THE POST-COVID 19 ERA, CHAT BOTS, AND THE HOMOGENIZATION OF EDUCATION: PLURALISM AS AN EPISTEMIC VIRTUE EXEMPLIFIED BY MATHEMATICS EDUCATION

**José Antonio Pérez-Escobar Deniz Sarikaya**

University of Geneva, Switzerland Vrije Universiteit Brussel, Belgium

Jose.Perezescobar@unige.ch deniz.sarikaya@vub.be

**Abstract**

This paper aims to raise awareness about a potential harmful consequence of teaching adaptations due to the Covid-19 pandemic and advanced AI-powered chat bots in the context of large language models. As video lectures might undermine pluralism in mathematics, we argue that such developments could be harmful for mathematical practice and society at large. We argue that the mathematical undergraduate curriculum should not be too hard codified and formalized via an abuse of online teaching tools. In order to show why a plurality of research practices constitutes a great resource for mathematical progress, we will discuss a historical situation regarding the developments of the Tripos in Cambridge. We will also discuss analogies with economics and ideas on mathematical pluralism and productive ambiguity. Finally, we suggest that pluralism in education may be desirable in other fields too.

**Keywords:** Mathematical pluralism; Education ethics; Mathematical communities; Productive ambiguity; Video lectures; Covid-19; Large language models

# 1 Introduction

In 2020, an unprecedented, generalized shift towards digital teaching formats happened throughout the world. The reason was the rapid spread of a new contagious disease: Covid-19. The lack of any previous immunity made it easy for the virus to spread exponentially fast, which would have overwhelmed the medical systems even of the strongest health-care systems if it was not for the adoption of social distancing measures. The educational sector was not spared from these contingencies and a lot of universities started to teach digitally via (sometimes pre-recorded) video lectures. Also, modern AI-powered tools can offer tailored replies to natural language inputs. In particular, systems like ChatGPT and Bard can help automate individual feedback for participants in online classes. This may have crucial implications for Academia. Online lectures might not be re-given again and again; instead, they can be “petrified” as standard recordings. After several terms of online teaching, canceled conferences, lockdowns and so forth, it seems clear that the pandemic, in synergy with AI-powered tools, will have long-lasting effects on our society. In this article we will focus on the impact that all this may have on teaching in the long run. Especially mathematics departments in the US justify their funding and the employment of a large part of their staff via teaching export to other faculties, especially to economics and STEM fields. *Prima facie*, this could lead to a strong reduction of teaching positions and a system of normalized standard lectures that would dominate the undergraduate curriculum in mathematics.

This paper argues that this would be highly problematic. To do this we discuss

1. a historical situation of mathematics at the University of Cambridge
2. general philosophical questions of pluralism in mathematics and logic

As established f.i. by Ernest (2003, 2018, 2019) and Kant and Sarikaya (2021) mathematics, mathematics education and our view on mathematics impacts society at large. Moreover, even pure mathematics finds mundane uses in social settings (Pérez-Escobar and Sarikaya 2022). This article is highly speculative and wants to spread awareness for this potential problem among mathematics educators. While the focus of our paper is pluralism in mathematics and mathematics education, we will also make some remarks about other areas in Section 5.

# 2. Winner takes it all economy and dying plurality

A winner-takes-all market is a concept from economics and refers to an economic system where the best performers in a competitive environment outrun the competition and monopolize the rest of the market. This is a term that was studied especially in the context of modern internet-based business models. As the cost of such an online business does not rise linearly as a function of the size of the customer base, the relative advantage of big companies over small companies increases until small companies disappear. A simple example is the following: A supermarket chain needs to open a new supermarket to service a new area and this entails cost. In contrast, a streaming service can serve anybody with a stable internet connection and the biggest can produce more original series than the rest at little scalable distribution cost.

We claim that a more automated way of teaching would imply a winner-takes-it-all market situation. Not every university could justify a high production cost, and with time there would be a few dominating learning environments. Those in charge of creating and curating the body of teaching materials would have a great impact in

1. The accessibility of mathematical fields
2. The way of framing the content

To give extreme examples: If the most convenient alternative has a focus on combinatorics but no courses on calculus, the next generation of mathematicians will be equipped with much more knowledge (and interest) in combinatorics than calculus. Similarly, if this provider decides to teach a very formal notation like a Bourbaki-like style and stresses that pictures are often pitfalls that do not depict real general cases, the use of pictorial proofs and the role of pictures in proofs in general would be significantly impacted.

