UPTAKE JUSTICE IN MATHEMATICS EDUCATION: PROOF, RECOGNITION, AND THE POLITICS OF EVIDENCE

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ABSTRACT

I argue that epistemic justice is constitutive of what counts as mathematics in school. The decisive moments occur at uptake—when a student's contribution must be recognized as a candidate reason, made legible across representations, and judged under public standards. I introduce uptake justice as a teachable orientation organized around three commitments: legibility, warrantability, and reparability. Recasting proving as a practice of public reason, the analysis shows how dialogic norms, multiple representational routes, and cultivated aesthetic judgment prevent the reduction of proving to verification and counter routine credibility deficits. Assessment is reframed as the politics of recognition, since what counts as evidence manufactures who can be recognized as a knower. The paper also examines datafication and policy imaginaries to show how platforms and indicators script what becomes hearable as learning, and why classrooms organized for public reasoning require evidence beyond dashboards. The conclusion sketches a minimal program—curricula that widen interpretive repertoires, recognition-forward assessments, and infrastructural scrutiny—so rigor is measured by the quality of reasons and the generosity of uptake rather than compliance with a single template.

Keywords: epistemic justice; public reason; recognition; translanguaging; datafication; policy imaginaries; classroom discourse.

I. Equity's Epistemic Turn

Epistemic justice is not an accessory to "equity" in mathematics education; it names how knowledge comes to count as mathematical at all. Fricker's distinctions between testimonial injustice—credibility deficits attached to speakers—and hermeneutical injustice—gaps in shared interpretive resources—travel directly into the grain of classroom life, where a contribution must first be recognized as a candidate reason, rendered in legible forms, and then judged against public standards (Fricker, 2007). When a diagram is waved off as "intuitive," when a counterexample is dismissed as "special," or when multilingual reasoning is required to be monolingual before it is audible, the wrong is not simply social. It is epistemic, because the community's criteria for what can be heard as mathematics have silently excluded a kind of reasoning. In this sense, questions of justice specify the conditions under which proof, modeling, representation, and authority become intelligible practices rather than mere procedures. (Fricker, 2007)

The field's readiness to speak in these terms owes much to the sociopolitical turn, which legitimized inquiries into identity, power, and ideology as constitutive—not peripheral—

dimensions of learning mathematics. That turn reframed equity beyond access and achievement, asking instead how classroom norms and institutional logics authorize some forms of knowing while silencing others (Gutiérrez, 2013). Epistemic justice extends this move inside the discipline's core activities. Tanswell & Rittberg (2020) bridge social-epistemic concerns to proving and justification, showing how credibility and intelligibility are produced within "apprenticeship" models of proof and how those models can reproduce injustice if standards remain tacit and narrow. These analyses shift the problem from who participates to how reasons themselves become publicly accountable, and thus who can be recognized as a knower when reasons are in play. (Gutiérrez, 2013; Tanswell & Rittberg, 2020)

Philosophy of mathematical practice has converged with this trajectory. Arguments for a "thick epistemology" of mathematics insist that exposure and attention—what ideas circulate, whose arguments receive uptake—are not merely sociological facts but epistemologically relevant, entangled with power and justice in ways that shape what knowledge becomes available to communities (Hunsicker & Rittberg, 2022). For classrooms, that claim is concrete: the norms by which students' contributions are taken up, translated across representations, and warranted are themselves part of what it means to know mathematics in school. (Hunsicker & Rittberg, 2022)

This essay offers a philosophy-first, practice-facing synthesis of that landscape. It reads across recent mappings of the field (Ernest, 2025) to argue that epistemic justice is enacted in three interdependent sites: in classrooms as standards of public reason during proving and modeling; in assessment as the politics of evidence that confers or withholds recognition; and in the datafied infrastructures that format what becomes visible as mathematical success. The aim is not another catalogue but a single argumentative thread: to show how testimonial and hermeneutical harms are made or repaired at the point of uptake, and how design for legibility, warrant, and reparability can make justice teachable without diluting rigor.

