MULTIFACETED CONCEPTIONS OF MATHEMATICS AMONG MATHEMATICS, ENGINEERING, AND ECONOMICS FACULTY

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ABSTRACT

This study investigates how faculty groups from mathematics, engineering, and economics conceptualize the nature of mathematics. Using comparative judgement, participants ranked paired statements representing six facets of mathematics: Quantities and Numbers; Patterns, Structures, and Relationships; Creativity and Beauty; Logic, Reasoning, and Proof; Tools and Applications; and Critical Thinking and Problem-Solving. Results show meaningful differences across disciplines—research mathematicians emphasized logical systems, while engineering and economics faculty prioritized practical applications. While some commonalities emerged, such as valuing critical thinking, the findings underscore disciplinary differences in mathematical perspectives with implications for teaching, curriculum design, and interdisciplinary collaboration.

Keywords: Conceptions of Mathematics; Nature of Mathematics

Introduction

It is impossible to create a brief dictionary-type definition of mathematics that encompasses all the different philosophies about the nature of mathematics, including formalism, Platonism, and constructivism. Instead, definitions of mathematics are often circular in nature with a common definition being that "mathematics is what mathematicians do" (Byers, 2020, p. 1). For something as complex as mathematics, it is better to see it as a multifaceted abstract object that necessitates using an encyclopedic style of definition that includes the different nuances and views of mathematics, some even conflicting with each other.

Using an empirical philosophy of mathematics approach to investigate how different groups define mathematics helps us to define mathematics as it bridges philosophical inquiry with lived experience and practice (Kant & Löwe, 2024, 2025). Rather than assuming a singular, abstract definition of mathematics, this approach recognizes that mathematical meaning is shaped by cultural, disciplinary, and contextual factors (Ernest, 1991; D'Ambrosio, 2006). By empirically examining how various communities—such as mathematicians, engineers, economists, or educators—conceptualize mathematics in their discourse and practice, researchers can uncover the pluralistic and situated nature of mathematical understanding (Restivo, 1992; Xu & Ball, 2024). This method allows for a nuanced exploration of mathematical epistemologies as they manifest in real-world settings, offering insight into how definitions of mathematics function within specific institutional and social frameworks.

Most studies examining beliefs about and the nature of mathematics at the tertiary level, often focus on students' perceptions of the usefulness of mathematics (Code et al., 2016), beliefs about the process of doing mathematics (Crawford et al., 1998; Sidney et al., 2024), assessment preferences (Iannone & Simpson, 2019), or sense of belonging in the mathematical community (Sidney et al., 2024). While these sometimes get at the multi-faceted nature of mathematics, this complexity is often ignored.

Within the university setting, mathematics is a foundational tool for many disciplines. Understanding how faculty members across disciplines think about and use mathematics fosters better communication and collaboration between mathematicians and other faculty members and has the potential to influence how mathematics is taught to students in various majors.

Research Question:

> How do different groups of tertiary-level faculty prioritize different conceptualizations of the nature of mathematics?

Literature Review

Conceptualizations of Mathematics

Using an open-ended questionnaire, Roberta Mura (1993) obtained responses to the question "How would you define mathematics?" from 106 faculty members in departments of mathematical sciences across Canada. From these responses over a dozen themes emerged that described different facets of mathematics. These facets included formal axiomatic systems, logical reasoning, a language of notation and symbols, models abstracted from reality, simplification of complexity, problem-solving, study of patterns, creative activity, a science, truth, specific topics, along with circular arguments such as, "Mathematics is what mathematicians do!" (p. 381). These themes underlie the desire that mathematicians have for secondary mathematics teachers to understand that mathematics is "(1) wide and varied, (2) lively and developing, (3) rich in connections, and (4) structured deductively" (Hoffmann and Even, 2024, p. 484). When expanding her survey to university teachers of mathematics education in Canada, Mura (1995) found that mathematics educators identified similar facets of mathematics but had an increase of describing mathematics as a study of patterns and seeing mathematics as culturally determined.