As we will see now via a historical case, such developments are not unprecedented. We will then look at more general points and aspects concerning the harmfulness of such developments.

# 3. A historical case: Continental mathematicians vs UK mathematicians

Before globalization, it was common to have different mathematical schools of thought, or traditions, delimited by geographical boundaries. Think of Newton’s calculus vs Leibniz’s calculus or, within the same country, the Gottingen school vs the Berlin school. Let us consider the case of continental mathematics vs UK mathematics, a historical division which eventually faded due to globalization. The division made the preservation of a strong, isolated community possible. On the other hand, results from outside the community were not sufficiently recognized.

Let us consider Andrew Warwick’s *Masters of Theory: Cambridge and the Rise of Mathematical Physics* (Warwick 2003) on the evolution of the study and practice of mathematics (and mathematical physics) at the University of Cambridge in the late 18th, 19th and early 20th Centuries. The book is divided into two halves. The first, covering a series of events between roughly 1770 and 1880, is concerned with the university’s pedagogical methods and traditions, its evolution and how it affected the mathematical practice and academic developments. On the other hand, the second part discusses how such pedagogical legacy hindered the incorporation of foreign achievements from the late 19th century and the early 20th century into the Cantabrigian academic scene.

At the time, the University of Cambridge was the most prominent institution for mathematics in all Britain, and as such, educated many famous mathematicians and gave birth to a vast amount of mathematical knowledge – including the inception of mathematical physics. This, coupled with the fact that the University of Cambridge has always advocated a high degree of competitiveness, led to the establishment of a very particular way of teaching and examining mathematical knowledge. Across centuries, this demanding and meritocratic Cantabrigian spirit has been well reflected in the examinations on mathematics sat by the students, commonly called “mathematical Tripos”.

Warwick argues that mathematics in Cambridge evolved together with the Tripos, so that modifications in the implementation of the latter were intensely reflected in the mathematical practice of the time. For instance, Warwick describes how in the 19th Century a big modification was implemented to the Triposes, whose focus steered away from oral disputation to meet a more rigorous written format. This allowed placing a stronger emphasis on concatenations of mathematical arguments, but this shift in the Tripos’ format was leveraged to as well adopt two key features: 1) a strong and particular interest in the multiple nuances of problem solving (analyzing and defining the problem, contemplating solutions, reaching meaningful results, etc), and 2) a preference for geometric, visual representations of problems and the rendering of solutions in diagrammatic form, which was believed to offer a better grasp of the nature of problems (as opposed to the leading-at-the-time French symbolic reasoning). As the complexity of the mathematical Tripos escalated in the 19th century, the problems presented became increasingly more abstract in their own way, following such a different path to those of mathematics in Continental Europe that each tradition sounded alien to each other. A very particular mathematical ecosystem began to consolidate, Cambridge mathematics became a language of its own and, as Gregory Moore (2005) notes, its influence would spread even to fields such as economics. Concerning the preference for the visual representations of problems, Moore claims the following:

It is, I believe, this preference that explains why those Cambridge men who migrated to the social sciences were so ready to represent their ideas diagrammatically (with Keynes being a famous exception). Stephen represented the laws of demand and supply diagrammatically in the early 1860s, Venn reconstructed Boole’s logical processes diagrammatically, Marshall found it entirely normal to translate passages from Mill’s Political Economy into relationships in two-dimensional space, and Bowley constructed extremely messy contract curves in the Edgeworth -Bowley box. (Moore, 2005, p. 96)

The reasons why the Tripos exerted so much influence in the mathematical practice in Cambridge as a whole are varied. The highly demanding exams prompted the students to hire private coaches to instruct and prepare them, and it so happened that these coaches were former students who succeeded in the Tripos and were therefore versed in the problem-solving approach to mathematics in question. Warwick even claims that it was very unlikely for a student to succeed in the Tripos without the assistance of one of these coaches. This way, the cycle is perpetuated in the educational sphere of mathematics in Cambridge, but there is more. More often than not, the faculty members themselves (who unsurprisingly were in their majority former Cambridge students) would elaborate their papers and present their research in the abstract, diagrammatic, problem-solving Tripos format, and in fact, their research and particular problem-solving techniques made their way into the Triposes very quickly. This process indirectly served as a normalization mechanism for new work.

