonic mathematics that you hate*, was born. Infinitesimals were declared inconsistent and chased out, along with Speculation.

The following is an eliciting example of how Understanding defines the tangent line to a curve. It is a sort of game that the reader may try to play during a conversation at a café. Understanding, as a hegemonic teacher, enters the classroom and announces:

> U: Today we will learn what a tangent line to a curve really is.

He asks a few students to turn their back to the blackboard, where he draws a curve and marks a point on it. Holding the chalk (or pencil) in one hand and a ruler in the other, he defies the students who had turned back.

> U: Without looking at what I am doing, tell me what to do with this ruler in order to draw line tangent to this curve through the marked point.

The teacher seeks to contradict every suggestion of the students. For instance, to the instruction 'the ruler touches the curve', the teacher knocks the screen with the ruler, etc. Finally, he asks one student to step on the platform, turns his back to the blackboard and starts instructing.

> U: Mark another point on the curve, about fifty centimeters away from the point already there. Let's call them points zero and one. With the ruler passing through both points, draw a line segment and extend it to about the length of the ruler.

The other students certify that the instruction has been executed.

> U: Now choose point two on the curve at about half the distance between points zero and one. Call it point two. Repeat the procedure of drawing a segment between points zero and two; extend it, as you did with points zero and one.

The students give their okay.

> U: Chose point three and repeat the procedure. Do this as long as your drawing abilities allow. Can you imagine the final position that you would reach if you could go on indefinitely? Well, *one says that* this final position is the tangent line to the curve through point zero. In other words, *one says that the tangent is the limit of the secant lines* as the second point approaches the given point.

Generally, the students take notes and remain silent. However, a student who is adept of infinitesimals expresses the subordinate position and dares to ask:

> S: Do the secant lines really arrive at the tangent or not?

The teacher starts explaining that this does not matter, as long as bla bla bla. The other students engage into the conversation. Finally, Speculation concludes:

> S: *Before it arrives, it becomes infinitely close.*

Most students agree. The hegemonic teacher continues his monolog, but the students cease paying attention. Their spontaneous conceptions have been satisfied. An example that is generally presented at the level of fifth grade in elementary school is whether 0.999… is equal to one. At the university, students generally answer that it is smaller. Among those who say that it is equal, most explain that they were told that the correct answer is 'equal' but that they do not really believe it. M20 has imposed its hegemonic view upon the students; there is no place to say that before it gets to one, it becomes infinitely close; M20 forbids infinitesimals. No wonder school has turned M20 into your object of hate.

This paper could end here; the final words would contain the question: do you think the discourse that we have sustained from the beginning is a discourse of Understanding or rather of Speculation? The answer is obvious, we have led M20 to confess: *I am your object of hate*. Only a speculative inquiry could have extracted this confession. Our discourse can stand as an example of the dialectical method.

Infinitesimals survive

Nevertheless, infinitesimals survive in other exact sciences, mostly as a kind of clandestine conception. Calculus textbooks do not mention them, but they remained alive, sometimes under the name of *differentials*. Paradoxically, the Leibniz notation for infinitesimals became the only one used in M20. Hegemonic Understanding cannot justify this notation to the students; he says that the symbols are due to cultural tradition; infinitesimals and infinite sums are considered meaningless. Instead, Understanding insists on *one says that* as a guarantee of truth.

Throughout his life, Cauchy supported infinitesimal thought. When he stated *on dit que*, Cauchy was probably looking for more rigor to speak about infinitesimals. Had he known the adverse effect of this dictum, he would probably have softened it. The way that Understanding tried to block off Cauchy and his thoughts is a paradigmatic example of *class-struggle in science*. Even when Cauchy explicitly wrote "if one designates by ε an infinitely small number"[5], Understanding is able to say that he had never actually thought of infinitesimals and that this statement was just "the standard lore for expressing an arbitrarily small number." (G. Schubring quoted in Bair et al, 2022, sec. 2.6)

However, in his attempt to reach greater precision when talking about infinitesimals, Cauchy introduced something new. "One says that a variable quantity <u>becomes</u> *infinitely small* when its numerical value decreases indefinitely so as to converge to the limit zero" (Cauchy, 1821, p. 26, added underline).[6]

A variable is for the mathematician as the hammer is for the carpenter. Now the hammer becomes something else. How can Understanding take this? The meaning of the French verb *devenir* is "To pass from one state to (another), to begin to be (what something was not)".[7] This verb is used to translate the German *werden* that Hegel employs referring to death-and-birth, like the flower that springs from the rotten trunk. *Werden* expresses the statement, followed by a misunderstanding of the listener and admitted by the speaker that what s/he has said has indeed a certain dose of contradiction. (Hegel, 1966, V. 1, p. 105). Admission (*aufheben*) of contradiction is the essence of Speculation. Hegel uses it to describe the four moments of dialectics pointed out above. This sort of becoming is an unavoidable consequence of the apparent misunderstandings of language.