These views about mathematics by mathematicians and mathematics educators are reflected in reports and policy documents related to the teaching of mathematics at the post-secondary level. For example, the CUPM curriculum guide from the Mathematical Association of America (MAA) (2004) states, "Mathematics is universal: it underlies modern technology, informs public policy, plays an essential role in many disciplines, and enchants the mind" (p. 3). This statement anchors the rest of the curriculum guide, emphasizing practical and aesthetic aspects of mathematics. Similarly, the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010) emphasizes mathematics for its "important 'processes and proficiencies'" (p. 5), reflecting an increased focus on mathematics as a system of problem-solving tools and thinking methods, corresponding to the belief that problem-solving is at the heart of mathematics.

Many disciplines with mathematics as a foundation have differing conceptions of the subject. Focusing on practical utility, an article about the relevance of mathematics to engineering states that “mathematics provides training in rational thinking as well as tools for undertaking analysis to obtain information about systems” (Flegg et al., 2012, p.1). Flegg et al. (2012) and Winkelman (2009) also found that engineering students believed the study of mathematics to be useful to their future careers and viewed the nature of mathematics as a gateway to engineering, solving closed problems, and a tool for other subjects of study and dealing with real-world problems. Economists frequently use mathematics in their research, with economics courses requiring strong mathematical foundations (Ely & Hittle, 1990; Hoag & Benedict, 2007). Nevertheless, the mathematics used in an upper-division economics course varies greatly from that in an upper-division mathematics course leading some economists to question the effectiveness of proof-based mathematics for economics (Velupillai, 2005). Conversely, fields like game theory in economics are highly linked to branches of mathematics (Buchanan, 2001), possibly resulting in conceptions similar to those of research mathematicians. These conceptions of mathematics are more purpose-oriented than those of creative problem-solving or development of mathematical structures.

Measuring Conceptualizations of Mathematics

Different conceptualizations about mathematics impact the mathematics curriculum throughout the educational trajectory. Understanding this relationship requires ways to measure these conceptualizations. Crawford et al. (1998) categorized conceptions of mathematics as either focusing on numbers and applications (fragmented) or logical systems to understand the world (cohesive), placing undergraduate students along a continuum between the two conceptualizations. This approach ignores the multifaceted nature of mathematics. To address this, Code et al. (2016) developed the Mathematics Attitudes and Perceptions Survey (MAPS) to measure multiple expert-like attributes and perceptions of mathematicians. Their survey was found to contain seven factors: Confidence, Growth Mindset, Real World, Persistence, Interest, Sense Making, and Answers. Most of these attitudes and perceptions are more about how the individual relates to mathematics rather than the underlying conceptualizations of the nature of mathematics itself.

Theoretical Construct

While prior work has identified diverse conceptions of mathematics, particularly among mathematicians and educators, fewer studies have explored how these views differ across disciplines. To address this gap, we introduce a framework that organizes mathematical conceptions into six key facets. Since we are expanding beyond faculty in departments of mathematics or those in mathematics education, we will combine some of the categories found by Mura (1993, 1995) and add some ideas related to applications to reflect the perspectives of partner disciplines. Due to the complexity and beauty of mathematics, we use a diamond analogy and describe the concept in terms of the different facets of the jewel. To simplify the study, we focus on six facets of mathematics and how the different groups view mathematics through these perspectives. Within the framework of Crawford et al. (1998), each individual facet of mathematics by itself could be considered a fragmented view of mathematics. However, when combined with other facets, the concepts of quantities and numbers can be a significant component of a very cohesive view of mathematics. Therefore, in our analysis regarding individuals’ and groups’ conceptions of mathematics, we will be analyzing the combinations and prioritizations of the different facets.

Quantities and Numbers. At a fundamental level mathematics studies and analyses numbers and quantities. The first exposure any individual has to mathematics is that of counting and numbering objects. Much of the literature surrounding elementary math focuses on developing children's belief and familiarity with numbers (Sowder, 2020; Wu, 2011), with basic mathematical operations such as addition, subtraction, multiplication, exponentiation, division, and more formalizing ways to understand how numbers relate to each other. Even at higher mathematical levels, sets are often considered in the context of number systems such as natural numbers, integers, rational numbers, real numbers, and complex numbers (Hamilton,1982; Glass, 2002). While this facet is not one of those identified as a category in the work of Mura (1993, 1995), the statements in this facet were identified in those studies under the category of "reference to specific mathematical topics" (p. 381).