It is not a lack of success of the Cambridge model what Warwick criticizes; in fact, it led to great scientific achievements. Rather, Warwick finds fault with both an inability and unwillingness to acknowledge and deal with foreign developments in the late 19th Century and early 20th Century, like the notable case of Einstein’s work on general relativity. However, even the work of other British scientists, like Poynting’s theory of energy flow – boasting the anti-Newtonian idea that energy can exist in empty spaces – found resistance within the academic community at Cambridge.

In any case, the Cambridge microcosmos was and is a great source of mathematical development. The mere existence of such a local style adds value to the landscape, especially if it is in touch with other communities, as happened during globalization. It is noteworthy that globalization already came with a certain degree of homogenization in mathematics, and as argued by Ernest (2008) this had some problematic consequences. The Covid-19 pandemic has and will likely exponentially increase these effects. Since online lectures are widely accessible all over the globe, standardization is much more common and recorded and YouTube lectures are progressively becoming the norm. The tools of modern AI are also at a point where such programmes can offer tailored elements at the fraction of a cost in comparison to the employment of many tutors. Such systems will naturally only increase in their capabilities. This will likely deprive us from mathematical oddities like the particular Cambridge style, and perhaps leave us with just a single, overarching style. This is deeply connected to questions of inclusivity, see (Masschelein & Simons 2005) and questions of decolonization, see e.g. (Parker 2021), (Hajir & Kester 2020), (Stein 2019), (Stengel 2019).

# 4. Pluralism: good or bad?

In the section above we saw that the lack of acknowledgement of achievements outside of the UK (or even outside of Cambridge) was highly problematic. This paper does not intend to promote the establishment of “intolerant”, local cultures. Instead, we want to make a case for pluralism, for a situation where different ideas and approaches interact and compete, and different epistemic needs are addressed efficiently. This idea is not unique to us. It is very common in other fields, like in economics: a key feature of free markets is that they are more efficient than planned economies because they imply a higher number and diversity of economic agents (Moroney and Lovell 1997).

A naive answer to this might be something like: but mathematics is a unified, objective body of knowledge, it does not require diversity; proving is not a matter of perspective and does not admit debate. Yet, the need for pluralism in mathematics has recently received a lot of attention in the academic landscape. This includes the issue of whether there is more than one true logic (logical pluralism) which was strongly influenced by the work of Beall and Restall (see, for instance, Beall and Restall 2000). But also, inner-mathematically, there is debate about the merit of different approaches, for instance with regards to competing theoretical foundations. We will explore these two issues in the next two subsections. It should be noted that not all worldviews need to be accepted, see (Horsthemke 2021).

Another factor to consider is the locality of practices. A one-size-fits-all solution might not honor different communities in the same manner, as each may have their own standards. For instance, tech companies “localize” their products to fit the market in question, but these products are still tools developed in one community superimposing values on other communities. Yet, in the context of mathematics education (like in the Realistic Mathematical Education tradition), an important aspect is that mathematics is tied to the localities where it emerges; see (Van Den Heuvel-Panhuizen 2015; Wijaya, Van Den Heuvel-Panhuizen, & Doorman 2015, or Wijaya, Van Den Heuvel-Panhuizen, Doorman, & Robitzsch 2014).

Moreover, mathematization and its rationality is connected to some kind of worldview, and some worldviews may be considered problematic by some people, see (Ernest 2010, or Ernest, Sriraman, & Ernest 2016).

Finally, it is worth mentioning that mathematical literacy empowers people to take part in societal discourse (f.i. on the social challenges presented by AI), fosters the general economy and plays a role in decision making in personal issues. Any phenomenon that makes one group more salient than another (f.i. due to a group being culturally closer to whom develop the tools, or being in a larger, more “attractive” market) is highly problematic regarding fairness concerning this empowerment, (see Nikolakaki 2010).