We argue that Cauchy's insistence on *devient* indicates the passage from what our finite universe is, to what it becomes in thought, an infinitesimal universe. Here, again, Understanding makes an effort to negate the transcendence: J. Grabiner sustains that an infinitesimal is just a variable tending to zero, (J. Grabiner ref. in Bair et al, 2022, sec. 2.1). Understanding cannot accept the infinite inside a finite being; he rejects the original meaning of the Leibniz notation. Understanding is even more pathetic when it sustains that Cauchy was a predecessor of Weierstrass because he used two Greek letters (ε, δ) that were later picked up by Weierstrass to refer to small, but finite numbers. How could Cauchy have guessed it? Understanding would be safer if he had argued that Weierstrass developed Cauchy's form *on dit que*, instead of searching for the relation between these two mathematicians in the content of their thoughts.

Class-struggle in calculus

Understanding is able to sustain fallacious arguments because it ignores the distinction of two relations that Speculation uses to connect past and future in the society. In German, there are two words for relation: *Beziehung* and *Verhältniss*. In French, these have been translated as *relation* and *rapport*, respectively. Portuguese and English conflate these meanings; perhaps we should say that English is the mother language of Understanding, and German is the mother language of Speculation. Marx used *Produktionsverhältnisse* to refer to a moving relation carried out by capital as an automatic subject (*automatisches Subjekt*), that is, an autonomous self growing process that recruits people as supporters. In *Verhältnis* the meaning of the related terms emerges from the movement; the relation comes before the related terms. In *Beziehung* the terms are there before and independently of the relation. *Dialetktische Verhältnisse* is a redundancy that Hegel uses once in his Logic (1981, p.244); dialectic *Beziehnung* is a contradiction in terms like *square circle* (Pais, 2016). Understanding cannot think the relation of Cauchy to Weierstrass as *Verhältnis*, only as *Beziehung*. Class-struggle is *Verhältnis*, but Understanding endeavors to make it a *Beziehung*, for instance, when Pope Francis I stresses the possibility of people renouncing class-struggle. “May people learn to fight for justice without violence, renouncing class-struggle.”[8] In the Pope’s sense, the classes are there before they clash into struggle, which can hopefully be avoided.

During the English bourgeois revolution, “[t]he two sides in the war established their bases at Oxford and in the Cambridge area, respectively. Of the two universities, Oxford became the ‘Royalist’ stronghold and Cambridge the ‘Puritan’ one (…).” (Evans, 2010, p. 319) In the aftermath of the revolution, Cambridge profited from students’ freedom to discuss and choose what they wanted to study. Isaac Barrow, one of Newton’s teachers, like other scholars, did not refrain from writing on subjects about which he did not have much information. “This remarkable freedom could even apply to the student studying for a profession. Isaac Barrow did not have to pursue a higher degree in medicine when he considered entering the medical profession. He merely had to read” (Evans, 2010, p. 303).

Taking advantage of his freedom, Barrow authorized himself to represent areas by lengths of line segments, which today is a triviality. For instance, in the graphs displayed by the media about the deforestation of the Amazon region, the height of the bars represents thousands of square kilometers. However, since Antiquity, areas were supposed to be compared to areas and lengths to lengths. Johannes Kepler (1571-1630) had been careful to say that the radius vectors of the planets sweep equal areas in equal times. It seems that the jump to stating that the *areolar speed* is constant was a too high stake for society at that time. This difficulty is confirmed by our students today. By representing areas by lengths, Barrow managed to unify the two distinct problems of calculating areas and determining tangent lines to curves. From Barrow’s theorem Newton and Leibniz developed calculus. It was not by chance that Calculus and capitalism share the same milestone.

In 1960, a mathematician, Abraham Robinson (1968), showed that infinitesimals can be reduced to legitimate M20 knowledge and is perfectly able to support any demand of rigor from Understanding. Nevertheless, after sixty years, infinitesimals remain outcast, surrounded by silence in calculus courses. Looking back into the initial milestone of capitalism and calculus, we can discern something else that has also been surrounded by silence, namely, surplus-value. It was there at the beginning of the new history, together with money and work. Why did Speculation have to wait two centuries for Marx to spell it out?