Patterns, Structures, and Relationships. One of the key distinctions found by Mura (1995) between mathematicians and mathematics educators in their definitions of mathematics is that the mathematics educators mentioned patterns 37% of the time while the mathematicians only mentioned this topic 5% of the time. We can further see the emphasis on this facet within the mathematics education community with the standard for mathematical practice of "look for and make use of structure" in the Common Core standards (National Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010).

Many fields of mathematics focus on the existence of structures and the patterns and relationships between them. This often takes the form of abstraction where mathematicians notice similarities between different mathematical objects and processes and is the basis for subjects such as abstract algebra and category theory, which focuses on objects and morphisms between these objects (Awodey, 2010).

Creativity and Beauty. In *A Mathematician's Apology*, Hardy (1940) states that in mathematics "beauty is the first test: there is no permanent place in the world for ugly mathematics" (p. 14). However, some recent empirical studies (Ingles & Aberdein, 2015, 2016, 2020) challenge the notion of universal agreement on mathematical beauty. This includes low levels of agreement on the aesthetics of proofs (Ingles & Aberdein, 2016), a lack of correlation between the simplicity and beauty of proofs (Ingles & Aberdein, 2015), and social influences on aesthetics (Ingles & Aberdein, 2020). Other studies have shown links between aesthetic judgments of proofs and epistemological properties (Starikova, 2018), brain activity aligned with beauty (Zeki et al., 2014), and understanding of the content (Hayn-Leichsenring et al., 2022), with these relationships being independent of regional culture (Sa et al., 2024). Therefore, while there may not be complete agreement regarding the beauty of individual proofs, we see that beauty is a significant facet of mathematics.

Logic, Reasoning, and Proof. The art of proof is fundamental in both modern and historical mathematics (Ko & Knuth, 2013; Reid & Knipping, 2010; Thurston, 1995) with a focus on the communicative role within the mathematical social construct (Aberdein, 2009; Dawkins & Weber, 2017) and the systematization of mathematics (de Villiers, 2010). In the strictest sense, proof incorporates four practical virtues of "Permanence, Reliability, Autonomy, and Consensus" (Berry, 2018) so that once something has been "proven" it becomes part of the mathematical knowledge that will never be changed. To apply the similar practices outside of the mathematics research community, Stylianides (2007) expanded the definition of proof to include "a connected sequence of assertions for or against a mathematical claim" (p. 291) using statements accepted by the community, within forms of reasoning accepted as valid by the associated community, and communicating with language appropriate to the audience. Overall, the facet of logic, reasoning, and proof in mathematics focuses on knowing how and why something is true. This emphasis on argumentation contrasts with the problem-solving focus prevalent in earlier mathematics education, causing students to struggle with formal proof due to misconceptions and inadequate learning of definitions (Moore, 1994).

Tools and Applications. "Mathematics is the tool used by physicists, engineers, biologists, neuroscientists, chemists, astrophysicists and applied mathematicians to investigate, explain, and manipulate the world around us" (Wilson, 2018, p. 345). This applicability of mathematics can cause challenges to different philosophies of mathematics and ways of defining the subject (Wilson, 2018). However, with our view of mathematics having multiple facets and individuals from different academic communities having different perspectives and priorities of these facets, the applicability of mathematics and considering mathematics as a tool or language of the sciences is an important facet to consider.

Critical Thinking and Problem-Solving. While acknowledging many of the other facets of mathematics, Halmos (1980) considered problem-solving to be the heart of mathematics. While the problem-solving process can be summarized by the four steps of Pólya (1945) of understand the problem, make a plan, carry out the plan, and look back at your work, the activity of problem-solving is extremely complex (Schoenfeld, 1985). This process of problem-solving is intertwined with the development of critical thinking skills throughout the educational process and has led to a large amount of research in mathematics education (Santos-Trigo, 2024).