II. Uptake Justice: Making Students' Reasons Hearable

The most consequential moments for epistemic justice in mathematics classrooms occur at uptake: the point at which a student's contribution becomes a candidate reason, is made legible in shared representational registers, and is judged against public standards of argument. Testimonial injustice appears when credibility deficits prevent a contribution from even entering that space; hermeneutical injustice appears when the classroom lacks the interpretive resources to render the contribution intelligible as mathematical. In proving and modeling, both harms are common and often invisible. A counterexample is dismissed as "special," a diagram is treated as merely intuitive unless translated into a privileged algebraic form, multilingual reasoning is required to pass through an English-only filter before it can count. These are not peripheral issues of participation but central questions about how mathematical reasons are recognized, translated, and warranted in the everyday life of the discipline at school. Tanswell & Rittberg (2020) show how such injustices are reproduced when proving is treated as tacit apprenticeship with narrow, unstated standards; it urges a shift toward explicit, publicly accountable criteria for what counts as a reason in the classroom community.

I use the term uptake justice to name a practical, teachable orientation for repairing these harms. Its center of gravity is threefold. Legibility concerns whether students' ideas can be expressed and heard across the semiotic resources of the classroom—speech, inscriptions,

diagrams, gestures, digital traces—without forcing premature translation into a single, prestige register. Warrantability concerns what counts as a sufficient reason here and why, making the criteria of adequacy explicit and discussable rather than leaving them to gatekeeping instincts. Reparability concerns routines for repairing misrecognition when uptake fails, so that a contribution waved aside can re-enter the space of reasons under fairer descriptions. From the standpoint of epistemic justice, these are not soft skills but conditions for mathematical knowledge in school, because they determine who can be recognized as a knower when reasons are at issue. Recent debates about who can know mathematics and about compensating epistemic exclusion show why the standards of recognition must be made visible and revisable within the practice of proving and modeling themselves, not only in participation formats (Walshaw, 2021; Baldino & Cabral, 2021).

Language is integral to uptake. For mathematics education, discourse is not a container for thought but its very medium: to enter a mathematical practice is to inhabit its ways of speaking, writing, diagramming, and justifying. Sfard's "thinking-as-communicating" thesis captures this point by showing how norms of discourse and the meta-rules of explanation regulate what may count as mathematical, and thus who can be heard as a knower (Sfard, 2001). In addition, research on language and communication has expanded rapidly across multilingual classrooms, teacher education, multimodality, and interactional analyses, while warning that loose conflations of "language" and "communication" risk obscuring what is at stake for mathematical intelligibility (Planas & Pimm, 2024). The classroom corollary is that uptake justice requires deliberate design of representational and linguistic repertoires, so that a contribution can travel from a student's initial register into a public proof or model without losing its epistemic force.

Translanguaging provides a concrete route for such design. Tai (2022) shows teachers orchestrating multilingual and multimodal resources—switches of language, inscriptions, diagrams, gesture, and digital annotation—to secure mutual intelligibility and to keep students' reasoning in view as mathematical rather than merely expressive. These pedagogies do not dilute rigor; they cultivate legibility by widening the channels through which candidate reasons can be offered and examined in public. They also support reparability, because a contribution that initially fails in one register can be revoiced or re-represented in another without disqualifying the knower (Tai, 2022). In this sense, translanguaging can be read philosophically as a technology of uptake justice, enabling the community to test warrants across registers rather than installing a single gate of admissibility.

Uptake justice thus resonates with the first and last passages of the epistemic-legitimation cycle—recognition and public validation—while its insistence on legibility and reparability aligns with interrogation and co-design as ongoing habits of collective reasoning (Yamaguchi, 2025). The translanguaging and multimodal routines are best seen as practical grammars of uptake: they enlarge the channels through which candidate reasons can travel, and they institutionalize repair when initial hearing fails, thereby converting equity talk into epistemic work

Taken together, these strands move epistemic justice from a general equity ideal to a disciplinary program for school mathematics. Uptake justice reframes what teachers already do—eliciting, revoicing, pressing for reasons—as work on the constitutive norms of mathematical knowing. It ties the micro-politics of proof and modeling to the design of semiotic resources in the room and prepares the ground for a wider rethinking of evidence and assessment, where credibility and intelligibility are continually made and remade. The next

section follows this thread by reconstructing proving as a practice of public reason whose standards can be taught, debated, and repaired without sacrificing mathematical depth.