##  4.1 Division of labor and the virtue of specialization

How researchers choose their research topic is hard to describe but, in principle, it is reasonable to believe that it is epistemically good when they freely choose their topics (cf Leonard 2002). A reason for this is that inner and outer mathematical questions may put a niche subject at the forefront, and rapid changes in the research landscape may be desirable. Let us give an example regarding foundational issues. There was a time when mathematics started to become more rigorous. An often-mentioned starting point is the theorems of Bolzano and Weierstrass proving seemingly obvious theorems. For instance, a continuous function, whose domain contains the interval [a,b] with negative and positive values in that interval must have a root in that interval as well (Bolzano’s Theorem, an immediate lemma of the intermediate value theorem). This endeavor runs into paradoxes like Russell’s Paradox and, to tackle those, a rigorous set theoretical foundation was developed and became the standard. But while these foundations solved all problems of the time, recent developments made it possible to revisit foundation issues (cf. Centrone et al. 2019). One such development was the creation of computers, which benefited greatly from type theoretical foundations. It was very convenient that there was a community ready to tackle the newly arising questions of semantics of programming language, etc. that the new area brought to us. Similarly, niche topics can become more important due to inner mathematical developments. To stay within the example, when Homotopy Type Theory established a connection between topology and type theory both disciplines could then profit from the expertise of the “other side”.

This makes it reasonable to seek that different universities focus on different areas, an endeavor that may be challenged if the teaching canon is normalized by dominating supplies of material.

## 4.2 Productive ambiguity

The digitalization and homogenization of mathematical notions would also undermine ambiguity. While in principle this may sound as a good thing, the truth is that a certain type of ambiguity is desirable from an epistemic perspective.

The inception of a mathematical theory usually starts from informal notions. Also, it may not be clear how to precisely define the structures alluded to by mathematicians and there may be different competing notions or clarifications for them. The *semantic view of theory* states that a theory can be identified with a collection of its models, rather than a set of its true statements. This idea is especially developed in the context of physics (see, for instance, van Fraasen 1987). Textbooks feature examples that influence the development of a field. Similarly, in mathematics, different *prototypes* of mathematical notions and proofs may make different conclusions or approaches more salient. Such prototypical elements have been recently studied in the context of the development of mathematical frames (see, for instance, Fisseni et al. 2019, Carl et al. 2021, and Fisseni et al. 2023). Furthermore, notions from Wittgensteinian philosophy, like rule-bending, also suggest that variety opens possibilities. One example could be how local decisions on what to do with a model (e.g., refine it when evidence does not match it, or tune measurement procedures instead; see Pérez-Escobar, 2023) also depends on tacit notions acquired in the training of scientists. For a general historical connection cf, Perez-Escobar 2022, forthcoming or Zeng 2022. For this reason, the homogenization of mathematics may undermine the variety of conclusions and approaches available to us. Consider the following statement on productive ambiguity:

When two or more traditions combine in the service of problem-solving, we find juxtaposed and superimposed notations and diagrams, as well as notations and diagrams that are subtly altered from their earlier uses, surrounded by prose in natural language that tries to explain how to use them in combination. Viewed this way, the texts often yield striking examples of language and notation that are irreducibly ambiguous and productive because they are ambiguous. (Grosholz, 2007, preface page 12)

A simple example is the following. The (mis)use of differential operators as simple fractions is sometimes beneficial because it saves time. Another example is the slight ambiguity between a certain set and its cardinality, like when we denote the numbers of pairs of a set with n elements by the respective binomial coefficient (see Wasserman 2019).

# 5. Does all this apply to other areas?

The considerations above easily lead to the question whether the homogenization of education would be harmful to other academic fields. For instance, one may argue that a plurality of epistemic intuitions is healthy in other fields because of similar reasons to those outlined above: it may lead to increased methodological variety, more discoveries/results, more readily available tools to face unexpected situations, more ways to use existing tools, and so forth.

In this line, the idea that ambiguity can be epistemically virtuous has also been explored in the contexts of how mathematics is used outside mathematics, in science. For example, in biology, a mathematical formula may be interpreted as a mechanism or as a teleological purpose (i.e., what a biological item is normatively supposed to do, if it is functioning correctly) depending on the context. This allows a single mathematical formula to capture two useful levels of analysis in the biological scientific practice, in an efficient manner. An extremely simplified example is: say we have an equation of the form A = B + CD. Imagine A stands for the electrical activity of a neuron, B stands for base activity, while C represents connections to that neuron and D represents synaptic strength. The equation can represent a mechanism in the sense that the activity of A is mechanistically determined by B + CD, but it also can represent the teleological purpose that A must conform to B + CD, else we would say that the neuron malfunctions. A purely mechanistic biological tradition in this context would deprive us of the teleological level of analysis, and its epistemic value. For a more detailed elaboration, see (Pérez-Escobar 2020).