Methods

Survey Development

Using the six facets of Quantities and Numbers; Patterns, Structures, and Relationships; Creativity and Beauty; Logic, Reasoning, and Proof; Tools and Applications; and Critical Thinking and Problem-Solving, we produced 22 statements reflecting different conceptions of mathematics building on previous instruments, research articles, commentaries, or websites. In the process of creating the statements, we made sure that each of the six categories contained at least five statements allowing for some statements to correspond to more than one category. Of these 22 statements about mathematics, seven were in "Patterns, Structures, and Relationships," five in "Creativity and Beauty," six in "Logic, Reasoning, and Proof," seven in "Tools and Application," five in "Critical Thinking and Problem-Solving," and five in "Quantities and Numbers." (A full list of statements is contained in the appendix.)

An online form was then used to randomly assign each participant 15 pairs of statements in a random order with the prompt to "Select the description of mathematics that you agree with more," following the format of comparative judgment (Davies et al., 2020, 2021; Jones & Davies, 2024; Jones et al., 2015; Thurstone, 1927). At the end of the survey participants ranked the six categories according to their priorities. The survey also included demographic information about their highest degree, gender identity, and racial/ethnic identity.

Participants and Setting

The study occurred at a large public university in the southeastern United States with the survey sent to all faculty members in the Department of Mathematics, the College of Engineering (other than Computer Science), and the College of Business whose research focus is listed as economics or finance. None of the faculty members in finance responded to the survey and so we will describe the group of business faculty who did respond as Economics. The make-up of the participants is given in Table 1.

Table 1: Participant Information

Faculty Group	Mathematics (Research)	Mathematics (Teaching)	Engineering	Economics
Male	7	3	10	5
Female	2	5	2	2
Total	9	8	12	7
Participation Rate	35%	33%	9%	18%

These participation rates are high enough to be a representative sample of the population of faculty at a single institution. While one needs at least 15 individuals in each group to guarantee that every possible pair of statements is evaluated by at least one individual, with 7 responses 45% of the possible pairs of statements are evaluated. This also provides approximately 10 comparisons for each statement within each of the groups providing a likelihood of a data set for each group that is adequate to provide an estimate of the Thurstone model for comparative judgement (Pollitt, 2012).

Analysis of Statements

Using the 15 comparisons per participant in each of the four groups, the 22 statements were ranked for each of the four groups using Thurstone's (1927) model of comparative judgement using a logistic model with the BradleyTerry2 package in R (Bradley & Terry, 1952; Turner & Firth, 2012). The Bradley-Terry model was used to generate comparative value estimates for each of the 22 statements for each of the four subgroups. So, for each of the subgroups, if statements A and B have comparative value estimates of v_A and v_B, respectively, the probability that statement A is chosen over statement B is given by

$$\frac{\exp\,(v_A - v_B)}{1 + \exp\,(v_A - v_B)}$$

Since the item parameters for the statements are continuous variables, the agreement between the four sets of parameters was analyzed using nonparametric correlations (Kendall's tau) to avoid the influence of outliers. We further analyzed the statements with the largest differences between groups to determine qualities of these differences and possible implications related to the group's conceptions of mathematics.

Analysis of Categories

Each participant ranked the six qualities of mathematics, "Patterns, Structures, and Relationships," "Creativity and Beauty," "Logic, Reasoning, and Proof," "Tools and Application," "Critical Thinking and Problem-Solving," and "Quantities and Numbers" based on their priorities about mathematics. The categories for each group were ordered using the mean of the ranks within each group. The groups were then compared pairwise using Kendall's coefficient of concordance to measure agreement.

Results

Analysis of Statements

For each of the four groups of faculty members, we used the Bradley-Terry model to create comparative value estimates for each of the 22 items. Since the value estimates only measure the distance between the pairs of items, we recentered the estimates so that the mean of the estimates of the 22 items for each of the groups was 0. Since the relationship between the estimates for each of the groups is not assumed to be linear, we used a Kendall's tau non-parametric correlation to better understand any possible relationships between the groups in how they ranked the items. We see in Table 2 that there are not statistically significant correlations at the $p<.05$ level between the rankings by the groups of faculty members.