III. Proving as Public Reason

Proving in school is not a checklist at the end of a unit but a practice of public reason in which claims are advanced, made legible, and judged under shared standards. When classrooms reduce proving to verification, testimonial injustices tend to follow almost mechanically: some students' contributions never become candidate reasons, and others are heard only after being translated into a narrow, prestige register. Recent syntheses show that research has moved beyond the old discovery–justification binary toward an integrated view of proving as inquiry that binds explanation, systematization, and communication to the cultivation of norms for what counts as a mathematical reason in a given community (Stylianides, Stylianides, & Moutsios-Rentzos, 2024). That shift reframes proving as a civic practice internal to the discipline, where credibility and intelligibility are continually at stake. In this sense, epistemic justice is not an add-on to equity; it is internal to the question of how reasons are recognized and warranted in classroom mathematics (Tanswell & Rittberg, 2020; Stylianides et al., 2024).

The integrated view has a history. Hanna and de Villiers (2012) frame school proof not as a mere chain of derivations but as a multifaceted practice in which explanation, systematization, conviction, and communication are constitutive. Building on that stance, Cabassut, Conner, İşçimen, Furinghetti, Jahnke, and Morselli (2012) show how debates over what counts as proof—product versus process, formal text versus argumentation—are negotiated in classrooms and teacher beliefs, making the criteria of admissibility inseparable from pedagogy. At the level of classroom practice, Dreyfus, Nardi, and Leikin (2012) document how visual, verbal, and dynamic representations, together with example-based and generic arguments, legitimately function as resources for proving. Read together, these chapters portray school proof as a public activity whose purposes and forms exceed verification. If proof in school inherits and reworks multiple functions of proof in mathematical practice, then the standards that govern uptake must be explicit and discussable. A diagrammatic insight, an example that disrupts a generalization, or a carefully crafted analogy should enter the arena of reasons as steps in a communal argument. When such contributions are discounted by default, the harm is epistemic as well as social: the criteria of mathematical hearing have silently excluded a kind of reason. Proving as public reason therefore demands sustained attention to the sites where recognition is granted or withheld (Hanna & de Villiers, 2012; Cabassut et al., 2012; Dreyfus et al., 2012).

Didactic transposition of proof sharpen why justice attaches to proof criteria. When proof travels into classrooms, its ontological and epistemic status changes: what students prove are school-objects, situated in didactic organizations and subject to the teacher's admissibility judgements. The warrants that count in this setting are not simply imported from research mathematics but are negotiated through tasks, representations, and institutional norms. This means that the standards of what may function as a reason—algebraic deduction, a carefully argued counterexample, a diagram supported by invariance arguments—are themselves part of what is being taught, and they bear directly on who can be recognized as a knower of mathematics. Making those standards visible and revisable is therefore a matter of epistemic justice as much as pedagogy (Balacheff, 2024).

The social texture of proof further complicates matters. Teacher–student interactions distribute attention and regulate which forms of representation acquire authority; what is revoiced, pressed, or bracketed shapes the epistemic economy of the lesson. When proving is orchestrated as dialogic inquiry, the ecology of examples, counterexamples, diagrams, and symbols can sustain shared criteria for adequacy; when proving is orchestrated as a private performance of technique, those criteria harden into opaque gatekeeping (Jones & Herbst, 2012). In a public-reason framing, language is not a neutral conduit but part of the proof itself: to justify is to make one's claim travel across registers and be answerable to others. Designing that travel—by inviting multiple representational paths to a claim and by naming the conditions under which each path suffices—is integral to fair uptake.