Understanding produces silence by talking a lot "around" what he wants to suppress, but never mentioning it directly. The active production of silence around surplus-value still goes on today. For instance, Thomas Piketty wrote a 500-page book entitled "Capital" without once mentioning surplus-value. The ideological interpellation naturalizes silences so that subjects *enjoy keeping silence* on certain things in order to be received as "educated". From social convenience, silence becomes an *unconscious imposition*. However, with respect to infinitesimals, Understanding's efforts to produce silence have failed. Most students keep them among their spontaneous conceptions, for instance, when they maintain that $0.999...$ is less than one.

Robinson opened a new vein for mathematicians to research, publish and develop their qualified-labor-power under the tolerance and surveillance of hegemonic Understanding. However, this does not mean that Speculation should relinquish infinitesimals to the waste basket. There is no reason for Understanding not to introduce them, especially in STEM courses. Infinitesimals are a stubbornly resistant part of the students' spontaneous conceptions; they are the natural language for speaking about areas and tangents. The action of Speculation to make Understanding confess its laziness to study this new branch of M20 is an instance of class-struggle in science. We are engaged in it and hope that you will join us, by taking infinitesimals to elementary school.

Four centuries of silence on labor-power

In this section, the reader will join us in assuming the role of Speculation, while Understanding will be endeavoring to produce silence. We will force Understanding to speak, though, and at the end we will collect the conclusions.

There are plenty of references online: people are talking freely about "the highest-paying college degrees".[9] There is no doubt that school is a place of economic production. As such, it must produce a commodity. Vulgar thinking suggests that school produces "degrees," whose owners sell them for higher salaries during their lifetime. However, "degrees" are not commodities. Whether intentionally or because it ignores Marxism, vulgar thinking forgets that *profit presupposes unpaid work in the production of a commodity*. A "degree" is not a commodity; it cannot be exchanged between two people and it is not worn out when "exchanged" for salary, as if it were divided into monthly vouchers.

"Degrees" have not always been so frenetically sought. For instance, in 1360, "in Cambridge, the earliest domestic arrangement was that students lived in lodgings, unless they were members of religious Orders and could therefore reside in a 'house' provided by the Order for the purpose" (Evans, 2010, p. 164). This situation changed very slowly: "In the seventeenth and eighteenth centuries, Cambridge, like Oxford, was still educating a high proportion of its students for ordination in the Church of England" (ibid, p. 285). The main event that clenched the rush for "profitability" was the English Revolution of 1641-45, the first bourgeois revolution in history. According to Marx, we can say that this event marks the beginning of a *new history*. Like the grain that makes the pile, this revolution was the result of the social unrest that had been accumulating during the sixteenth century.

> But the *mere presence of monetary wealth*, and even the achievement of a kind of supremacy on its part, is in no way sufficient for this *dissolution into capital* to happen. Or else ancient Rome, Byzantium etc. would have ended their history with free labor and capital, or rather begun a new history (*eine neue Geschichte begonnen*)" (Marx, 1973, p. 506)

Commodities started been produced by waged workers to be sold in the market, instead of supplying family needs. The production in England was dominated by the so-called *domestic system:* "Wool or yarn was supplied by the merchant to be spun or woven by the laborer and his family in his own home” (Hill, 1940, p. 17). In times of depression, the merchant failed to supply the raw material, forcing the family heads into debt. One century earlier, during the expulsion of the Jews from Spain in 1492, the richest among them were assimilated into Italian cities as bankers (Attali, 2003, p. 293-304). The Church had stopped considering usury a sin. By lending money, one could passively watch his capital grow. Was that a sin?

> Occasionally, indeed, a small master managed to “better himself” by fortunate borrowing of the capital which was indispensable if one was to get on, but far more were unlucky. Hence the small producers joined in the clamour of the feudal landlords against “usury.” They could not do without loans, and yet were crippled by the high rates of interest which could be exacted in a pre-capitalist society. *“Usury” was to ordinary people what wage-labour is to their successors to-day.* (…) Hence there was coming into existence a petty-bourgeois class with specific economic interests of its own, but changing in composition as its most enterprising and lucky members rose to become capitalists, and the unfortunate sank to be wage-laborers (Hill, 1940, p. 17, added emphasis).

Either exploit or be exploited; wage labor was born alongside class-struggle, dictated by capital as an automatic subject amidst a debate on usury. The concern about usury could have been extended to waged labor. Would it have be a sin to passively watch a waged worker increasing their capital? However, the moral similarity of usury and surplus value seems to have fallen under silence, despite Oxford and Cambridge being at the center of the ideological war between the papal church and the Calvinists; for these, labor and profit were positive values. The labor-power, with its inherent unpaid work, was already there, but only with Marx, two centuries later, was it recognized as a commodity with use and exchange values.