Table 2: Kendall's tau correlations of item parameters between groups

	Mathematics Teaching	Mathematics Research	Economics	Engineering
Mathematics Teaching	-	.290	-.143	.238
Mathematics Research	.290	-	.255	.238
Economics	-.143	.255	-	-.143
Engineering	.238	.238	-.143	-

To better understand the places of agreement and disagreement, we used a radar chart to see how the different groups ranked the items (See Figure 1).

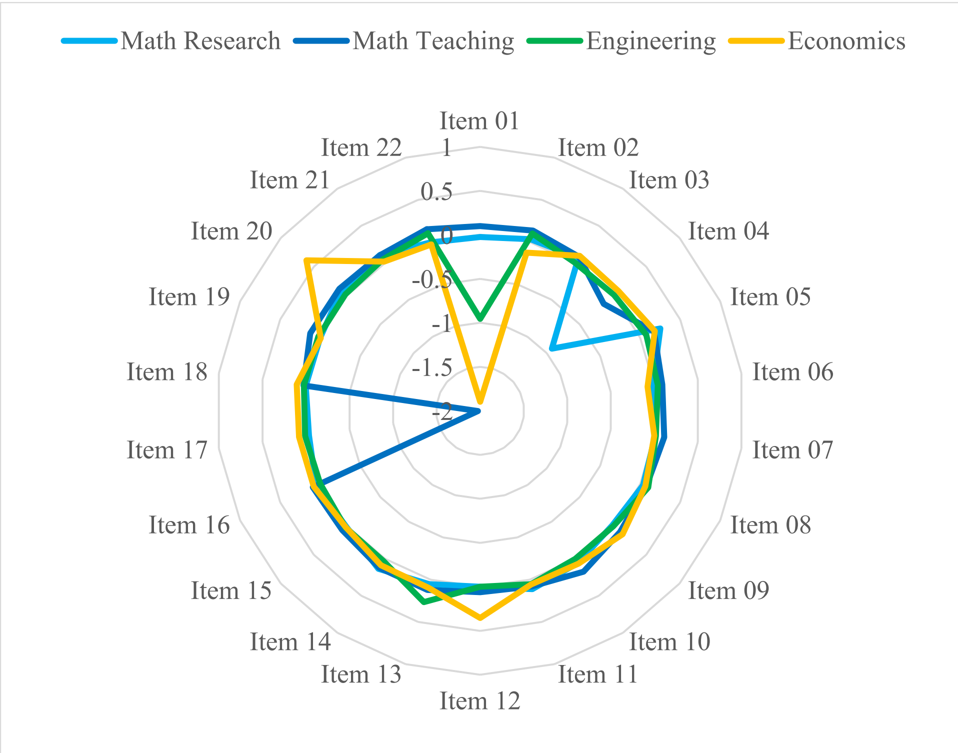

Figure 1: Radar chart of item strengths

The statement ranked highest by the overall population was "It focuses on logical systems which have been developed to explain the world and relationships in it" (Item 05). This statement was ranked first by the mathematics research faculty, second by the mathematics teaching and engineering faculty, and third by the economics faculty. This agreement contrasts with the disagreement among the groups about the statement, "Provides knowledge and tools that can be applied to practical problems and real-world questions" (Item 13). Both statements refer to the applicability of mathematics, with the former statement adding a focus on the development of logical systems. The latter statement was ranked fourteenth by the mathematics research faculty, while the engineering faculty ranked it first, the mathematics teaching faculty ranked it fourth, and the economics faculty ranked it sixth. When taking the rankings of both statements into account, we see that all the groups valued the "tools and applications" facet of mathematics, but the research mathematicians seem to see the applications as a motivating factor for the other facets rather than as a primary facet.

Similarly, the role of numbers differentiated between the mathematics faculty and the other groups. The statement describing mathematics as "The study of numbers" (Item 01) ranked at the bottom for all the groups other than the mathematics teaching faculty. They ranked the statement in the middle of the pack. On the other hand, the engineering and economics faculty placed the statement "It is about finding answers through the use of numbers and formulas" (Item 04) in the middle of their rankings, while the mathematics faculty placed it at the bottom of their rankings.