# 6. Conclusion

We argued for the benefits of a plurality of approaches in mathematics education (at least at the university level). It is essential that we protect this diversity against the generalization of recent developments in teaching caused by social distancing due to the global Covid-19 pandemic and the development of large language models. Otherwise, educational and scientific practices will suffer from a lack of pluralism which, we have argued, is epistemically virtuous. If one accepts that education has the goal of shaping the character of people in society, then a lack of pluralism in education would lead to homogeneous societies with less diverse viewpoints and preparedness for future problems. While this article focused on this problematic side, one should balance the caution for which we advocate with profiting from the positive aspects that come with these developments, like an increase of inclusivity of students from disadvantaged geographical locations. In this regard, Imperial, Phipps & Fassetta (2021) study, for instance, the connection between hospitality and online education.

**Acknowledgements:**

The first author is grateful for a grant from the Swiss National Science Foundation: *Mathematizing biology: measurement, intuitions, explanations and big data* (P500PH\_202892). The second author would like to thank the Research Foundation Flanders (FWO) who funds his postdoc within the project “[*The Epistemology of Big Data: Mathematics and the Critical Research Agenda on Data Practices*](https://researchportal.be/en/project/epistemology-data-science-mathematics-and-critical-research-agenda-data-practices)“ (Project number FWOAL950). The paper was finalized during a visit of the second author in Denmark (University of Copenhagen & Technical University of Denmark) funded by a DAAD-Kurzzeitstipendium with the project *“Theoretical virtues of conjectures and open questions in mathematical practice”.*

**The Authors**

José Antonio Perez-Escobar (University of Geneva) <https://orcid.org/0000-0002-3728-6896> & Deniz Sarikaya (Vrije Universiteit Brussel) <https://orcid.org/0000-0001-8951-8161>

Both authors contributed equally and share first authorship. Both authors serve as corresponding authors. Author’s emails are: Jose.Perezescobar@unige.ch & deniz.sarikaya@vub.be

# 7. Literature

Beall, J. C. & Restall, G. (2000). Logical pluralism. *Australasian Journal of Philosophy*, *78*(4), 475–493.

Carl, M., Cramer, M., Fisseni, B., Sarikaya, D. & Schröder, B. (2021). How to Frame Understanding in Mathematics: A case study using extremal proofs*,* *Axiomathes, 31*(5)*,* 649–676.

Ernest, P. (2003). Images of mathematics, values and gender: A philosophical perspective. In *Mathematics Education*, pp. 21-35. Routledge.

Ernest, P. (2008). Epistemological issues in the internationalization and globalization of mathematics education. In: Allen, B. & Johnston-Wilder, S. (Eds.) *Internationalisation and globalisation in mathematics and science education* (pp. 19-38). Dodrecht: Springer.

Ernest, P. (2010). The scope and limits of critical mathematics education. In: Alrø, H., Ravn, O., & Valero, P. (Eds.) *Critical mathematics education: Past, present and future* (pp. 65-87). Brill.

Ernest, P. (2018). The ethics of mathematics: Is mathematics harmful? In: P. Ernest (Ed.), *The philosophy of mathematics education today* (pp. 187–216). Cham: Springer.

Ernest, P. (2019). Privilege, power and performativity: The ethics of mathematics in society and education. *Philosophy of Mathematics Education Journal,* *35*, 1-19.

Ernest, P., Sriraman, B., & Ernest, N. (Eds.) (2016). *Critical mathematics education: Theory, praxis and reality*. Information Age Publishing.

Fisseni, B., Sarikaya, D., Schmitt, M. & Schröder, B. (2019). How to Frame a Mathematician. In: Centrone, S., Kant D. & Sarikaya, D. (Eds). *Reflections on the Foundations of the Mathematics*. (417–436) Cham: Springer.

Fisseni, B., Sarikaya, D. & Schröder, B. (2023). How to frame innovation in mathematics. *Synthese,* *202*, Article: 108.