Aesthetic judgment belongs here too, not as ornament but as part of the rigor we teach. Mathematicians and students alike experience certain arguments as fitting, economical, or illuminating. Sinclair (2004)'s account of the aesthetic roles in mathematical inquiry—generative, motivational, evaluative—shows that such sensibilities guide the production and appraisal of reasons. To cultivate them is to render explicit why some proofs "sound right," anchoring taste to shared criteria rather than to elitist tacit knowledge. When students learn to articulate why a proof feels economical or explanatory, aesthetic talk becomes epistemic talk: it constrains which inferences are acceptable and elevates explanation over mere derivation (Sinclair, 2004). Positioned in this way, aesthetics expands the resources of public reason rather than narrowing them to a single canonical form.

A classroom image can gather these strands. Imagine a brief proof conference midway through a unit, where a conjecture is put to the class not for immediate verdict but for staged uptake. The proponent narrates the claim in ordinary language, sketches a diagram that exhibits its invariants, and offers an example–nonexample pair to delimit scope. Peers pose challenges and request clarifications; the teacher presses for warrants, asking what would make this argument sufficient for us, here, and whether an alternative representation would strengthen or weaken its force. The discussion ends not with anointing the one right form but with an explicit summary of criteria that have emerged: which representations made the claim legible, which inferences satisfied our standards, and where gaps remain that call for repair. In such a scene, proving functions as public reason: credibility is distributed through accountable uptake, and rigor is measured by the quality of warrants and the generosity of translation, not by compliance with a single template. That is what it means to treat proof not as a checklist of verification, but as a shared practice in which epistemic justice is enacted.

IV. Credibility by Design: Assessment as Recognition

Assessment does not simply mirror what students know; it manufactures who can be recognized as a knower of mathematics. From a philosophical standpoint this is not a moral flourish added after the technical work is done. It follows from the kinds of questions assessment is built to answer and the evidential forms it privileges. Biesta (2007)'s critique of "what works" makes the fault line visible: when educational decision-making is narrowed to efficacy claims answerable by standardized evidence, the democratic and epistemic purposes of education are displaced. The result is an evidentiary imaginary that prizes effect sizes and comparability while muting the goods internal to a practice—here, the public giving and judging of mathematical reasons (Biesta, 2007). In mathematics education this narrowing is acute, because dashboards and exams render speed and accuracy visible and make other forms of knowing—like the quality of warrants, the repair of misunderstandings, or the capacity to

translate across representations—hard to see. To mistake such visibility for neutrality is already to take a position on who counts as a knower.

Nieminen and Lahdenperä (2024) reframe assessment's role in producing not only knowledge but knowers. They show how students co-construct the dominance of examinations by treating them as the natural measure of mathematical competence; at the same time, experiences with alternative assessments enable students to redefine what counts as valid knowledge and how it can be shown. Their analysis, cast explicitly in terms of epistemic injustice, argues that assessment practices do not merely record learning but organize epistemic resources, distributing credibility toward certain displays of understanding and away from others (Nieminen & Lahdenperä, 2024). If this is right, then the ethics of assessment cannot be separated from its epistemology: to change what evidence is admissible is to change who can stand as a credible knower in mathematics.

The Philosophy of Mathematics Education Journal has become a key venue for naming this politics of credibility. Walshaw (2021) presses beyond participation counts to the norms through which classroom communities authorize knowing, foregrounding the entanglement of identity, authority, and disciplinary standards. On the same stage, Baldino and Cabral (2021) interrogate "rewarding effort" as a compensatory tactic: good intentions can become another mechanism of exclusion when the currency of recognition—what counts as effort, how it is evidenced, and who is positioned to display it—is already unequally distributed. Both pieces insist that assessment regimes are sites where epistemic standing is actively conferred and denied, not neutral backdrops to instruction (Walshaw, 2021; Baldino & Cabral, 2021). Read together, they invite a re-description of assessment tasks, rubrics, and feedback as instruments that either widen or constrict the interpretive resources through which mathematical claims can be heard.