At the bottom of the rankings, the statements "The starting point of beauty in fields such as art, music, architecture, and proof" (Item 17) and "The process of creating proofs that are in essence beautiful" (Item 18) were ranked very low by most of the faculty groups. The former statement was ranked as 19th, 21st, or 22nd by all the groups, while the latter statement was ranked 5th by the mathematics research faculty and 19th or 20th by the other groups. Since all the groups deemphasized the beauty aspects of mathematics, the facet of creating proofs within mathematics elevated the related latter statement among the mathematics research faculty.

Another important facet of mathematics involves reasoning to answer questions or prove assertions. The two statements rated highest by the economics faculty were "Employs the use of deductive reasoning and critical analysis to answer proposed questions" (Item 20) and "Uses logically coherent arguments to establish the truth of an assertion from a known and agreed base" (Item 12). The other groups placed both statements in the middle of their rankings.

Category Rankings

To determine if each of the groups had similar ideas about the rankings of the categories, we computed Kendall's coefficient of concordance and Spearman's rank correlation coefficients between all possible pairs of rankings (Kendall & Gibbons, 1990) to assess the agreement among the raters of each group. We found that the groups had coherence in their rankings, with the research focused faculty in mathematics having a very strong coherence (see Table 3).

Table 3: Inter-rater Reliability for Categories

	Math Teaching	Math Research	Engineering	Economics and Finance
W	.468	.692	.440	.513
p-value	.002	<.001	<.001	.003
r	.392	.648	.389	.431

We then found rankings of the categories for each group by taking the average of the ranks (see Table 4 and Figure 2). The rankings of the categories aligned with the rankings of the statements in many ways. We see that the economics faulty prioritize the reasoning aspects of mathematics, while the mathematics faculty deemphasize the "Tools and Applications" and "Quantities and Numbers" facets. One surprising result was that the engineering faculty did not emphasize the Tools and Quantities facets of mathematics above the Problem-solving and Reasoning aspects.

Table 4: Average Category Rankings by Group

	Problem-Solving	Patterns/ Structures	Reasoning	Tools	Quantities	Beauty
Math Research	2.11	2.11	2.56	4.56	4.56	5.11
Math Teaching	1.65	2.38	3.63	4.50	5.00	3.88
Engineering	3.42	2.00	3.58	2.92	3.33	5.75
Economics	2.57	2.71	1.86	3.86	4.71	5.29

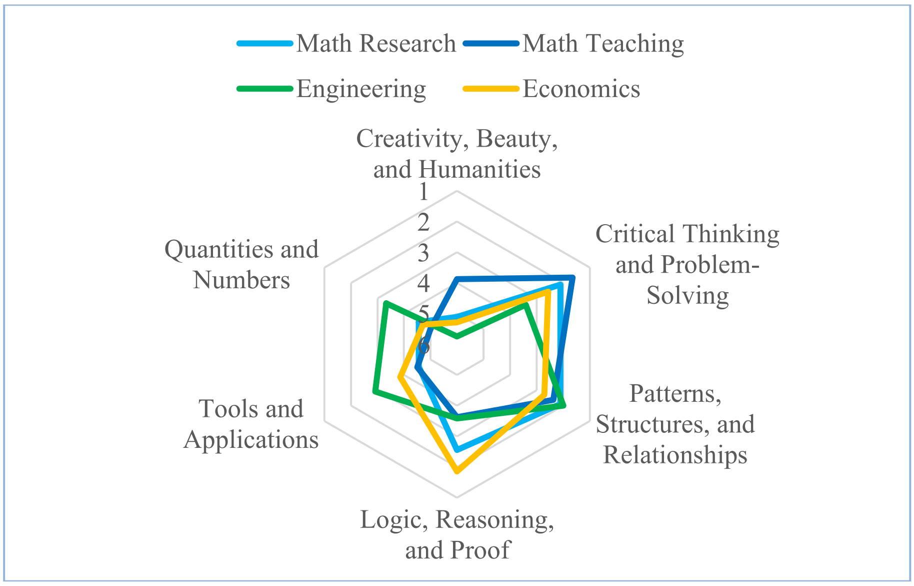

Figure 2: Radar chart of group category rankings

Conclusions and Discussion

This study examined how faculty members from mathematics, engineering, and economics conceptualize the nature of mathematics through a comparative judgement framework. The findings reveal significant disciplinary differences in the prioritization of mathematical facets, underscoring the multifaceted and context-dependent nature of mathematical understanding.