In the aftermath of the revolution, during the ten years of Oliver Cromwell’s dictatorship, university students enjoyed a period of great freedom to decide what to study. “This was a period of self-education and intellectual self-reliance for Cambridge students” (Evans, 2010, p. 302). Educated people started taking responsibility for the profitability of their “degrees”. They found that a certain value should be preserved: “personal reputation proved to be portable” (ibid, p. 204). Barrow had been bold enough to represent areas by lengths of line segments, in defiance of classic tradition. Newton was reluctant to publish some of his findings; he never left his position in

Trinity College. Leibniz worked for the same family during the last forty years of his life. Scholars were discovering that, like any other workers, they too possessed a labor-power; they realized that, beyond its portability, they should take care to preserve its value. More recently, after their deed, Einstein and Gödel accepted tenure at Princeton Institute and remained there for the rest of their lives, silently recognizing that their labor-power had a value to be kept in a safe, just as gold is kept in Fort Knox.

We call this special kind of labor-power that has undergone schooling, *qualified-labor-power* (Baldino and Cabral, 2013). We will argue that the four centuries of silence around this special form of commodity has the same source as the silence about general labor-power and surplus-value.

The double character of the labor-power was one of the main discoveries of Marx. "The economists, without exception, have missed the simple point that if the commodity has a double character – use-value and exchange-value – then the labor represented by the commodity must also have a two-fold character"[10]. This means that the produced commodity can be sold in the market because it is somehow useful – has a use-value; its price is the amount of human work crystallized in it during its production. Labor-power has an *exchange value,* which is the human work necessary to reproduce the worker; its price is the salary regulated by the market of labor-power. Purchase and sale occur between commodities of equal value. All this was well known in the classical political economy of Smith and Ricardo. What Marx added was that this *double character replicates itself in the production of labor-power*.

The labor-power also has a use-value for the capitalist, otherwise he would not hire the worker, that is, would not buy this commodity. Upon using the labor-power, the capitalist becomes owner of the worker's production. Surplus-value is simply the difference of the produced value *collected* by the capitalist and the cost of production, *paid* by the worker to reproduce the labor power spent in a day's work or in the production of a number of pieces, just as if it were divided into vouchers. This was Marx's contribution.

Since qualified-labor-power is a special kind of labor-power, it also must have the double character of use and exchange values. As an already produced commodity, it is sold throughout the lifetime of its owner − the graduate − there is nothing to add to the vulgar thinking that clogs the internet with the "profitability of degrees"; once qualified by the school process, the labor-power can be sold for a higher salary, greater than the room, board and tuition paid by its owner during schooling. At each stage, from elementary and secondary education to college, university, post-grad and post-doctorate, the less qualified labor-power climbs step by step towards greater qualification. Its use value may also be a function of other factors, such as experience, and its market price may be influenced by subjective values like the sign-value (Baldino and Cabral, 2015).

However, as Marx stresses, the labor employed in the production of the labor-power used in the process of qualification must also display a double character: a *use-value* for the capitalist who collects the production and an *exchange-value*, necessary to reproduce the worker. Here is the critical character of the qualified-labor-power: *both capitalist and worker are the same, namely, the student.* In Baldino and Cabral (2013) we have been careful to ground this concept on Mature Marx's theory, identifying the student with the free autonomous worker who possesses the means of production, produces and collects surplus value, the "fixed capital being man himself" (Marx, 1973, p.712). Elsewhere, Marx is more specific:

> The independent peasant or handicraftsman is cut into two. "In the small enterprises ... *the entrepreneur is* often his *own worker* — (Storch, Vol. II St. Petersburg edition, [p.] 242). As owner of the means of production he is a capitalist, as worker he is his own wage laborer. He therefore pays himself his wages as a capitalist and draws his profit from his capital, i.e. he exploits himself as wage laborer and pays himself in *surplus value* the tribute labor owes to capital (Max, 1863, p. XXI, 1329)

In summary, our concept of qualified-labor-power is well founded in Mature Marx: the student is "cut into two". This concept elicits the student's double class position: as owner of the qualified-labor-power resulting from the educational enterprise, the student is in the *capitalist's position*; as owner of the less qualified labor power applied in the school process, the student is in the *worker's position*. Hence, in daily actions, the student is subjected to the interpellation of both supporting ideologies. No wonder some students come to hate school. Upon receiving the certificate, the student is authorized to collect the surplus-value of the joint enterprise with the school owner, be it a public or a private institution. Essentially, this collection is present at all steps of the school credit system. At the final stage it is celebrated in the graduation solemnity, with students throwing hats up and, sometimes, burning books.