An ethics-focused turn in mathematics education amplifies the point. In his opening chapter, Ernest (2024a) argues that the aims, content, pedagogy, and assessment of mathematics are intrinsically ethical and political choices, because they shape persons and communities rather than merely report learning. He shows how decisions about what counts as acceptable evidence already enact commitments to goods such as autonomy, beneficence, and justice. In a companion chapter, Ernest (2024b) interrogates authority and control—from overt classroom power to subtler ideological forms—and traces how these dynamics circulate through curricular standards, assessment rubrics, and the tacit criteria by which reasons are admitted or excluded. His analysis clarifies why ostensibly neutral procedures are value-saturated: methodological scripts and evidence norms position some voices as credible knowers and others as peripheral. When assessment systems reduce learning to what is easily counted, they risk moral harm by excluding forms of reasoning vital to mathematical agency and public life; when they invite students to justify, explain, and critique, they enact a view of persons as capable of public reason and deserving of uptake (Ernest, 2024a, 2024b). The ethical vocabulary here—responsibility, recognition, repair—therefore maps directly onto epistemic concerns about who may speak for mathematics and under what descriptions.

What might assessment look like if its central task were recognition rather than selection? In a classroom oriented to proving as public reason, evidence would be gathered not only from products but from the circulation and uptake of reasons. A student's diagram that makes an invariant visible would not need to be sanitized into a single algebraic form to count; its adequacy would be judged by whether peers can see why the conclusion follows and under which conditions it fails. Feedback would name the warrant, not just the result, and would include invitations to repair misrecognitions by revoicing an argument in another register or

by offering an example–nonexample pair that narrows a claim's scope. Grading would still aim at validity and reliability, but those technical virtues would be in service of epistemic ones: legibility across semiotic resources, warrantability made explicit, and reparability when uptake goes wrong. Such "assessment for recognition" does not romanticize opacity; it aims to stabilize standards that keep reasons at the center while widening who can be recognized as offering them. In doing so, it answers Biesta's challenge by reattaching evidence to educational goods that are openly contestable and properly democratic, and it answers the PMEJ interventions by designing credibility, not presuming it.

V. Datafication and policy imaginaries: Numbers That Script Learning

Metrics and platforms do not merely measure mathematics in schools; they help decide what can count as mathematics and who can appear as mathematically capable. The point is not rhetorical. Policy devices such as international assessments manufacture a horizon of intelligibility within which teachers, students, and curriculum planners come to see what matters. PISA's comparative apparatus is exemplary: by installing a portable, policy-ready definition of mathematical literacy, it organizes pedagogical attention around skills that travel well as indicators, inaugurating a utilitarian grammar that privileges what can be aggregated and ranked. In classrooms, that grammar descends as pacing pressures, task selection, and assessment templates that keep explanation secondary to performance, even when teachers value reasoning. The authority of the indicator reaches down into lesson time, not only reporting outcomes but helping to enact them (Grek, 2009).

The digital turn amplifies this formatting power. Platforms, dashboards, and analytics promise real-time insight into learning, but they also relocate epistemic authority to the interface. What becomes visible are signals that systems can reliably capture: speed, accuracy, item completion, and streaks of engagement. Such visuals are not neutral windows; they are instruments that script what teachers and students will notice and respond to, and they normalize the idea that mathematics is best evidenced by streams of micro-performances. As Williamson (2017) has argued, data infrastructures shape educational governance by specifying in advance the units of evidence and thus the kinds of judgment considered legitimate. The result is a politics of evidence in which reason-giving, dialogic repair, and the slow work of making warrants explicit struggle to register as "learning" because they are hard to count and awkward to visualize on a dashboard (Williamson, 2017).

When automation becomes the horizon of improvement, pedagogical values are folded into optimization problems, and the space for public reasoning narrows. Williamson, Macgilchrist, & Potter (2023) warn that the grammar of datafication can colonize what counts as educational success, producing a tight coupling between what is measurable and what is deemed worthwhile. In mathematics education the effect is acute: explanation loses status relative to correct output; modeling is appraised for answer matching rather than for the scrutiny of assumptions; multilingual contributions are filtered to the one language of analytics. In such conditions, the epistemic standing of learners is quietly redistributed toward those whose knowledge is legible to the system (Williamson, Macgilchrist, & Potter, 2023).