Research-focused mathematicians prioritized abstract facets such as logic, reasoning, and proof, consistent with traditional disciplinary norms that emphasize internal coherence and formal rigor. In contrast, engineering and economics faculty emphasized the instrumental value of mathematics, highlighting its role as a tool for solving practical, real-world problems.

Mathematics teaching faculty demonstrated a more balanced distribution, acknowledging both foundational and structural facets alongside applications.

These results illustrate that while some convergence exists, particularly around the importance of problem-solving and reasoning, substantial variation remains in how mathematics is defined and valued across disciplines. The absence of strong inter-group correlations in item rankings further confirms the presence of distinct epistemological frameworks that shape disciplinary engagement with mathematics.

These findings carry several important implications for teaching. The disciplinary differences in how faculty conceptualize mathematics suggest that a one-size-fits-all approach to curriculum design may not meet the needs of diverse student populations. For instance, engineering and economics faculties prioritized practical applications, while mathematics faculty, especially those focused on research, emphasized logic, reasoning, and proof. As such, courses serving multiple majors should balance theoretical depth with practical relevance. Faculty might benefit from engaging in interdisciplinary dialogue to better understand how mathematics is used across fields and to inform the design of contextually meaningful instruction. Furthermore, instructors should be encouraged to explicitly discuss the multifaceted nature of mathematics in the classroom, helping students develop richer, more connected understandings of the subject. Faculty development opportunities could also support instructors in reflecting on their own mathematical beliefs and aligning instructional practices with a more comprehensive view of mathematics. Finally, assessment practices should reflect the valued facets of mathematics, incorporating tasks that go beyond procedural fluency to include problem-solving, argumentation, and creative reasoning.

Future research could investigate how these conceptions influence teaching practices and student outcomes, as well as explore broader institutional or cultural factors that shape faculty beliefs. Longitudinal studies examining how conceptions evolve across academic careers or in response to professional development may also offer valuable insights.

Overall, this study reinforces the importance of recognizing mathematics as a complex, multidimensional discipline. Acknowledging and integrating diverse faculty conceptions can support more inclusive, relevant, and effective mathematics education, particularly in interdisciplinary and applied contexts.

Declarations

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Research study was approved by the Institutional Review Board of ******.

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Appendix: Items from Survey

Item	Statement
01	The study of numbers.
02	The classification and study of all possible patterns.
03	A set of logical principles that a person follows to generate a solution to a proposed problem.
04	It is about finding answers through the use of numbers and formulas.
05	It focuses on logical systems which have been developed to explain the world and relationships in it.
06	The craft of creating new knowledge from old, using deductive logic and abstraction.
07	Helps students gain mathematical ways of thinking and an appreciation for the place of the subject relative to other academic pursuits.
08	A language for expressing physical, chemical and engineering laws.
09	The study of the measurement, properties, and relationships of quantities and sets, using numbers and symbols.
10	An area of knowledge that includes the topics of numbers, formulas and related structures, shapes and the spaces in which they are contained, and quantities and their changes.
11	The logical dovetailing of a carefully selected sparse set of assumptions with their surprising conclusions via a conceptually elegant proof.
12	Uses logically coherent arguments to establish the truth of an assertion from a known and agreed base.
13	Provides knowledge and tools that can be applied to practical problems and real world questions.
14	Studies sets and groups of objects, and the patterns and relationships that emerge within them.
15	Examines the ways that things interact with each other.
16	A creative generative field of knowledge.
17	The starting point of beauty in fields such as art, music, architecture, and proof.
18	The process of creating proofs that are in essence beautiful.
19	Helps students develop critical thinking skills.
20	Employs the use of deductive reasoning and critical analysis to answer proposed questions.
21	A process of building, analyzing, and interpreting analytical models of real world situations.
22	It is about understanding the world through studies of quantity, structure, pattern, and change to create logical solutions that make life more meaningful and more beautiful.