We argue that the relatively small impact of our 2013 paper is explained by the secular difficulty that "well educated gentlemen" have in admitting that they are the best selected in an economic practice of producing and seizing surplus-value. The same difficulty was there at the beginning of the new history. Subjects are not born ready; they constitute themselves by insertion in the field of the Other. Their daily social representation is conditioned by the two driving forces of ideology: *repression* to remain within bounds, and *jouissance*[11] for corroborating the social repressive norm (Butler, Laclau, and Žižek, 2000, p. 134).

Of course, the first argument against an unpleasant conclusion is silence. However, since the dominant ideology is the ideology of the dominant class (Althusser), this silence of the upper-class spreads to the related general concepts of labor-power and surplus-value. It would be a kind of social misdemeanor for an Oxford or Cambridge "educated gentleman" to refer to the ROI[12] of his "degree".

To produce silence on a subject, Understanding talks a lot around its edges without ever mentioning it. The active production of silence around surplus-value is on-going today. Thomas Piketty wrote a 500-page book entitled "Capital" without once mentioning surplus-value. The official media can display lengthy discussions blaming Russia for the war in Ukraine without mentioning NATO; it can discuss Gaza, Israel and Hamas without mentioning "genocide"; it can discuss electronic cigarettes without mentioning "tar"…

Final words

Having made Understanding speak up and reveal the source of its silence, here are our three main conclusions. 1) The credit system in the capitalist school, from elementary to post-grad, is built on the purpose of leading *students to participate in an economic practice of production and seizure of surplus-value.* This is the true importance of the so-called "education". 2) This conclusion is also the answer to Althusser's question: *Why is the educational apparatus in fact the dominant Ideological State Apparatus in capitalist social formations, and how does it function?* (Althusser, 1970, p . 93). 3) In the absence of failure in school, there would be no appropriation of surplus-value; *failure is necessary for the school credit system to function*.

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[1] We are indebted to our colleague Alexandre Pais for many interesting remarks on a draft of this paper.

[2] Understanding has recently been refusing to enter into the dialog; it takes refuge in repeated doctrines, as the fascist tide mounts in most countries. The imminent defeat of NATO in Ukraine may raise a self-destructing aggressive answer from Understanding, beyond the point of survival of humanity. See, for instance, https://www.youtube.com/watch?v=b7HTZ6IaOck (in Portuguese).

[3] https://www.marxists.org/archive/marx/works/1867-c1/p3.htm

[4] https://encyclopedia.pub/entry/32030

[5] Si l’on désigne par ε un nombre infiniment petit.

[6] On dit qu’une quantité variable devient *infiniment petite* lorsque sa valeur numérique décroît indéfiniment de manière à converger vers la limite zéro (Cauchy, 1821, p. 26, added underline).

[7] Passer d'un état à (un autre), commencer à être (ce qu'on n'était pas) (Grand Robert). https://www.lerobert.com/dictionnaires/francais/dictionnaire-langue/dictionnaire-le-grand-robert-de-la-langue-francaise-edition-abonnes-3133099010289.html

[8] https://catholicfreepress.org/news/message-of-pope-francis-for-world-day-of-peace (Accessed 20 April, 2024).

[9] https://www.indeed.com/career-advice/finding-a-job/which-majors-make-the-most-money

[10] Karl Marx and Frederick Engels, *Selected Correspondence* (Progress Publishers, Moscow, 1975). Scanned and prepared for the Marxist Internet Archive by Paul Flewers. Available at: https://www.marxists.org/archive/marx/works/1868/letters/68_01_08.htm (Accessed: 20 April, 2024).

[11] From the French: enjoyment with sexual connotation.

[12] Return on investment.

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Authors and correspondence details

Roberto Ribeiro Baldino and Tânia Cristina Baptista Cabral have been working together for over 40 years. He got his Dr. Sc. from L. Nachbin at IMPA, Rio de Janeiro, in 1972. She got her Dr. in Education degree from A. Villani in University of São Paulo (USP), São Paulo, in 1998. Today, both work at Universidade Estadual do Rio Grande do Sul, Guaíba, Brazil (UERGS), Guaíba, Rio Grande do Sul, Brazil, where the students began to call them the *Cabraldinos*; they assumed the name by which they became known in the community of mathematics educators. They are accessible by e-mail: rrbaldino@terra.com.br e cabral.taniacb@gmail.com