There is a philosophical cost to this imaginary. Rømer (2019)'s critique of "Visible Learning" names a tendency to displace educational judgment with effect-size reasoning, mistaking statistical generality for pedagogical normativity. The worry is not that evidence is unwelcome, but that the will to visibility can occlude the goods internal to a practice. Mathematics education's central good is public reason: the giving and assessing of warrants in

a community bound by shared yet revisable standards. A classroom organized around proving and modeling as public practices therefore requires forms of evidence that are not reducible to dashboard signals. It needs records of arguments as they mature, of translations across representations, of repairs after misrecognition, and of the moments when an initially marginal contribution becomes a reason everyone can answer to. Such evidence is slower and messier, but it better fits the object of concern.

To insist on evidence beyond the dashboard is not to reject validity or reliability; it is to reattach those technical virtues to epistemic ones. Validity means that the traces we collect actually index the quality of reasons and the generosity of uptake, not only the frequency of correct responses. Reliability means that judgments about arguments can be made shareable and stable enough for accountability without being flattened into a single metric. In this counter-imaginary, numbers still have work to do, but they do not monopolize what is hearable as learning. The authority that matters returns to the seminar-like space of the classroom, where students and teachers practice making claims answerable to others. That is a different vision of mathematics in school, and it is one the present datafied regime cannot deliver by itself. It must be philosophically argued for and pedagogically designed against the grain of the screens that would make learning permanently obvious.

VI. A Minimal Program for Uptake Justice: Legibility, Warrantability, Reparability

Epistemic justice in mathematics is teachable because it lives in norms and designs, not only in attitudes. A classroom can be constituted as a small public where mathematical claims are made answerable to others and where credibility is earned through reasons that can travel across voices and representations. The curriculum that serves this aim widens students' interpretive repertoires so that argument can move among speech, symbol, diagram, gesture, and multiple languages without losing force; thinking-as-communicating is treated as a disciplinary commitment rather than a convenience (Sfard, 2001; Planas & Pimm, 2024). Assessment is reattached to recognition: evidence is gathered for what it ought to mean to know mathematics—explaining, systematizing, and justifying—rather than only for what can be tallied. That redirection answers the democratic worry that "what works" imaginaries eclipse educational goods, and it resists assessment cultures that produce knowers by privileging one narrow display of understanding (Biesta, 2007; Nieminen & Lahdenperä, 2024). Infrastructures, finally, are treated as ethical matters. Indicators, dashboards, and platforms are judged by how they keep reasons visible, not by how quickly they sort performances; governing by numbers remains under philosophical scrutiny when it threatens to monopolize what is hearable as learning (Grek, 2009; Williamson, 2017; Williamson, Macgilchrist, & Potter, 2023; Rømer, 2019).

Within this program, uptake justice names teachable standards. Legibility requires designing linguistic and representational routes by which a contribution can enter the common space of reasons. Warrantability requires making criteria of sufficiency explicit and discussable, so that proving and modeling become practices of public reason rather than private performances; the recent synthesis on proof and the didactic reflections on transposition show how such criteria can be taught without diluting mathematical depth (Stylianides, Stylianides, & Moutsios-Rentzos, 2024; Balacheff, 2024). Reparability requires routines that reopen uptake when misrecognition occurs, so that a diagram, a counterexample, or a multilingual explanation can be revoiced and judged on its merits. Together these norms relocate justice inside the practice of school mathematics, answering the question of who can know by reshaping how reasons

are heard. The research agenda that follows is necessarily design-oriented: articulate and test classroom ecologies—proof conferences, model audits, recognition-forward assessment—that redistribute epistemic authority while sustaining rigor, and study how such ecologies travel across topics, languages, and policy regimes (Tanswell & Rittberg, 2020; Walshaw, 2021; Ernest, 2024).

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