Van Fraassen, B.C. (1987). The Semantic Approach to Scientific Theories. In: Nersessian, N.J. (Ed.) *The Process of Science. Science and Philosophy*, vol 3. Springer, Dordrecht. https://doi.org/10.1007/978-94-009-3519-8\_6Grosholz, E. R. (2007). *Representation and productive ambiguity in mathematics and the sciences*. Oxford University Press.

Hajir, B. & Kester, K. (2020). Toward a Decolonial Praxis in Critical Peace Education: Postcolonial Insights and Pedagogic Possibilities. *Studies in Philosophy and Education*, *39*, 515–532.

Horsthemke, K. (2021). Diversity and Epistemic Marginalisation: The Case of Inclusive Education. *Studies in Philosophy of Education, 40*, 549–565.

Imperiale, M.G., Phipps, A. & Fassetta, G. (2021) On Online Practices of Hospitality in Higher Education. *Studies in Philosophy and Education*, *40*, 629–648.

Kant, D., & Sarikaya, D. (2021). Mathematizing as a virtuous practice: different narratives and their consequences for mathematics education and society. *Synthese* *199*, 3405–3429

Leonard, T. C. (2002). Reflection on rules in science: an invisible-hand perspective, *Journal of Economic Methodology, 9*(2), 141–168.

Masschelein, J. & Simons, M. (2005). The Strategy of the Inclusive Education Apparatus. *Studies in Philosophy and Education*, *24*, 117–138.

Moore, G. (2005). Masters of Theory and its relevance to the history of economic thought. *History of Economics Review*, *42*, 77–99.

Moroney, J. R., & Lovell, C. A. K. (1997). The relative efficiencies of market and planned economies. *Southern economic journal, 63*(4), 1084–1093.

Nikolakaki, M. (2010). Investigating critical routes: The politics of mathematics education and citizenship in capitalism. *Philosophy of Mathematics Education Journal, 25*(1), 1–16.

Parker, L. (2022). The Skin as Seen: Thinking Through Racialized Subjectivities and Pedagogy with Levinas. Studies in Philosophy and Education, 41(2), 227–242

Pérez-Escobar, J. A. (2020). Mathematical Modelling and Teleology in Biology. In: Zack, M. & Waszek, D. (Eds.) *Research in History and Philosophy of Mathematics* (pp. 69-82). Cham: Birkhäuser.

Pérez-Escobar, J. A. (2022). Showing Mathematical Flies the Way Out of Foundational Bottles: The Later Wittgenstein as a Forerunner of Lakatos and the Philosophy of Mathematical Practice. *KRITERION–Journal of Philosophy,* *36*(2), 157–178.

Pérez-Escobar, J. A., & Sarikaya, D. (2022). Purifying applied mathematics and applying pure mathematics: how a late Wittgensteinian perspective sheds light onto the dichotomy. *European Journal for Philosophy of Science*, *12*(1), Article 1.

Pérez-Escobar, J. A. (2023). A new role of mathematics in science: Measurement normativity. *Measurement, 223*, 113631.

Pérez‐Escobar, J. A. (Online first). The role of pragmatic considerations during mathematical derivation in the applicability of mathematics. *Philosophical Investigations.*

Sinha, S., & S. Rasheed. (2018). Introduction to deconstructing privilege in the classroom: teaching as a racialized pedagogy. *Studies in Philosophy and Education*, *37*, 211–214

Stein, S. (2019) Beyond Higher Education as We Know it: Gesturing Towards Decolonial Horizons of Possibility. *Studies in Philosophy and Education* (38), 143–161.

Stengel, B.S. (2019) Com-Posting Experimental Futures: Pragmatists Making (Odd)Kin with New Materialists. *Studies in Philosophy and Education*, *38*, 7–29.

Van Den Heuvel-Panhuizen, M. (2005). The Role of Contexts in Assessment Problems in Mathematics. *For the Learning of Mathematics, 25*(2), 2–23.

Wasserman, N. (2019). Duality in Combinatorial Notation. *For the Learning of Mathematics*, *39*(3), 16–21.

Wijaya, A., van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). *The Mathematics Enthusiast, 11*(3), 555–584.

Wijaya, A., van den Heuvel-Panhuizen, M. & Doorman, M. (2015) Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics, 89*, 41–65.

Zeng, W. (2022) Lakatos’ Quasi-Empiricism Revisited. *KRITERION – Journal of Philosophy*, *36*(2), 227–246.